Math 16A (Fall 2020)
Kouba
Exam 2

Printing and signing your name below is a verification that no other person assisted you in the completion of this Exam.

PRINT your name ____________ SIGN your name KEY

Show clear, organized supporting work for your answers. Correct answers without supporting work may not receive full credit. Use of unapproved shortcuts may not receive full credit. There are 13 pages. You must submit exactly 12 pages to Gradescope.

1.) Differentiate each of the following four functions. DO NOT SIMPLIFY ANSWERS.

a.) (7 pts.) $y = x^3 - 5x^{-7} + 9^{100}$

$\xrightarrow{D} y' = 3x^2 + 35x^{-8} + 0$

b.) (7 pts.) $f(x) = (2x+3) \cdot (4x - 3 + x^2)$

$\xrightarrow{D} f'(x) = (2x+3)(4+2x) + (2)(4x-3+x^2)$

c.) (7 pts.) $f(x) = \dfrac{x^5 + 2x}{5x^4 - 8}$ $\xrightarrow{D}$

$$f'(x) = \frac{(5x^4-8)(5x^4+2)-(x^5+2x)(20x^3)}{(5x^4-8)^2}$$

d.) (7 pts.) $f(x) = \dfrac{x \sec x}{4 + x^2}$ $\xrightarrow{D}$

$$f'(x) = \frac{(4+x^2)\,[x \cdot \sec x \tan x + (1)\sec x] - x \sec x\,(2x)}{(4+x^2)^2}$$

2.) (10 pts.) Solve $f'(x) = 0$ for x, where $f(x) = 3x^4 + 4x^3 - 12x^2$, and set up a Sign Chart for f'.

$$\xrightarrow{D} f'(x) = 12x^3 + 12x^2 - 24x$$

$$= 12x(x^2 + x - 2)$$

$$= 12x(x-1)(x+2) = 0$$

$$\longrightarrow \quad x = 0,\ x = 1,\ x = -2$$

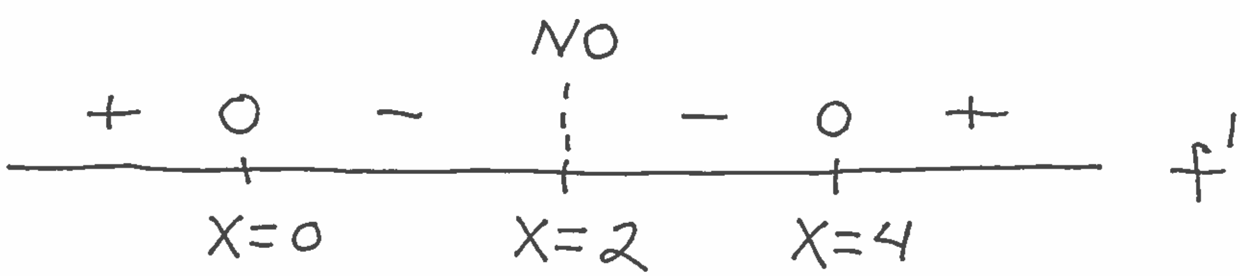
$-$	0	$+$	0	$-$	0	$+$	f'
	$x=-2$		$x=0$		$x=1$		

3.) (10 pts.) Solve $f'(x) = 0$ for x, where $f(x) = \dfrac{x^2}{x-2}$, and set up a Sign Chart for f'.

