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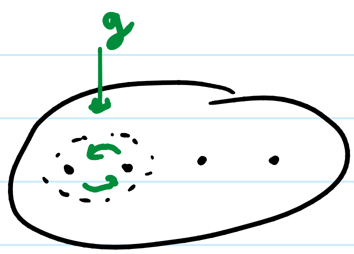
Σ - surface, $\chi(\Sigma) < 0$

$\Gamma(\Sigma) = \mathrm{Diffeo}^+(\Sigma)/_{\text{isotopy}}$ - the mapping class group

$\Gamma(\Sigma) \circlearrowleft \{$simple closed (multi)curves on $\Sigma\}$

$\rotatebox{90}{$\in$}$

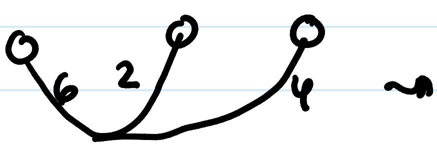
g

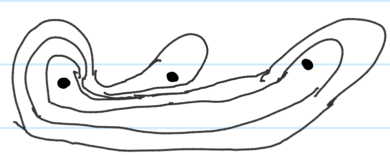
e.g. $\Sigma =$ 

$\Gamma(\Sigma) \cong Br_4$

$g \in \Gamma(\Sigma)$

(I) Q: How do curves grow under g?



$\rightsquigarrow$ 

Say $g = \sigma_1^{-1}\sigma_2$ a composition of Dehn twists

preserves curves of the form 

and transforms

by $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

__Warning__ Action of B_4 on $\mathbb{Z}_+^2$ by these is only __piecewise__

Warning: Action of B_4 on $\mathbb{Z}^2_+$ by thm is only piecewise linear.

(II) $\Gamma(\Sigma) \curvearrowright \mathrm{Teich}(\Sigma) = \left(\text{hyperbolic metrics on } \Sigma\right)/\sim$

$$\begin{matrix} \mathrm{Teich}(\Sigma) \\ \cong \\ \text{open ball} \end{matrix}$$

(I) and (II) are closely related

$$\Gamma(\Sigma) \curvearrowright \overline{\mathrm{Teich}} = \mathrm{Teich} \sqcup PMF$$

$$\overline{\mathrm{Teich}} \cong \text{closed ball}, \quad \mathrm{Teich} \leftarrow \text{open ball}, \quad PMF \leftarrow \text{sphere}$$

$PMF = \{\text{proj-ve measured foliations}\}$

A simple closed curve gives a point in PMF

Let $S = \{\text{simple closed multicurves on } \Sigma\}$

$\gamma \in \mathrm{Teich}$, can think $\gamma \in \mathbb{R}^S_{>0}$, so

$$\begin{aligned} \mathrm{Teich} &\longrightarrow \mathbb{R}^S_{>0} \\ \gamma &\longmapsto (c \longmapsto l_\gamma(c)) \end{aligned}$$

$$\begin{aligned} S &\longrightarrow \mathbb{R}^S_{>0} \\ c &\longmapsto (c' \longmapsto \#\mathrm{int}(c \cap c')) \end{aligned} \quad \text{(Not signed!)}$$

In fact,

$$\mathrm{Teich} \overset{i}{\hookrightarrow} \mathbb{P}(\mathbb{R}^S_+)$$

In fact,

$$\begin{tikzcd} \mathrm{Teich} \arrow[r, hook, "j"] & \mathbb{P}(\mathbb{R}^S_+) \\ S \arrow[ur, hook, "k"] & \end{tikzcd}$$

im(k) = rational pts. in bdry

Thurston compactification: $\overline{im(j)} = im(j) \cup \overline{im(k)}$

$\Gamma(\Sigma) \curvearrowright \overline{\mathrm{Teich}} = \mathrm{Teich} \sqcup \mathrm{PMF}$

obtained via considering ↑ ↗ as functionals on $\mathbb{P}(\mathbb{R}^S_+)$

<u>IMPORTANT PTS</u>

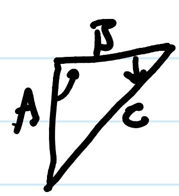
1) Teich maps homeomorphically onto its image

2) $\overline{\mathrm{Teich}}$ is a closed f.d. Euclidean ball

3) $\Gamma(\Sigma)$ acts $\underbrace{\text{p.}}_{\text{piecewise}}$linearly on the bdry PMF.

