THEORY OF NUMBERS, Math 115 B
Homework 1 Due Friday January 17

1. For each $\epsilon > 0$ below find lattice points (p, q) whose distance from the line $y = \sqrt{2}$ is less than ϵ:

   a) $\epsilon = 1/2$, b) $\epsilon = 1/5$, c) $\epsilon = 1/10$

2. Are there infinite strips between parallel lines that are completely lattice-point free?

3. Prove that every triangle with three non-collinear vertices at lattice points, but no other lattice points on its boundary and no interior lattice points, has an area of $1/2$.

4. Prove that for any triangulation of a polygon having n holes the sum $V + E + F + n$ is odd. V, E, F denote the number of vertices, edges, and triangles in the triangulation.