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Geometry behind some nabla formulas

(Procesi) P $\quad$ B (tautological)

$$P \to \mathrm{Hilb}^n(\mathbb{C}^2) \leftarrow B$$

Haiman: $H^i(P \otimes B^{\otimes l}) = \begin{cases} 0, & i \geq 1. \\ R(n,l), & i = 0. \end{cases}$

Where $R(n,l)$ is the cokernel of

$$\pi : \mathbb{C}[x,y,u,v] \longrightarrow \bigoplus_{f:[l]\to[n]} \mathbb{C}[x,y]\, P_f$$

(free $\mathbb{C}[x,y]$-module)

$$\pi(g_1(x,y)\, g_2(u,v)) = g_1(x,y)\, g_2(x_{f(i)}, y_{f(i)})\, P_f$$

$$R(n,l) = \mathrm{coker}(\pi) = \mathrm{im}(\pi)$$

For $l = n$, we can consider

$$(*)\quad \mathbb{C}[x,y,u,v] \longrightarrow \bigoplus_{\sigma \in S_n} \mathbb{C}[x,y]\, P_\sigma = \bigoplus_{\sigma, a \in S_n \times \mathbb{Z}_{\geq 0}^n} \mathbb{C}[x]\, y^a\, P_\sigma$$

↑ Secretly homology of some space $H_*^T X_\gamma$ - affine Springer fiber

O. Kivinen: Need negative degrees on Y's. For the case of affine Grassmannian

$J = \langle$ anti-invariant poly. $/ \mathbb{C}[x,y] \rangle$

Check $J[y^{-1}]$ does the work and (+) behaves like localization to fixed pts.

Pick one variable x, y, z, w, say y. Let $y^a \leq_{deg} y^b$ if

1- $\mathrm{sort}(a) <_{lex} \mathrm{sort}(b)$, or
2- $a \leq_{lex} b$.

$F_a J = \langle y^b : b \leq_{deg} a \rangle_{\mathbb{C}[x,z,w]}$. Then ∇ formula for J is

$$ch_{q,t} J = \sum_a ch_{q,t} F_a J / F_{a-1} J$$ where $a-1$ is previous term

Now take $J = \Gamma(\mathbb{S}_\lambda(\mathcal{B}))$ (Schur), $F_a J = \langle \gamma^b : b \leq a \rangle$.

Conj $$ch_{q,t} F_a J / F_{a-1} J = \frac{1}{(1-q)^n} \sum_\nu q^{*} t^{|a|} [Q_a] \, s_\lambda Q_\nu$$

What we do know is that $ch_{q,t} J = \sum_a \mathrm{RHS}$.

Haiman: $DR_n \cong \Gamma(P \otimes \mathcal{O}_{Z_n})$

<u>Thm</u> (C-Oblomkov) $F_a DR_n = \langle y^b \rangle$.

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(a) $\mathrm{ch}_{q,t} F_a DR_n / F_{a-1} DR_n = \begin{cases} t^{|a|} \prod q\text{-numbers} & \text{if } y^a \text{ is a GS descent poly} \\ 0, & \text{else} \end{cases}$

(b) $F_a / F_{a-1} \cong (g) \subseteq R_n(x)$

Proof Not $DR_n \cong H_*(X_\gamma)$.

$$\Gamma(P \otimes P) = \nabla h\left[\frac{-XY}{(1-q)(1-t)}\right]$$

$$R[x,y,z,w] \longrightarrow \bigoplus_{\sigma \in S_n} \mathbb{C}[x,y] P_\sigma = \bigoplus_{m,\sigma \in \mathbb{Z}^n_{\geq 0} \times S_n} \mathbb{C}[x] y^m P_\sigma$$

Thm (C, Mellit)

1.- $[m_\mu] \nabla h\left[\frac{-X}{(1-q)(1-t)}\right] = \sum_{[w] \in S_\mu \backslash W / S_\lambda} t^{|m|} q^{dinv(w)}$

$w: \mathbb{Z}^n_{\geq 0} \to \mathbb{Z}^n_{\geq 0}$ $\quad w_{i+n} = n + w_i$ $\quad$ (max-l in Swhat)

$$dinv(w) = \# \{ 1 \leq i \leq n, i < j \mid w_i < w_j < w_{i+n} \}$$

Notes: a.- This could be a defn

b.- Each summand is a polynomial

2.- $[m_\nu] \nabla h_\mu \left[\frac{X}{1-q}\right] = \sum ???$

$H_*(X_\gamma) = H^T_*(\Lambda \backslash X_\gamma)$

$\gamma = (a_i t, \ldots)$ $\quad a_i \neq a_j \neq 0$

$H_*(X_\gamma) = H_*(\Lambda \backslash X_\gamma)$

$\gamma = \begin{pmatrix} a_1 t & & \\ & \ddots & \\ & & a_n t \end{pmatrix} \quad a_i \neq a_j \neq 0,$

$$X_\gamma = \{ gI \in \widetilde{Fl}_n \mid g^{-1}\gamma g \in Lie(I) \}$$

$W \subset H_*(X_\gamma) \supseteq W$

$\cup$ $\quad\quad$ $\cup$

S_n $\quad\quad$ S_n

Action of lattice + corresponds to q-variables

Conj $ch_{q,t} F_a J / F_{a-1} J$ = summand on RHS, $J = \Gamma(P \otimes P)$

<u>Proof of C-Mellit thm</u> + : Combinatorics + counting bundles on $\mathbb{CP}^1$ over $\mathbb{F}_q$.