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sexta-feira, 21 de junho de 2019 14:00

jt. wt. E. Gorsky

I.- Motivation

$$\underset{\uparrow \atop \mathbb{B}r_n}{H_n} = \mathbb{Z}[q^{\pm 1}]\mathbb{B}r_n \Big/ \langle \overbrace{\times - \times = (q - q^{-1}) \uparrow\uparrow}^{\text{skein rdn}} \rangle$$

More generally, M = cpt oriented 3-mfld, $P \subseteq \partial M$ equipped w/ framings

$$SK(M,P) = R\langle \text{framed } \overset{\text{up to reg isotopy}}{\text{tangles}} \text{ in } (M,P)\rangle \Big/ \langle \overset{\text{skein rdn}}{\overset{\uparrow}{\circ} = -a \uparrow} \rangle$$

R contains $q^{\pm 1}, a^{\pm 1}$. If $q - q^{-1}$ is invertible, it follows that

$$\bigcirc = \frac{a - a^{-1}}{q - q^{-1}}$$

$$\{\text{framed tangles in } (M,P)\} \longrightarrow SK(M,P)$$

Mapping class group $\circlearrowright SK(M,P)$

e.g. $H_n = SK\Bigg($  $\Bigg)$ (only take tangles oriented up!)

Rmks ① Excellent gluing properties

② if $M = \Sigma \times I$, $P = P' \times \{0\} \sqcup P' \times \{1\}$ then

$SK(\Sigma, P') = SK(M,P)$ is an algebra w.r.t. stacking

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② $(D^2, n\text{ pts}) \Rightarrow H_n$

( , n pts) $\Rightarrow H_{n,ext}$

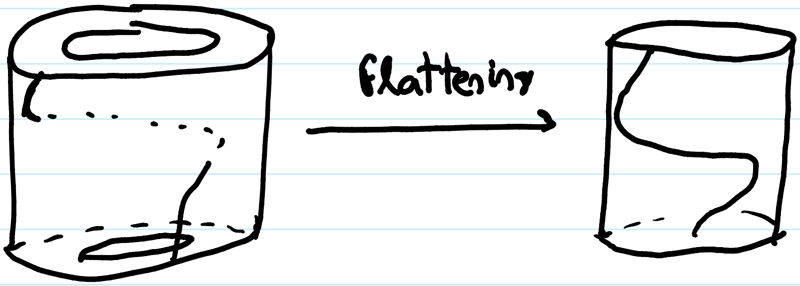
Guide for categorification

higher cat-y

$$\begin{tikzcd}
\left\{\begin{array}{l}\text{objects: tangles in } (M,P)\\ \text{mor: tangle cob. in } (M,P)\times I\\ \quad \text{modulo isotopy}\end{array}\right\} \arrow[r, "\text{functor}"] \arrow[d, "\text{isoclasses}"] & CatSk(M,P) \arrow[d, "K_0"] \\
\{\text{tangles in } (M,P)\}/\text{isotopy} \arrow[r] & Sk(M,P)
\end{tikzcd}$$

Examples ① $(D^2, n\text{ pts})$

$$\begin{tikzcd}
Braid_n \arrow[r, "\text{Rouquier}"] \arrow[d] & K^b(SBim_n) \arrow[d, "K_0"] \\
Br_n \arrow[r] & H_n
\end{tikzcd}$$



Combinatorial?

braids rotated ↑

$$\text{II}.\quad SK^+(A) = \frac{\mathbb{Z}[q^{\pm1}]\langle \text{braid closures}\rangle}{\langle \times - \times = (q-q^{-1})\,)(\,\rangle}$$

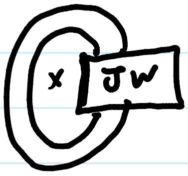
"Positive HOMFLY-PT skein"

$$= \bigoplus_{n\geq 0} H_n/[H_n, H_n]$$

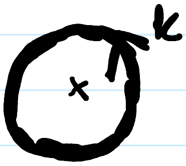
Turaev $SK^+(A) \cong \Lambda_q$ (symm. functions)

