Math 16A (LEC 004), Fall 2008.
November 19, 2008.

MIDTERM EXAM 2

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ______________________

ID#: ________________

Instructions: Each of the first four problems is worth 15 points, while problems 5 and 6 are each worth 20 points. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor in determining credit. Calculators, books or notes are not allowed.

Make sure that you have a total of 7 pages (including this one) with 6 problems. Read through the entire exam before beginning to work.

1	
2	
3	
4	
5	
6	
TOTAL	

1.

1. Compute the derivatives of the following two functions. *Do not simplify!*

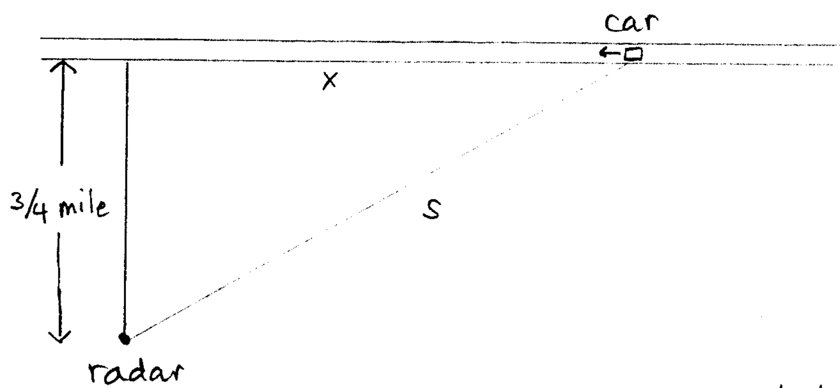
(a) $y = \dfrac{\cos x}{5x^2 - 1}$

$$y' = \frac{-\sin x \cdot (5x^2 - 1) - \cos x \cdot 10x}{(5x^2 - 1)^2}$$

(b) $y = (\sin(7x))^8$

$$y' = 8(\sin(7x))^7 \cdot \cos(7x) \cdot 7$$

2. A car is driving on a straight highway and its speed is checked by a police radar which is at distance 3/4 of a mile from the road. At one point, the radar measures the car's distance from itself as 5/4 miles and also that this distance is decreasing at the rate of 40 miles/hour (see the picture). How fast is the car driving along the road?



When $s = \frac{5}{4}$, $\frac{ds}{dt} = -40$

and

$x^2 = \left(\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = 1$

so $\underline{\underline{x = 1}}$.

$$s^2 = x^2 + \left(\frac{3}{4}\right)^2$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$$

Plug in $s = \frac{5}{4}$, $x = 1$, $\frac{ds}{dt} = -40$;

$$\underline{\underline{\frac{dx}{dt}}} = -\frac{5}{4} \cdot 40 = \underline{\underline{-50}} \quad (\text{miles/hr})$$

3. Find the equation of the tangent line to the curve $x^4 + y^4 + 3xy^2 - y = 6$ at the point $(1, -1)$. (You may leave the equation of the line in the point-slope form.)

$$4x^3 + 4y^3 \frac{dy}{dx} + 3y^2 + 6xy \frac{dy}{dx} - \frac{dy}{dx} = 0$$

Plug in $x = 1,\ y = -1$:

$$4 - 4 \frac{dy}{dx} + 3 - 6 \frac{dy}{dx} - \frac{dy}{dx} = 0$$

$$11 \frac{dy}{dx} = 7, \quad \frac{dy}{dx} = \frac{7}{11} \leftarrow \text{slope}$$

Line:

$$\underline{\underline{y + 1 = \frac{7}{11}(x - 1)}}$$

4. You are standing on top of an 80 ft. tall tower. You throw a rock straight *up* with velocity 64 ft/sec. How fast is the rock traveling in the moment when it hits the ground? Assume the acceleration of the rock is constantly -32 ft/sec^2.

$$s = -16t^2 + 64t + 80$$

$$\frac{ds}{dt} = -32t + 64$$

$s = 0$ when

$$-16(t^2 - 4t + 5) = 0$$

$$(t - 5)(t + 1) = 0$$

$$t = 5 \text{ (sec)}$$

At $t = 5$,

$$\frac{ds}{dt} = -32 \cdot 5 + 64 = -96 \text{ (ft/sec)}$$

5. Consider the function $f(x) = \dfrac{(x-2)^2}{x^2+4}$.

(a) Determine the domain of this function. All x.

