Math 16A — 002, Fall 2009.
Nov. 20, 2009.

MIDTERM EXAM 2

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ______________________________

ID#: ______________________

Instructions: Each of the first four problems is worth 15 points, while problems 5 and 6 are each worth 20 points. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor in determining credit. Calculators, books or notes are not allowed.

Make sure that you have a total of 8 pages (including this one) with 6 problems. Read through the entire exam before beginning to work.

1	
2	
3	
4	
5	
6	
TOTAL	

1. Compute the derivatives of the following two functions. *Do not simplify!*

(a) $y = (2 + \sqrt{x})^7$

$y' = 7(2 + \sqrt{x})^6 \cdot \frac{1}{2} x^{-1/2}$

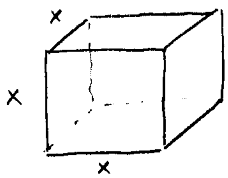
(7)

(b) $y = x^2 \cdot \sin(5x)$

$y' = 2x \cdot \sin(5x) + x^2 \cdot \cos(5x) \cdot 5$

(8)

2. All edges of a cube are expanding (increasing in length) at the same rate. When the volume of the cube is 8 m^3, the volume is increasing at the rate of 2 m^3/s. Find the rate of the expansion of the edges at this time.



$V = x^3$ (3) When $V = 8$, $x = 2$, (2)

$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$ (5)

Plug in $x = 2$, $\frac{dV}{dt} = 2$, to get:

$2 = 12 \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{6}$ (m/sec) (5)

3. Find the equation of the tangent line to the curve $x^3y^2 - y^3 + x^2 + 3 = 0$ at the point $(1, 2)$. (You may leave the equation in the point-slope form.)

$$3x^2y^2 + x^3 \cdot 2y\frac{dy}{dx} - 3y^2\frac{dy}{dx} + 2x = 0 \qquad (10)$$

Plug in $x = 1,\ y = 2$:

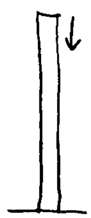
$$12 + 4\frac{dy}{dx} - 12\frac{dy}{dx} + 2 = 0$$

$$14 = 8\frac{dy}{dx} \qquad \frac{dy}{dx} = \frac{7}{4} \qquad (3)$$

slope

Line: $\underline{\underline{y - 2 = \frac{7}{4}(x-1)}}$ (2)

4. You are standing on top of a 96 ft tall tower. You throw a rock straight *down* with velocity 16 ft/sec. How fast is the rock traveling in the moment when it hits the ground? Assume the accelleration of the rock is constantly -32 ft/sec^2.



Position:

$$s = -16t^2 - 16t + 96 \qquad (5)$$

$$\frac{ds}{dt} = -32t - 16 \qquad (5)$$

Rock hits the ground:

$$-16t^2 - 16t + 96 = 0$$

$$t^2 + t - 6 = 0$$

$$(t+3)(t-2) = 0, \quad t = 2 \ (\text{sec}) \qquad (2)$$

Velocity then:

$$-32 \cdot 2 - 16 = \underline{\underline{-80}} \ (\text{ft/sec}) \qquad (2)$$

6

5. Consider the function $f(x) = \dfrac{x^2+1}{(x+1)^2}$.

(a) Determine the vertical asymptote of this function and the limits at the vertical asymptote.

$\underline{x = -1}$, $\quad \lim\limits_{x \to -1^+} f(x) = \dfrac{+\infty}{\underset{(\text{small} > 0)}{(\approx 2)}}$, $\quad \lim\limits_{x \to 1^-} f(x) = +\infty$ (5)

(b) Determine the intervals on which $y = f(x)$ is increasing and the intervals on which it is decreasing.

$$f'(x) = \frac{2x(x+1)^2 - (x^2+1)\cdot 2(x+1)}{(x+1)^4} = \frac{2(x+1)(x^2+x-x^2-1)}{(x+1)^4}$$

$$= \frac{2(x-1)}{(x+1)^3} \qquad \text{Critical nos: } x = -1, 1$$ (5)

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
sign of f'	+	−	+
	↗	↘	↗

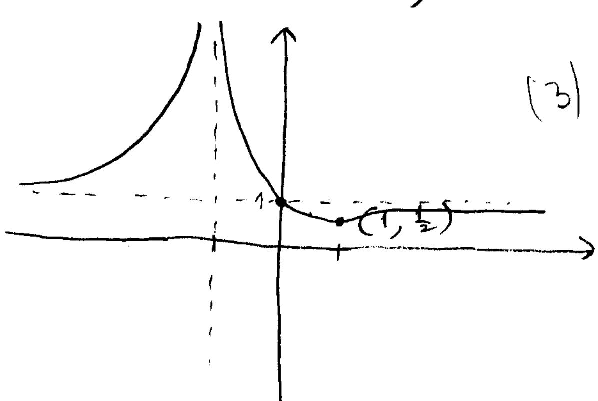
$(1, \frac{1}{2})$ local min (3)

(c) Determine the horizontal asymptote of this function.

$$\lim_{x \to \infty} \frac{x^2+1}{x^2+2x+1} = \lim_{x \to \infty} \frac{x^2}{x^2} = 1 \qquad \underline{y = 1}$$ (2)

(d) Sketch the graph of $y = f(x)$ and determine the range of this function.

y-intercepts $(0, 1)$; no x-intercepts.



(3)

Range: $[\frac{1}{2}, \infty)$, i.e. $y \geq \frac{1}{2}$. (2)

6. You own a coffee shop and one of the items you sell, by the pound, are coffee beans which go by the brand name Resurrection. Currently you sell 60 pounds per month of these, for \$10 per pound. You are told that every \$0.20 decrease in price, down to price 0, will increase the number of pounds sold by 4 pounds.

Each montly order of Resurrection has a fee of \$160 regardless of its size. Each pound ordered carries a price of \$4.

(a) Determine the monthly demand function for Resurrection, assuming it is linear. Identify the proper interval for the order size x.

x	p
60	10
64	9.8

$$p - 10 = \frac{-0.2}{4}(x-60)$$

(5)

$$p - 10 = -\frac{1}{20}x + 3$$

$$\underline{p = -\frac{1}{20}x + 13}, \quad \underline{0 \le x \le 260}$$

$$\left(p = 0 \text{ when } -\frac{1}{20}x + 13 = 0, \quad x = 260\right)$$

(b) Express your shop's montly profit P, as a function of x.

$$C = 160 + 4x, \quad R = x\left(-\frac{1}{20}x + 13\right) = -\frac{1}{20}x^2 + 13x$$

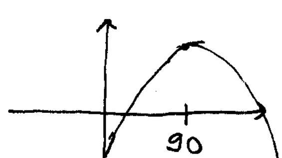
(5)

$$\underline{P = -\frac{1}{20}x^2 + 9x - 160}$$

(c) Compute the marginal profit and determine the intervals on which the profit P increases and decreases.

$$\frac{dP}{dx} = -\frac{1}{10}x + 9 = 0 \quad \text{when} \quad x = 90$$

(5)



	$(0, 90)$	$(90, 260)$
$\frac{dP}{dx}$	+	−
P	↗	↘

(d) Determine the sales level of Resurrection which maximizes the profit, and compute the price and the profit at this sales level.

$$\underline{x = 90}, \quad \underline{p} = -\frac{90}{20} + 13 = \underline{8.5},$$

(5)

$$P = 90 \cdot 8.5 - 160 - 4 \cdot 8.5$$

$$= 245 \ (\$)$$