Triangulated cat-s are "2-diml" (Dyk-Kapranov)

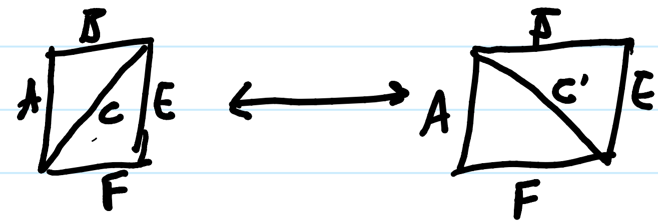
$$A \to B \to C \to A[1]$$



Octahedral axiom

B B

Octahedral axiom



Replace $\Gamma(\Sigma)$ with $\mathrm{Aut}(\mathcal{T})$ ← triang. cat-y

Important: Teich ⇝ Stab($\mathcal{T}$) - moduli space of Bridgeland stability conditions

Stability conditions on $\mathcal{T}$

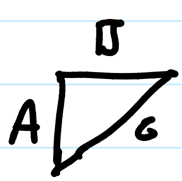
A stability condition on $\mathcal{T}$, $\sigma \in \mathrm{Stab}(\mathcal{T})$ specifies

(1) (Semi-)stable objects $\{E_\lambda\}$

To each object σ assigns: mass $m(E_\lambda) \in \mathbb{R}_{>0}$
phase : $\phi(E_\lambda) \in \mathbb{R}$

Fix $t \in \mathbb{R}$, $\boxed{m_t(E_\lambda) = m(E_\lambda)\, e^{i\pi t \phi(E_\lambda)}}$

(2) Every object $X \in \mathcal{T}$ has a canonical Harder-Narasimhan filtration

Important  $\Rightarrow \overset{m(A)}{\overset{\shortparallel}{|m_t(A)|}} \leq |m_t(B)| + |m_t(C)|$

<u>Important</u> $A \diagup_{C} \Rightarrow |m_t(A)| \le |m_t(B)| + |m_t(C)|$

Shifting doesn't change mass, phase moves by 1.

Bridgeland ⇝ moduli space of stability conditions
$\mathrm{Stab}(T)$ - a complex mfld

$\mathrm{Aut}(T) \circlearrowright \underbrace{\mathrm{Stab}(T)}_{\text{Contractible}}$ continuously

in all cases that
have been computed

e.g. $T = \underset{\sim}{2CY}$ cat-y assoc. to ADE quiver
¿cluster category?
$\mathrm{Stab}(T)$ is an open Euclidean ball

<u>WANT</u> Compactify $\mathrm{Stab}(T)$

Let $S = \{$semistable objects for some stability condition in $T\}$

$\sigma \in \mathrm{Stab}(T)$, $t \in \mathbb{R}$

$\sigma \mapsto f_\sigma : S \to \mathbb{R}$
$f_\sigma(X) = m^\sigma(X)$

if not stable add HN constituents

$\mathrm{Stab}(T) \to \mathbb{R}^S_{\geq 0} \longrightarrow \mathbb{P}(\mathbb{R}^S_{\geq 0})$

(jt. wt. A. Bapat - Deopurkar)

Prop $\overline{\mathrm{Stab}(T)} = \overline{\iota(\mathrm{Stab}(T))} \subset \mathbb{P}(\mathbb{R}^S_{\geq 0})$

Propose $\overline{\mathrm{Stab}(T)} = \overline{\mathrm{im}(\mathrm{Stab}(T))} \subseteq \mathbb{P}(\mathbb{R}^S_{\geq 0})$

Expectation (Conjecture for 2CY cats assoc to ADE quivers)

$\overline{\mathrm{Stab}(T)}$ is a closed Euclidean ball
$\quad \| $
$\mathrm{Stab}(T) \sqcup$ sphere

WTF is this?

Some pts (for $T = $ 2-CY cat-y of ADE quiver)
so $S = \{$spherical objects$\}$

$E \in S$ gives a functional

$E \mapsto (f_E : Y \mapsto \dim \mathrm{Hom}^\cdot(E, Y))$

$\mathcal{D}^b$(proj/rj. alg)

these belong to that weird sphere