

Math 16A

Section 2.2

Careful Reading Problems

Example: Find all points (x, y) on the graph of $y = x^3 - 3x^2$ with Horizontal Tangent Lines:

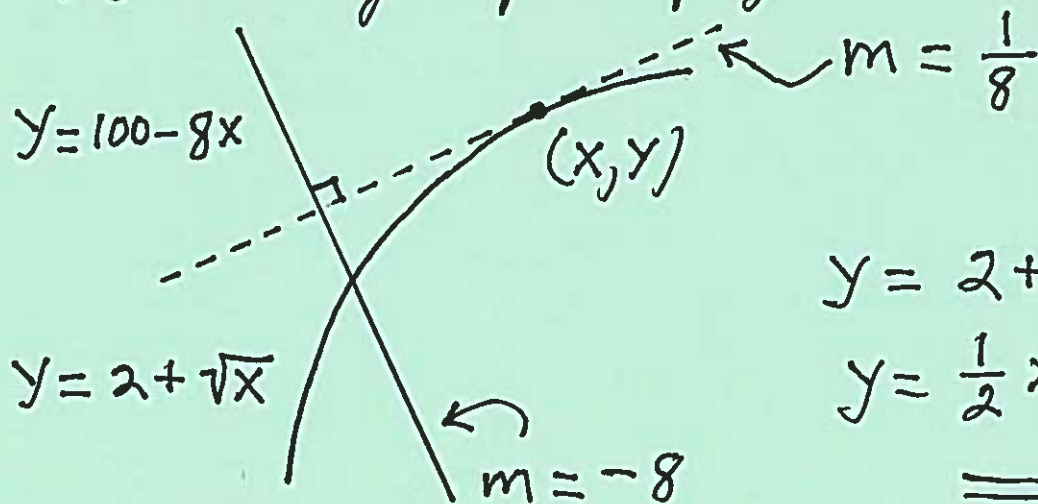
$$y = x^3 - 3x^2 \xrightarrow{D} y' = 3x^2 - 6x \\ = 3x(x-2) = 0 \rightarrow$$

$$x=0, y=0$$

and

$$x=2, y=-4$$

Example: Find all points (x, y) on the graph of $y = 2 + \sqrt{x}$ with Tangent Lines Perpendicular to the graph of $y = 100 - 8x$:



$$y = 2 + \sqrt{x} \xrightarrow{D} \\ y = \frac{1}{2} x^{-1/2} = \frac{1}{8}$$

$$\rightarrow \frac{1}{\sqrt{x}} = \frac{1}{4} \rightarrow \sqrt{x} = 4 \rightarrow$$

$$\boxed{x = 16, y = 2 + \sqrt{16} = 6}$$

Example: Consider the graphs of $f(x) = -x^2$ and $g(x) = \frac{1}{x^2}$. Find all x -values where the SLOPES of the graphs of f and g are equal at the same x -value:

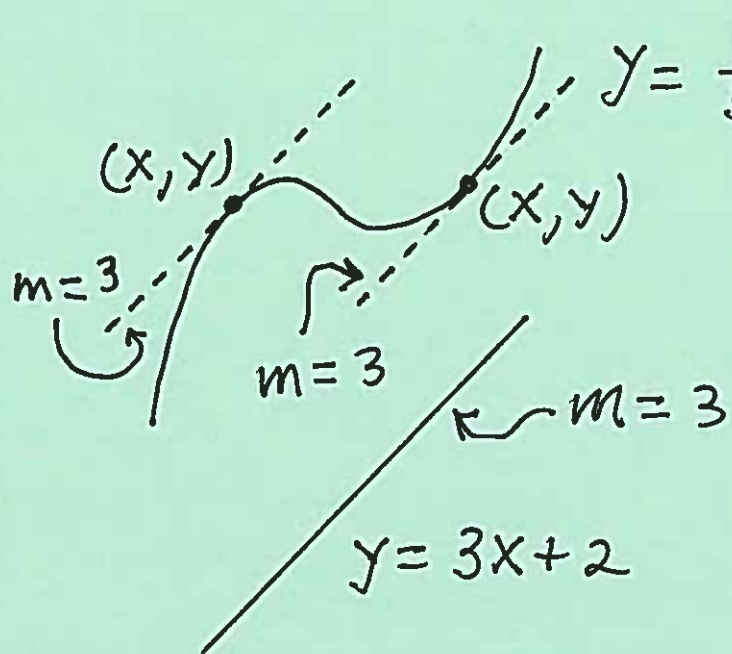
$$\left. \begin{array}{l} f(x) = -x^2 \xrightarrow{D} f'(x) = -2x \\ g(x) = x^{-2} \xrightarrow{D} g'(x) = -2x^{-3} \end{array} \right\} \begin{array}{l} \text{SET} \\ \text{EQUAL} \end{array}$$

$$\rightarrow -2x = -2x^{-3} \rightarrow x = \frac{1}{x^3} \rightarrow x^4 = 1 \rightarrow$$

$$x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1) = 0$$

$$\rightarrow \boxed{x = 1, x = -1}$$

Example: Find all points (x, y) on the graph of $y = \frac{1}{3}x^3 - x^2$ with Tangent Lines Parallel to the line $y = 3x + 2$:



$$y = \frac{1}{3}x^3 - x^2 \xrightarrow{D}$$

$$y' = \underline{\underline{x^2 - 2x = 3}} \rightarrow$$

$$x^2 - 2x - 3 = 0 \rightarrow$$

$$(x-3)(x+1) = 0 \rightarrow$$

$$\boxed{x=3, y=0},$$

$$\boxed{x=-1, y=-\frac{4}{3}}$$