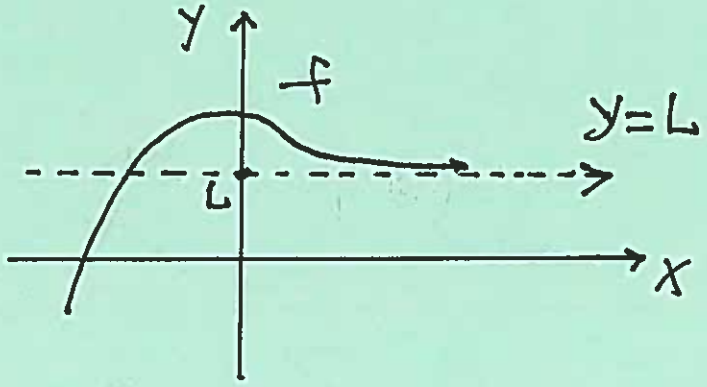
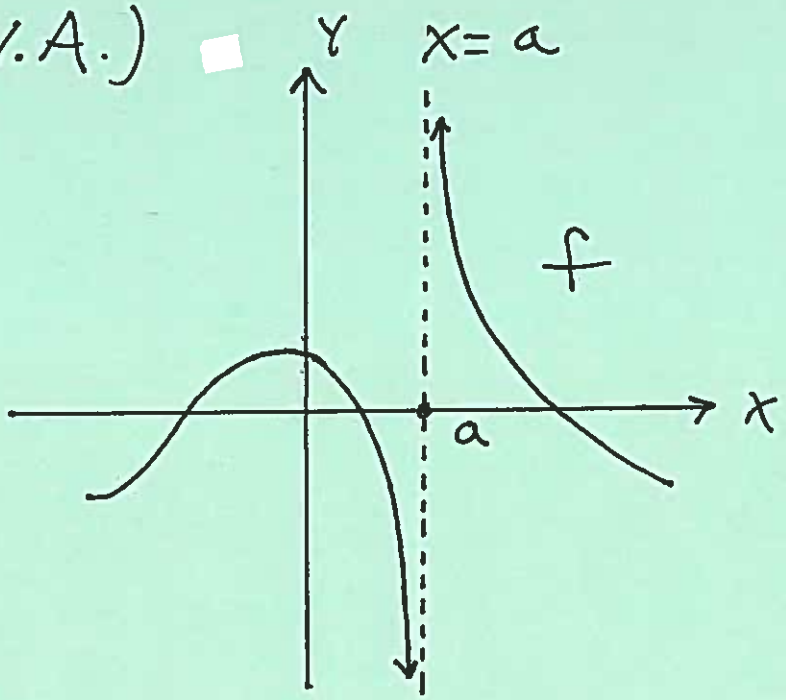
Asymptotes (Linear)

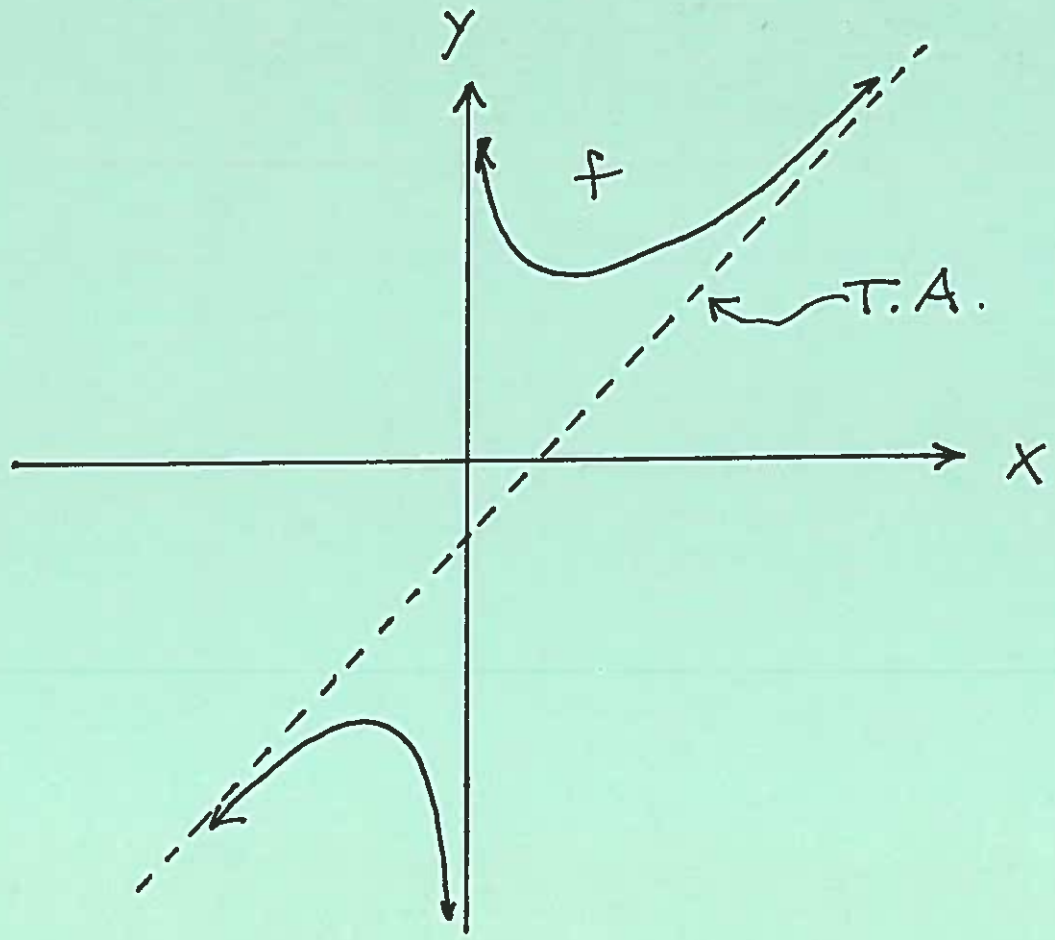
I.) If $\lim_{x \to \pm\infty} f(x) = L$ (a finite #), then the graph of f has a horizontal asymptote (H.A.) at $\boxed{y = L}$:



II.) If $\lim_{x \to a^{\pm}} f(x) = \pm\infty$, then the graph of f has a vertical asymptote (V.A.) at $\boxed{x = a}$:



III.) If $f(x) = \frac{\text{polynomial } A}{\text{polynomial } B}$ and (degree A) − (degree B) = 1, then the graph of f has a tilted/slant asymptote (T.A.), which can be found using POLYNOMIAL DIVISION.



Ex: Use limits to find all H.A.'s for each function.

1.) $f(x) = \dfrac{3x+4}{x-5}$

2.) $f(x) = \dfrac{x-x^2}{2x^2+1}$

(*) 3.) $f(x) = \dfrac{\sqrt{x^2+4}}{x+4}$

(*) 4.) $f(x) = \sqrt{x^2+4} - x$

Ex: Use limits to find all V.A.'s for each function.

1.) $f(x) = \dfrac{x+2}{x-1}$

2.) $f(x) = \dfrac{x^3}{x+3}$

3.) $f(x) = \dfrac{x}{x^2-4}$

4.) $f(x) = \dfrac{x^2+2x-3}{x^2+4x+3}$

Ex: Use polynomial division to find all T.A.'s for each function.

1.) $y = \dfrac{x^2-x+5}{x+1}$

2.) $y = \dfrac{3x^2+2x-4}{x-2}$

3.) $y = \dfrac{x^3-x^2+1}{x^2-1}$

4.) $y = \dfrac{x^5+3x^4}{x^4+1}$