

Enumerative Combinatorics, Math 245
Homework five (and take home final)

1. Problems from Stanley's book Chapter 4: 10, 27, 29, 30,
2. Let $P_3(r)$ be the number of 3×3 semi-magic squares that are symmetric to their main diagonal and have line sum r . is $P_3(r)$ a polynomial?
3. Show a bijection from the set of $n \times n$ Latin squares onto that of $n \times n \times n$ semi-magic cubes with magic line sums all equal to one.
4. Let P be a full-dimensional integer polytope and let f be its Ehrhart polynomial. Prove that the highest term of f is equal to the volume of P .
5. Let P be the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(1, 1, n)$, where $n > 0$ is an integer parameter. prove that the Ehrhart polynomial of P is

$$p(k) = (n/6)k^3 + k^2 + (12 - n)k/6 + 1$$