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quarta-feira, 19 de junho de 2019 09:57

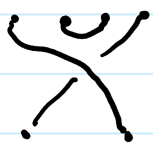
Plan: 1) General philosophy (example: tangles)

2) Another diagrammatic cat-y (sticky braids)
(jt. wt. T. Logvinenko)

1) Topological cat-y

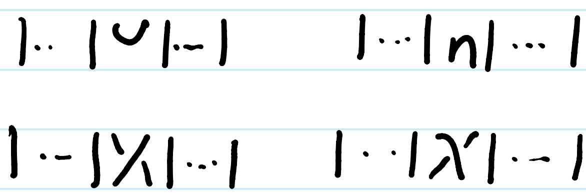
Morphisms: top. up to isotopy

e.g. Tangles: Objects: Sets of 2n points: [0] [2], [4], . .
Morphisms: Tangles



Diagrammatic description:

Generators:



Relations: Reidemeister ~ , braid
Pitchfork ~
Commutations

Weak cat. rep: To each object: Cat-y
morphism: Isoclass of functors

Usual way to do it. Associate a functor to
each generator, and on iso

Usual way to do it: Associate a functor to each generator, and an iso to each relation.

4) Triangulated repr w/ restrictions

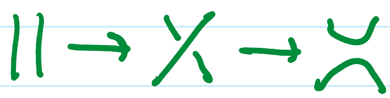
= Adjunction $(\cap[1], \cup, \cap[-1])$ is adjoint triple

By adjunction

$$\boxed{\mathrm{id}[1] \to \bigcirc \to \mathrm{id}[-1]}$$

Require it is an exact Δ.

Also,



is an exact triangle (think as Skein relation!)

= May replace by saying $\cup$ is spherical

$$\begin{tikzcd} \mathcal{C} \arrow[loop left, "{\{\mathrm{id} \to RF\}}{\{LF \to \mathrm{id}\}}"] \arrow[r, "F"] & \mathcal{D} \arrow[l, bend right=50, "L"'] \arrow[l, bend left=50, "R"] \arrow[loop right, "{\{FL \to \mathrm{id}\}}{\{\mathrm{id} \to FL\}}"] \end{tikzcd}$$

F is spherical iff 4 cones are autoeq.

$R \simeq C_L[1]$, where $C \to \mathrm{id} \to RF$

#

$\forall$

C cotwist

$C[1] \cong \mathrm{id}[-2]$, $\quad R[1] \cong L[-1]$

<u>Thm</u> If we have a collection $\mathcal{T}_{2n}$ of Δ cats

$$F_{i,2n-2} : \mathcal{T}_{2n-2} \longrightarrow \mathcal{T}_{2n}$$

spherical with cotwist $\cong [-3]$ and R_0 holds:

$$\cap \cong 1$$

and far away commute

$\Rightarrow$ Get tangle rep!

This is pretty strict. Recall the cotwist is defined by

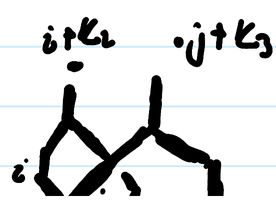
$$C \to \mathrm{id} \to RF \to C[1]$$

In most known cat's $RF \cong \mathrm{id} \oplus \mathrm{id}[-2]$.
So the condition on C is satisfied

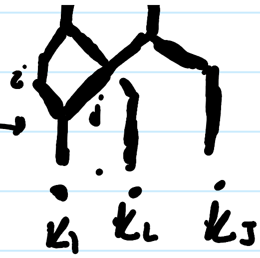
Now instead of tangles...

Objects $\quad k_1 \quad k_2 \quad \cdots \quad k_m \qquad \sum k_i = n$ fixed

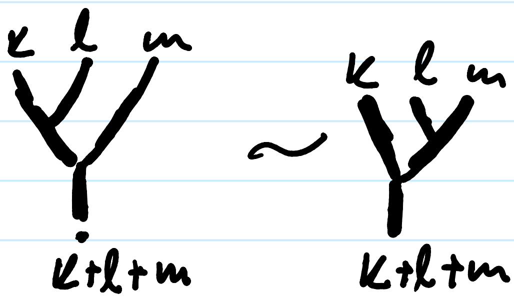
Morphisms



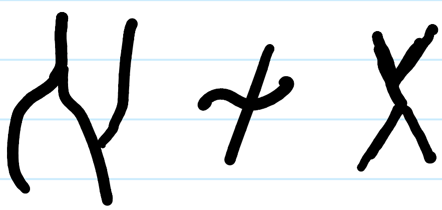
Sticky together can separate and join



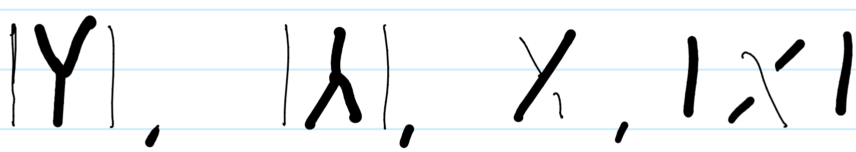
Relations: Isotopy

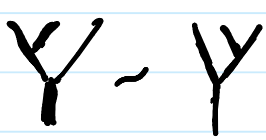


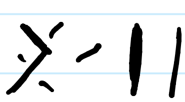
But

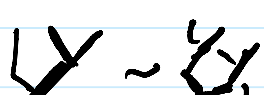


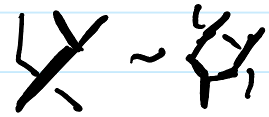
We can describe via generators & relations

Generators 

Relations 

R2  R3: Braiding

Pitchfork Commutation, 

Pitchfork, Commutation, 

Example of rep

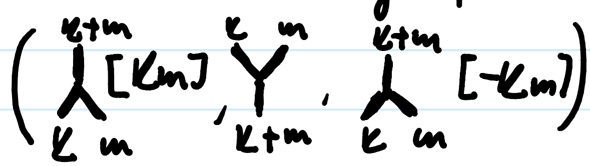
$\underline{k} = (k_1, \dots, k_m)$, $\mathcal{T}_{\underline{k}} = D^b_X(\mathrm{Coh}\,Y)$

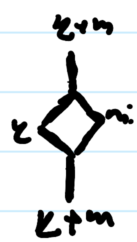
X = partial flag variety $(k_1, k_1+k_2, \dots, n)$

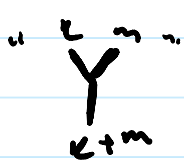
$Y = T^*X$

(Cautis-Kamnitzer-Licata) Prove studying FM kernels

Restrictions on triang. reps

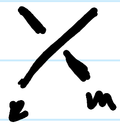
1)  adjoint triple

 $: T \to T$ $\quad$ RF[km]

 "Gr(k, k+m)-functor" ¿?

("dg-dg whose coh. is $H^* Gr$ ¿?)

Skein Relations

 : convolution of the complex (CKL)