Enumerative Combinatorics, Math 245
Homework three

1. Problems from Stanley's book Chapter 2: 1, 2, 3ab, 6, 7

2. Without getting your hands dirty with awful analytic arguments prove that inside the ring of formal power series $(e^x)(e^{-x}) = 1$, and that $\log(e^x) = x$.

3. Prove the following identity on Stirling numbers of the second kind:

$$S(n,r) = \frac{1}{r!}\sum_{k=0}^{r}(-1)^{r-k}\binom{r}{k}k^n$$

   One of your proof should not use inclusion-exclusion methods.

4. Let $A_1, A_2, ..., A_k$ be distinct subsets of $[n]$ with the property that A_i and A_j have a non-empty intersection for all pairs i, j. Show that $k \leq 2^{n-1}$. ¡p¿

5. Let $A_1, \ldots, A_n$ be arbitrary events of a probability space (Ω, P). For each $I \subset \{1, \ldots, n\}$, let $A_I = \Pi_{i \in I} A_i$, $A_\emptyset = \Omega$. Define

$$\sigma_k = \sum_{|I|=k} P(A_I), \ \sigma_0 = 1.$$

   Prove that $P(A_1 + \ldots + A_n) = \sum_{j=1}^{n}(-1)^{j-1}\sigma_j$

6. Using the inclusion-exclusion principle obtain a summation formula for the number of permutations in S_n with no cycles of length l. Can you give a proof without inclusion-exclusion?

7. Determine the number of seatings of n couples at a table of length $2n$ with no couple sitting side-by-side, but with the additional restriction that men and women must take alternate seats.

8. Find a formula for $C(n, k, s)$ be the number of k-subsets of $[n]$ that contain no run of s consecutive integers.