One such iso:

$\longmapsto e_1 = h_1 = p_1 = s_\square$



$\longmapsto h_n$



$\longmapsto e_k$

$SK^+(A)$ has a basis given by monomials in either of these guys

$$\begin{tikzcd} H_n \arrow[r] & SK^+(A) \arrow[r] & \Lambda_q \\ x \arrow[rr, mapsto] & & \sum_{\lambda\vdash n} Tr(x, V_\lambda)\, s_\lambda \end{tikzcd}$$

Turaev Uses dg generator

$C_\varepsilon =$

Winding n times

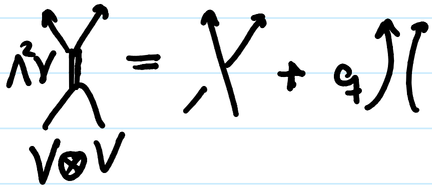
n-1

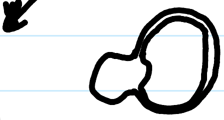
$C_\varepsilon =$  Winding n times

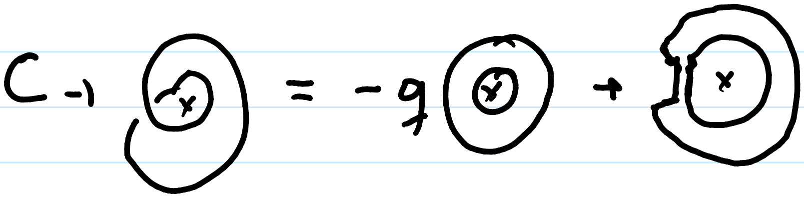
$\underbrace{\varepsilon \in \{\pm 1\}^{n-1}}_{\text{one for each } n}$

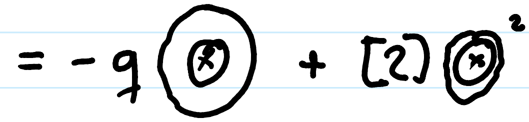
with some choice of crossings

Example



same thing as 





$$= -q e_1^2 + [2] e_2$$

$$= -q\, s_{\square\square} + q^{-1} s_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$$

$$C_{-1,-1,\dots,-1} \longmapsto (-1)^{n-1} q^{n-1} s_{\square\square\cdots\square} + \cdots - q^{3-n} s_{\begin{smallmatrix}\square&\square\\\square\\\vdots\end{smallmatrix}} + q^{1-n} s_{\begin{smallmatrix}\square\\\vdots\\\square\end{smallmatrix}}$$

$$= \frac{e_n[X(q^{-1}-q)]}{q^{-1}-q}$$

Where we use plethystic transformation

$$\Lambda_q \longrightarrow \Lambda_q$$
$$f \longmapsto f[X(q^{-1}-q)]$$
$$p_k \longmapsto (q^{-k}-q^{k}) p_k$$

$$p_k \longmapsto (q^{-k} - q^{k}) p_k$$

Prop $\varepsilon \in \{\pm 1\}^{n-1} \rightsquigarrow$ Skew Young diag, eg

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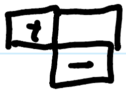


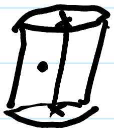
$$\frac{s_{\lambda/\mu}[X(q^{-1}-q)]}{q^{-1}-q} \in \Lambda_q \longleftarrow\!\shortmid \overline{C_\varepsilon} \in Sk^+(A)$$

III - Categorified

$$\begin{tikzcd}
\mathrm{Braid}_n \arrow[r] \arrow[d] & K^b(\mathrm{SBim}_n) \arrow[d] \\
h\mathrm{Tr}(\mathrm{Braid}_n) \arrow[r, "\substack{\text{Annular}\\ \text{KR homology}}"] \arrow[d] & K^b(h\mathrm{Tr}(\mathrm{SBim}_n)) \simeq K^b(\hat{P}) \arrow[d] \\
\{\text{braids in } A\} \arrow[r] & Sk^+(A) \cong \Lambda_q
\end{tikzcd}$$