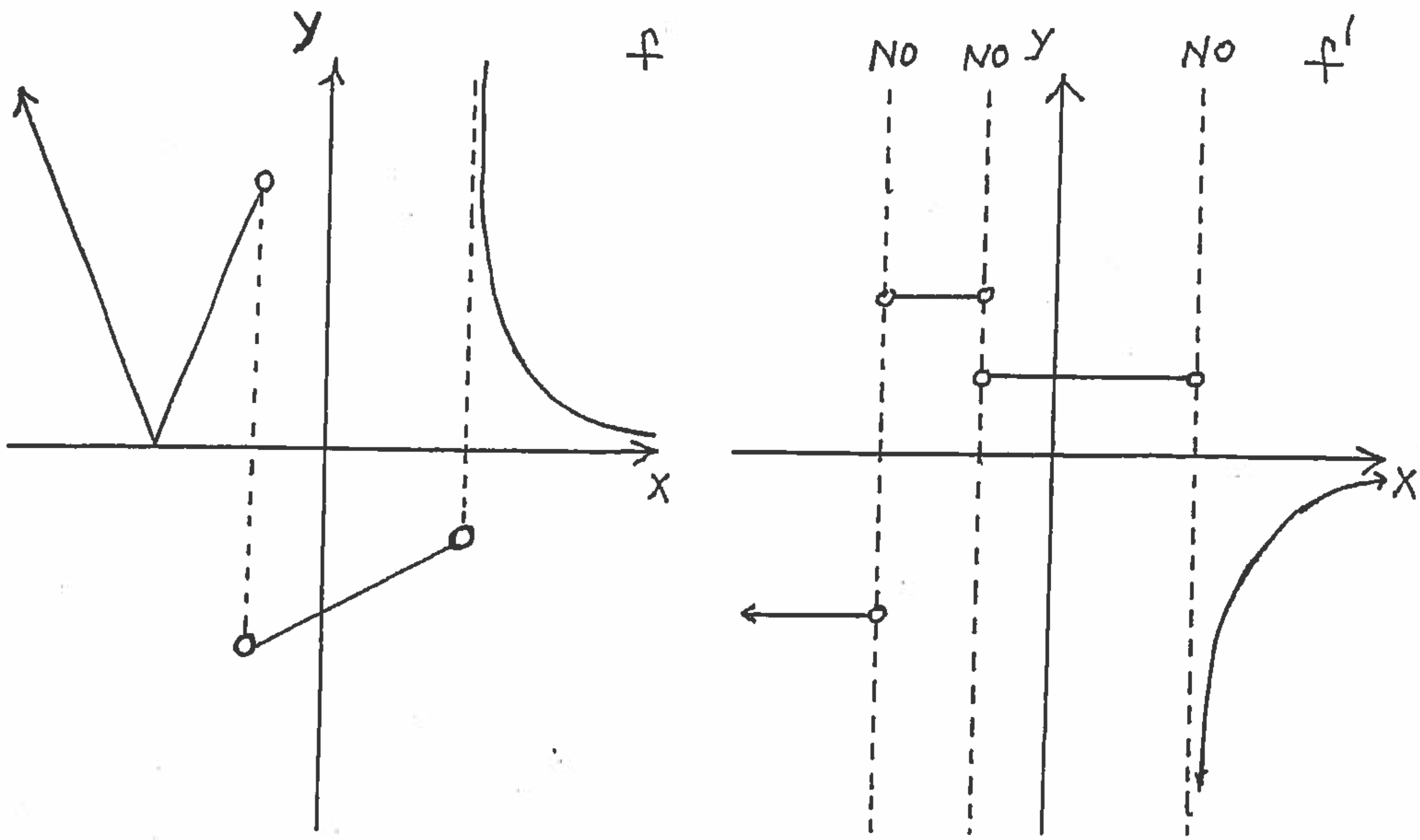
$$\xrightarrow{D} f'(x) = \frac{(x-2)(2x) - x^2(1)}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2}{(x-2)^2} = \frac{x^2 - 4x}{(x-2)^2}$$

$$= \frac{x(x-4)}{(x-2)^2} = 0 \longrightarrow x = 0, \; x = 4$$



4.) (8 pts.) Sketch a graph of the derivative f' using the given graph of f.



5.) (9 pts.) Use $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to differentiate $f(x) = 2 + \sqrt{x+3}$.

$$f'(x) = \lim_{h \to 0} \frac{\cancel{2} + \sqrt{x+h+3} - (\cancel{2} + \sqrt{x+3})}{h}$$

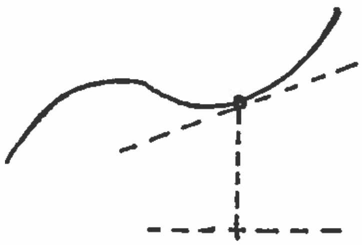
$$= \lim_{h \to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}}$$

$$= \lim_{h \to 0} \frac{(\cancel{x}+h+\cancel{3}) - (\cancel{x}+\cancel{3})}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \frac{1}{\sqrt{x+0+3} + \sqrt{x+3}} = \frac{1}{2\sqrt{x+3}}$$

6.) (9 pts.) Find an equation of the line *tangent* to the graph of $f(x) = x^2(x-1)$ at $x = 2$.



$f(x) = x^2(x-1) = x^3 - x^2 \xrightarrow{D}$

$f'(x) = 3x^2 - 2x$, then SLOPE

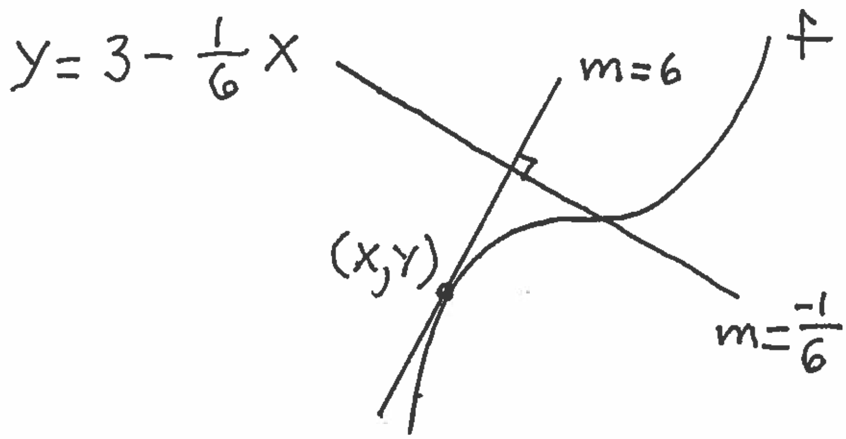
$m = f'(2) = 3(2)^2 - 2(2) = 8$;

if $X = 2$, $Y = f(2) = (2)^3 - (2)^2 = 4$;

tangent line is $Y - 4 = 8(X-2)$

or $Y = 8X - 12$

7.) (9 pts.) Find all points (x, y) on the graph of $f(x) = 2x^3 + 1$ with *tangent lines perpendicular* to the line $y = 3 - (1/6)x$.



