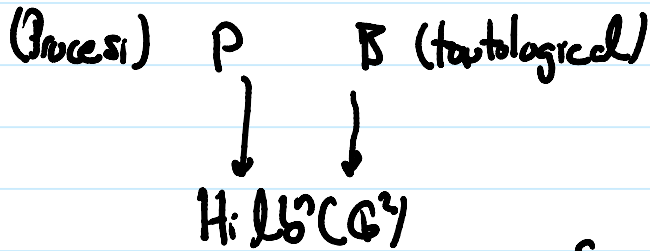


Geometry behind some notable formulas



Haiman: $H^i(P \otimes B^{\otimes l}) = \begin{cases} 0, & i \geq 1. \\ R(n, l), & i = 0. \end{cases}$

Where $R(n, l)$ is the cokernel of π free $\mathbb{C}[x, y]$ -module

$$\pi: \mathbb{C}[x, y, u, v] \longrightarrow \bigoplus_{f: [l] \rightarrow [n]} \mathbb{C}[x, y] P_f$$

$$\pi(g_1(x, y) g_2(u, v)) = g_1(x, y) g_2(x_f(u) - y_f(v)) P_f$$

$$R(n, l) = \text{coker}(\pi) = \text{im}(\pi)$$

For $l = n$, we can consider

$$(*) \mathbb{C}[x, y, u, v] \longrightarrow \bigoplus_{\sigma \in S_n} \mathbb{C}[x, y] P_\sigma = \bigoplus_{\sigma \in S_n, k \in \mathbb{Z}_3^3} \mathbb{C}[x, y] y^k P_\sigma$$

Secretly homology of some space $H^T \rightarrow X_{\mathbb{Z}_3}$ - affine Springer fiber

O. Kivinen: Need negative degrees on Y^* . For the case of affine Grassmannians

$$J = \langle \text{anti-invariant poly.} / \mathbb{C}[x, y] \rangle$$

Check $J[y^{-1}]$ does the work and (+) behavior like localization to fixed pts.

Pick one variable x, y, z, w , say y . Let $y^a \leq_{\deg} y^b \neq$

- 1- $\text{sort}(a) \leq_{\text{lex}} \text{sort}(b)$, or
- 2- $a \leq_{\text{lex}} b$.

$F_a J = \langle y^b : b \leq_{\deg} a \rangle_{\mathbb{C}[x, z, w]}$. Then ∇ formula for J is

$$ch_{q,t} J = \sum_a ch_{q,t} F_a J / F_{a-1} J \text{ where } a-1 \text{ is previous term}$$

↙ Schur

Now take $J = \Gamma(S_2(B))$, $F_a J = \langle v^b : b \leq a \rangle$.

$$\text{Conj } ch_{q,t} F_a J / F_{a-1} J = \frac{1}{(1-q)^n} \sum_v q^{*t} t^{|\text{lot}|} [Q_a] S_2 Q_v$$

What we do know is that $ch_{q,t} J = \sum_a \text{RHS}$.

Haiman: $DR_n \cong \Gamma(P \otimes \mathcal{O}_{Z_n})$

Then (C-Oblomkov) $F_a DR_n = \langle y^b \rangle$.

Thm (C-Oblomkov) $F_a DR_n = \langle y^b \rangle$.

$$(a) \text{ ch}_{q,t} F_a DR_n / F_{a-1} DR_n = \begin{cases} t^{|a|} \prod q\text{-numbers if } j^a \text{ is a GS descent poly} \\ 0, \text{ else} \end{cases}$$

$$(b) F_a / F_{a-1} \cong (q) \subseteq R_n(x)$$

Proof Note $DR_n \cong H_*(X_S)$.

$$\Gamma(P \otimes P) = \nabla h \left[\frac{-XY}{(1-q)(1-t)} \right]$$

$$R(x, y, z, w) \rightarrow \bigoplus_{\sigma \in S_n} \mathbb{C}(x, y) P_\sigma = \bigoplus_{m, \sigma \in \mathbb{Z}_{\geq 0}^n \times S_n} \mathbb{C}(x, y)^m P_\sigma$$

Thm (C, Mellitt)

$$1. [m_p] \nabla h \left[\frac{-X}{(1-q)(1-t)} \right] = \sum_{[w] \in S_p \setminus W/S_n} t^{|m|} q^{\text{dir}(w)}$$

$w: \mathbb{Z}_{\geq 0}^n \rightarrow \mathbb{Z}_{\geq 0}^n \quad w_{i+n} = a + w_i \quad \uparrow \text{max-l in } \text{Duhat}$

$$\text{dir}(w) = \# \{ 1 \leq i \leq n, i \leq j \mid w_i < w_j < w_{i+n} \}$$

Notes: a. This could be a defn

b. Each summand is a polynomial

$$2. [m_2] \nabla h_p \left[\frac{X}{1-q} \right] = \sum ???$$

$$H_*(X_S) = H_*^T(\Lambda \setminus X_S)$$

v - lat. .) $a: \neq a \pm 0$

$$H_*(X_\gamma) = H_*(\mathbb{A}^n \setminus X_\gamma)$$

$$\gamma = (a_1 t, \dots, a_n t) \quad a_i \neq q, \neq 0,$$

$$X_\gamma = \left\{ gI \in \widetilde{\mathcal{F}}\ell_n \mid g^{-1}\gamma g \in \text{Lie}(I) \right\}$$

$$\begin{array}{ccc} \text{WC} & H_*(X_\gamma) & \text{W} \\ \uparrow & & \uparrow \\ \text{S}_n & & \text{S}_n \end{array}$$

Action of lattice
corresponds to q -variables

$$\text{Conj } ch_{q,t} F_n J / F_{n-1} J = \text{sum mod on RHS, } J = \Gamma(P \otimes P)$$

Proof of C-Mellit thm: Combinatorics + counting order on $\mathbb{C}P^1$ over \mathbb{F}_q .