$\hat{P} = \mathrm{Kar}(P)$,

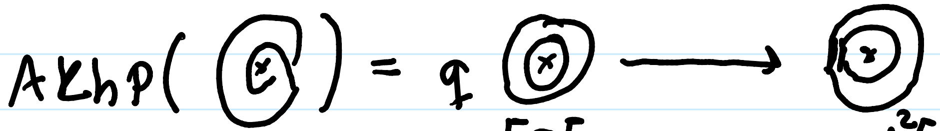
P = free graded strictly symmetric monoidal $\mathbb{C}$-linear cat-y w/

- obj $E \longleftrightarrow$ 
- deg 2 end $X: E \to E \longleftrightarrow$ 

Ind. objects in $\hat{P} \sim S^\lambda E$ (Schur functors)

$\hat{P}$ ‖ Proj-ve modules are $\bigoplus_n \mathbb{C}(x_1 .., x_n) \rtimes S_n$

Pr qj-rt modules de $\bigoplus_n \mathbb{C}[x_1, \dots, x_n] \rtimes S_n$



$$q E \otimes E \qquad\qquad q\Lambda^2 E \oplus q^{-1}\Lambda^2 E$$

$$\|$$

$$q\Lambda^2 E \oplus q S^2 E$$

$$= \begin{pmatrix} q\Lambda^2 E \\ \oplus \\ q\, S^2 E \end{pmatrix} \longrightarrow \begin{pmatrix} q\Lambda^2 E \\ q^{-1}\Lambda^2 E \end{pmatrix}$$

$$\simeq q\, S^2 E \longrightarrow q^{-1}\Lambda^2 E$$

$$\begin{tikzcd} \hat{P} \arrow[r, bend left, "\pi^*"] & \hat{P}[t] \arrow[l, bend left, "\pi_*"] \end{tikzcd}$$

t : deg 2

$$\bigoplus_{n\geq 0} E \longleftarrow E[t]$$

$$K(E, x) = [\pi^* E \xrightarrow{x-t} \pi^* E]$$

Define $\hat{P} \longrightarrow K^b(\hat{P})$

$$\phi : F \longmapsto F(K(E,x)) \longmapsto \pi_*(F(K(E,x)))$$

$$\| \qquad\qquad \|$$

$$S^\lambda E \qquad S^\lambda K(E,x)$$

If F decategorifies to f, on K_0, $f \longmapsto \dfrac{f[X(q^{-1}-q)]}{q^{-1}-q}$

$$\phi(E) : \pi_*(\pi^* E \xrightarrow{x-t} \pi^* E)$$

$$\simeq \begin{tikzcd} E \arrow[r, "x"] \arrow[dr, "t"] & E \\ \oplus & \oplus \\ {} \arrow[r, "x"] & E \end{tikzcd} \simeq E$$

$$\simeq \begin{tikzcd} E \arrow[r, "\wedge"] \arrow[rdd, "t"] & E \\ \oplus & \oplus \\ E \arrow[r, "x"] \arrow[rdd, "t"] & E \\ \oplus & \oplus \\ E \arrow[r, "x"] & E \\ \vdots & \end{tikzcd} \simeq E$$

$$\phi(\Lambda^2 E) \simeq [q^2 S^2 E \longrightarrow q^{-1} \Lambda^2 E]$$

$$\phi(E^{\otimes n}) \cong (\Lambda^{\bullet} U_n \otimes E^{\otimes n}, D) = \mathrm{Cube}(n)$$

U_n = refl. rep of S_n

$U_n = \langle v_1, \dots, v_{n-1} \rangle$, $D \cdot v_i \mapsto x_i - x_{i+1}$

<u>Thm</u> (Solomon) $\mathbb{C}[S_n] \cong \bigoplus_{\varepsilon \in \{\pm 1\}^{n-1}} \mathbb{C}[S_n] s_\varepsilon \overline{s_\varepsilon}$

$\Rightarrow \mathrm{AKhR}(\overline{C_\varepsilon}) \cong \mathrm{Cube}\, s_\varepsilon \overline{s_\varepsilon}$