Math 16A — 002, Fall 2009.
Oct. 21, 2009.

MIDTERM EXAM 1

NAME(print in CAPITAL letters, *first name first*): ________________________________

NAME(sign): ________________________________

ID#: ________________________

Instructions: Each of the four problems is worth 25 points. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor in determining credit. Calculators, books or notes are not allowed.

Make sure that you have a total of 5 pages (including this one) with 4 problems. Read through the entire exam before beginning to work.

1	
2	
3	
4	
TOTAL	

1.

(a) A line has y–intercept $(0,3)$ and slope $-1/3$. Find its x–intercept.

$$y - 3 = -\frac{1}{3}x$$

$$y = -\frac{1}{3}x + 3 \qquad\qquad y = 0 \text{ when } -\frac{1}{3}x + 3 = 0$$

$$\underline{\underline{x = 9}}$$

(b) A circle has center at $(0,5)$ and goes through the origin. Find the equation of this circle.

$$(x-0)^2 + (y-5)^2 = 25$$

$$\underline{\underline{x^2 + (y-5)^2 = 25}}$$

(c) In addition to the circle from (b), consider the line $x + y = 12$. Find all points of intersection (if there are any) between the line and the circle.

$$y = 12 - x$$

$$x^2 + (7-x)^2 = 25$$

$$x^2 + 49 - 14x + x^2 = 25$$

$$2x^2 - 14x + 24 = 0$$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0, \quad x = 3, 4$$

Answer: $\underline{(3,9),\ (4,8)}$

2. Consider the function $f(x) = \dfrac{x^2 - x}{x^2 - 2x - 3}$. Determine its domain, intercepts, and vertical asymptotes. Determine left and right limits at all vertical asymptotes. Determine also its horizontal asymptote and find any points where the graph of $y = f(x)$ intersects its horizontal asymptote. Then sketch the graph of this function on which all obtained points and asymptotes are clearly marked.

$$f(x) = \frac{x(x-1)}{(x-3)(x+1)}$$

<u>Domain</u> $x \neq 3,\ x \neq -1$

<u>Intercepts</u>: $(0,0),\ (1,0)$

<u>Vertical asymptotes</u>: $x = 3,\ x = -1$

$$\lim_{x \to 3^+} f(x) = \infty \quad \frac{(\approx 6)}{(\text{small} > 0)(\approx 4)}; \qquad \lim_{x \to -1^+} f(x) = -\infty \quad \frac{(\approx 2)}{(\approx -4)(\text{small} > 0)}$$

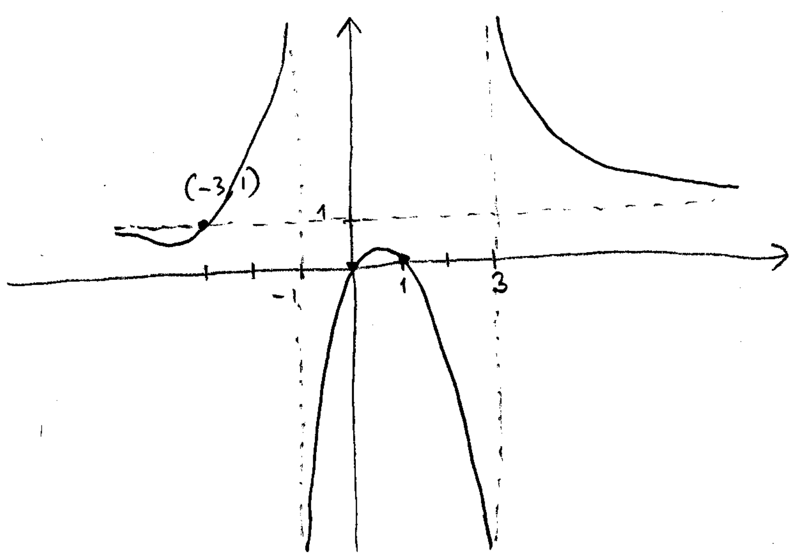
$$\lim_{x \to 3^-} f(x) = -\infty \quad \frac{(\approx 6)}{(\text{small} < 0)(\approx 4)}; \qquad \lim_{x \to -1^-} f(x) = \infty \quad \frac{(\approx 2)}{(\approx -4)(\text{small} < 0)}$$

