Math 16A
Kouba
The Derivative of Sin x

FACTS: 1.) $\lim\limits_{w \to 0} \dfrac{\sin w}{w} = 1$

2.) $\lim\limits_{w \to 0} \dfrac{\cos w - 1}{w} = 0$

3.) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

Let $f(x) = \sin x$. Then $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$

$$= \lim\limits_{h \to 0} \dfrac{\sin(x + h) - \sin x}{h}$$

$$= \lim\limits_{h \to 0} \dfrac{\sin x \cdot \cos h + \cos x \cdot \sin h - \sin x}{h}$$

$$= \lim\limits_{h \to 0} \dfrac{\sin x \cdot (\cos h - 1) + \cos x \cdot \sin h}{h}$$

$$= \lim\limits_{h \to 0} \left\{ \sin x \cdot \left(\dfrac{\cos h - 1}{h}\right) + \cos x \cdot \left(\dfrac{\sin h}{h}\right) \right\}$$

$$= \sin x \cdot (0) + \cos x \cdot (1)$$

$$= \cos x$$