Math 16A (LEC 004), Fall 2008.
October 22, 2008.

MIDTERM EXAM 1

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ____________________

ID#: ____________________

Instructions: Each of the 4 problems is worth 25 points. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Calculators, books or notes are not allowed.

Make sure that you have a total of 5 pages (including this one) with 4 problems. Read through the entire exam before beginning work.

1	
2	
3	
4	
TOTAL	

1.

(a) Find the equation of the circle with center at the point (2,1) which goes through the point (3,4).

$$(x-2)^2 + (y-1)^2 = (3-2)^2 + (4-1)^2$$

$$(x-2)^2 + (y-1)^2 = 10$$

(b) Find the equation of the line that goes through the two points (2,1) and (3,4).

$$y - 1 = \frac{4-1}{3-2}(x-2)$$

$$y - 1 = 3(x-2)$$

$$y = 3x - 5$$

(c) Find the points of intersection (if there are any) between the circle from (a) and the line from (b).

$$(x-2)^2 + (3x-5-1)^2 = 10$$

$$(x-2)^2 + (3x-6)^2 = 10$$

$$(x-2)^2 + 9(x-2)^2 = 10$$

$$10(x-2)^2 = 10$$

$$(x-2) = \pm 1$$

$x-2 = 1$, $x = 3$:

$x-2 = -1$, $x = 1$:

$$\boxed{\begin{matrix}(3,4)\\(1,-2)\end{matrix}}$$

2. Consider the function $f(x) = \dfrac{2x^2 - 2}{(x-2)^2}$. Determine its domain, its intercepts, and its vertical asymptotes. Determine its horizontal asymptote and find any points where the graph of $y = f(x)$ intersects its horizontal asymptote. Then sketch the graph of this function on which all obtained points and asymptotes are clearly marked.

Domain: $x \neq 2$

Intercepts: $(0, -\frac{1}{2})$, $(1, 0)$, $(-1, 0)$

Vertical asympt.: $x = 2$

$$\lim_{x \to 2^+} \frac{2x^2-2}{(x-2)^2} = \infty$$

$$\frac{(\approx 6)}{(\text{small} > 0)}$$

$$\lim_{x \to 2^-} \frac{2x^2-2}{(x-2)^2} = \infty$$

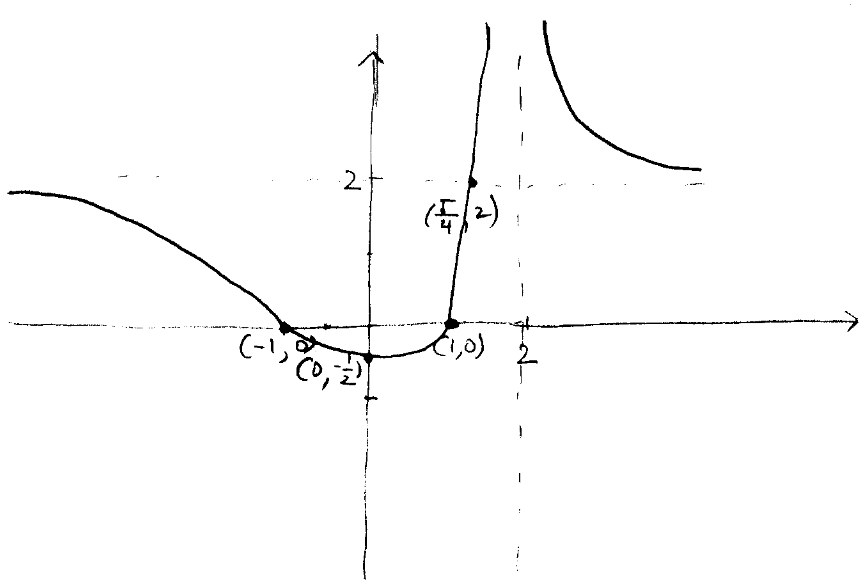
Horizontal asympt. $\lim_{x \to \infty} \dfrac{2x^2-2}{x^2-4x+4} = 2$

$y = 2$

Intersection: $\dfrac{2x^2-2}{x^2-4x+4} = 2$, $\cancel{2x^2}-2 = \cancel{2x^2} - 8x + 8$

$8x = 10$, $x = \frac{5}{4}$

$(\frac{5}{4}, 2)$



3. Compute the following limits. Give each answer as either a finite number, or $+\infty$, or $-\infty$.

(a) $\lim_{x\to 1} \frac{x+2}{2+\sqrt{5x-1}} = \frac{3}{2+2} = \underline{\underline{\frac{3}{4}}}$

(b) $\lim_{x\to 2-} \frac{\sqrt{x+7}-4}{(x-2)\sqrt{x+7}} = \underline{\underline{+\infty}}$

$$\frac{(\approx -1)}{(\text{small} < 0)(\approx 3)}$$

(c) $\lim_{x\to 2} \frac{x-2}{x+1-\sqrt{x+7}} = \lim_{x\to 2} \frac{(x-2)(x+1+\sqrt{x+7})}{(x+1-\sqrt{x+7})(x+1+\sqrt{x+7})}$

$$= \lim_{x\to 2} \frac{(x-2)(x+1+\sqrt{x+7})}{(x+1)^2-(x+7)}$$

$$(x+1)^2-(x+7) = x^2+2x+1-x-7 = x^2+x-6 = (x+3)(x-2)$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+1+\sqrt{x+7})}{\cancel{(x-2)}(x+3)} = \underline{\underline{\frac{6}{5}}}$$

(d) $\lim_{x\to\infty} \frac{6x^2+7x+\sqrt{x}}{3x^2+5x\sqrt{x}+7x} = \lim_{x\to\infty} \frac{6x^2}{3x^2} = \underline{\underline{2}}$

4. Consider the functions $f(x) = 2|x| + x + 1$ and $g(x) = 2x\sqrt{x} + x + 2$. $= 2x^{3/2} + x + 2$

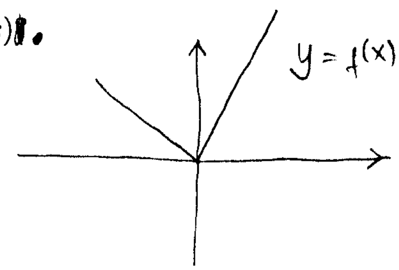
(a) Determine the domains of $y = f(x)$ and $y = g(x)$.

Domain of f: all x.

Domain of g: $x \geq 0$.

(b) Discuss continuity and differentiability of $y = f(x)$.

$$f(x) = \begin{cases} 3x+1, & \text{if } x \geq 0; \\ -x+1, & \text{if } x < 0. \end{cases}$$



This function is continuous for all x, and differentiable everywhere except at $x=0$, where left and right slopes are different, -1 and 3.

(c) Determine the point at which the tangent to $y = g(x)$ is parallel to the line $y - 7x + 17 = 0$.

slope $= 7$

$$g'(x) = 3x^{1/2} + 1$$

$$3x^{1/2} + 1 = 7$$

$$x^{1/2} = 2$$

$$x = 4$$

Point: $(4, 2\cdot 8 + 4 + 2) = \underline{(4, 22)}$

(d) Compute $g(f(0))$ and $f(g(0))$.

$f(0) = 1$, $g(0) = 2$,

$g(f(0)) = g(1) = \underline{5}$, $f(g(0)) = f(2) = \underline{7}$.