<u>Horizontal asymptote</u>:

$$\lim_{x \to \infty} \frac{x^2 - x}{x^2 - 2x - 3} = 1 \qquad y = 1$$

Intersection with h.a. $\dfrac{x^2 - x}{x^2 - 2x - 3} = 1$, $\cancel{x^2} - x = \cancel{x^2} - 2x - 3$, $\underline{x = -3}$

$\underline{(-3, 1)}$



3. Compute the following limits. Give each answer as a finite number, $+\infty$ or $-\infty$.

(a) $\lim_{x\to 3} \frac{x+5}{\sqrt{x+1}} = \frac{8}{\sqrt{4}} = \underline{\underline{4}}$

(b) $\lim_{x\to 3} \frac{x-3}{x-1-\sqrt{x+1}}$

$$= \lim_{x\to 3} \frac{(x-3)\cdot(x-1+\sqrt{x+1})}{(x-1-\sqrt{x+1})\cdot(x-1+\sqrt{x+1})}$$

$$= \lim_{x\to 3} \frac{(x-3)(x-1+\sqrt{x+1})}{(x-1)^2-(x+1)}$$

$$= \lim_{x\to 3} \frac{(x-3)(x-1+\sqrt{x+1})}{\underbrace{x^2-2x+\cancel{1}-x-\cancel{1}}_{=x(x-3)}}$$

$$= \lim_{x\to 3} \frac{x-1+\sqrt{x+1}}{x} = \frac{2+2}{3} = \underline{\underline{\frac{4}{3}}}$$

(c) $\lim_{x\to 3+} \frac{x-6}{(x-3)\cdot\sqrt{x+6}} = \underline{\underline{-\infty}}$

$$\frac{(\approx -3)}{(\text{small} > 0)(\approx 3)}$$

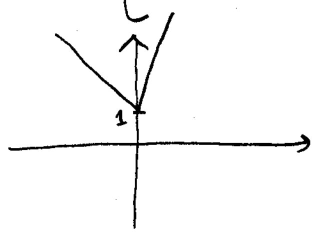
(d) $\lim_{x\to\infty} \frac{x^3-4x^2+4}{3x^3+x^2+x+4} = \frac{1}{3}$

4. Consider the functions $f(x) = 4\sqrt{x} + 2x - 15$ and $g(x) = 2|x| + x + 1$.
(a) Determine the domains of the functions $y = f(x)$ and $y = g(x)$.

Domain of f: $x \geq 0$
Domain of g: all x

(b) Discuss continuity and differentiablity of $y = g(x)$.

$$g(x) = \begin{cases} 3x+1, & x \geq 0 \\ -x+1, & x < 0 \end{cases}$$



The function is continuous for all x.

It is differentiable everywhere but at $x=0$, because the right and left derivatives there are 3 and -1, respectively.

(c) A line is tangent to the graph of $y = f(x)$ and perpendicular to the line $x + 3y + 7 = 0$. Determine the equation of this line.

$y = -\frac{1}{3}x - \frac{7}{3}$ has slope $-\frac{1}{3}$, so perp. line has slope 3.

$$f'(x) = \frac{4}{2\sqrt{x}} + 2 = \frac{2}{\sqrt{x}} + 2$$

$$\frac{2}{\sqrt{x}} + 2 = 3$$

$$\frac{2}{\sqrt{x}} = 1, \quad \sqrt{x} = 2, \quad x = 4$$

Point: $(4, 4 \cdot \sqrt{4} + 2 \cdot 4 - 15) = (4, 1)$

Line: $y - 1 = 3(x-4)$, or $y = 3x - 11$

(d) Compute $f(g(1))$ and $g(f(1))$.

$g(1) = 4$ $\quad f(g(1)) = f(4) = 1$

$f(1) = -9$ $\quad g(f(1)) = g(-9) = 2 \cdot 9 - 9 + 1 = 10$