(b) Determine the intervals on which $y = f(x)$ is increasing and the intervals on which it is decreasing.

$$f'(x) = \frac{2(x-2)(x^2+4) - (x-2)^2 \cdot 2x}{(x^2+4)^2} = \frac{2(x-2)\left(x^2+4 - x(x-2)\right)}{(x^2+4)^2}$$

$$= \frac{2(x-2)(2x+4)}{(x^2+4)^2} = \frac{4(x-2)(x+2)}{(x^2+4)^2} = 0 \text{ at } x = 2, -2$$

	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
f'	+	−	+
	↗	↘	↗

$(-2, 2)$ local max

$(2, 0)$ local min

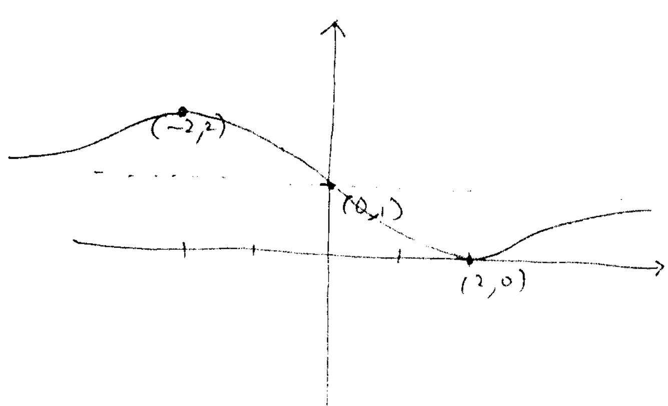
(c) Determine the horizontal asymptote of this function.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$$

$y = 1$

(d) Sketch the graph of $y = f(x)$ and determine the range of this function.

Intercepts: $(2, 0)$, $(0, 1)$



Range: $[0, 2]$

6. You are the head manager of a store that sells flat-screen TV's. A new TV called $L2$ is coming on to the market. You want to order a certain number of $L2$'s. You know that when you make the order, the company that makes $L2$'s will first charge you a flat sum of \$50,000 (as your share of marketing costs) and then \$1,000 for every $L2$ you order.

The market research department at your store tells you that you can sell 1000 $L2$'s for \$1500, and 500 $L2$'s for \$2000. You want to sell every $L2$ you order.

(a) Determine the demand function for $L2$, assuming it is linear. Identify the proper interval for the order size x.

x	p
1000	1500
500	2000

$$p - 1500 = -\frac{500}{500}(x - 1000) = -x + 1000$$

$$\underline{p = -x + 2500}$$

$$0 \le x \le 2500$$

(b) Express the store's profit P as a function of x.

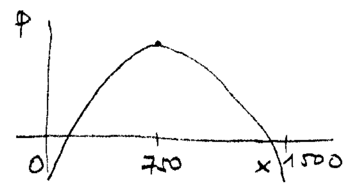
$$P = R - C = xp - C = x(-x + 2500) - 1000x - 50000$$

$$= \underline{-x^2 + 1500x - 50000}$$

(c) Compute the marginal profit and determine intervals on which P increases.

$$\frac{dP}{dx} = -2x + 1500 = 0 \quad \text{at} \quad x = 750$$

	$[0, 750)$	$(750, 1500]$
$\frac{dP}{dx}$	$+$	$-$
	$\nearrow$	$\searrow$



(c) To maximize the profit, what is the number of $L2$'s that your store should order and at what price should you be selling them?

Order size : $\underline{x = 750}$.

Price : $p = -750 + 2500 = \underline{1750}$.