$$\xrightarrow{D} f'(x) = \boxed{6x^2 = 6}$$

$$\longrightarrow 6x^2 - 6 = 0$$

$$\longrightarrow 6(x-1)(x+1) = 0$$

$x = 1$ OR $x = -1$

$Y = 3$ $\quad$ $Y = -1$

8.) The amount of money M (in dollars) in your little sister's savings account after t months is given by $M = 3t^2 - 5t + 1000$.

a.) (4 points) How much is in your sister's account when i.) $t = 1$ month ? ii.) $t = 10$ months ?

$$t=1 : M = 3(1)^2 - 5(1) + 1000 = \$998 \ ;$$

$$t=10 : M = 3(10)^2 - 5(10) + 1000 = \$1250$$

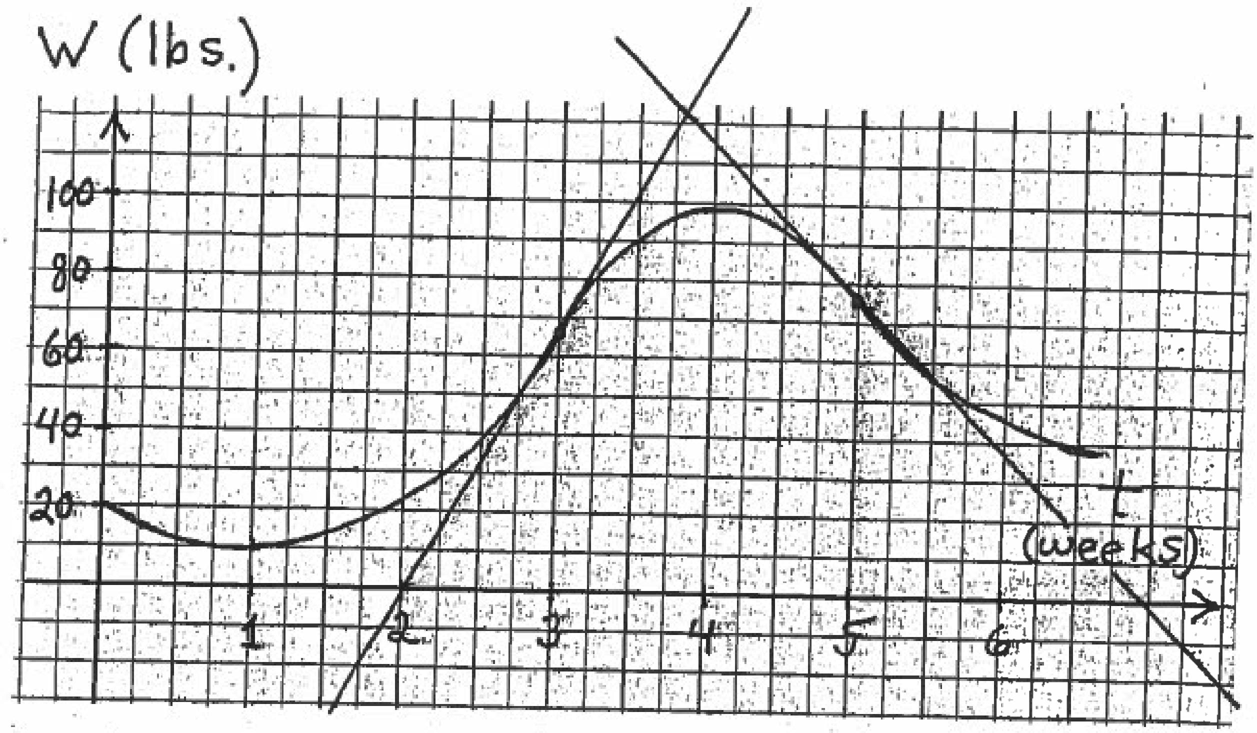
b.) (6 points) At what rate is the amount of money in your sister's account changing when $t = 6$ months ?

Rate is $\frac{dM}{dt} = D(3t^2 - 5t + 1000)$

$= 6t - 5 \ ;$

$t=6 : \frac{dM}{dt} = 6(6) - 5 = \$31/\text{month}$

9.) The given graph represents the weight W (in pounds) of a newborn calf during the first 6 weeks of its life.



a.) (4 pts.) Estimate the average rate of weight change (pounds/week) for $t = 1$ to $t = 4$ weeks.

$$ARC = \frac{W(4) - W(1)}{4-1} \approx \frac{98-10}{3} = 29\tfrac{1}{3} \ \frac{\text{lbs.}}{\text{wk.}}$$

b.) (4 pts.) Estimate the instantaneous rate of weight change (pounds/week) when $t = 5$ weeks.

$$IRC = W'(5) \approx \frac{\text{rise}}{\text{run}} = \frac{-75}{2} = -37.5 \ \frac{\text{lbs.}}{\text{wk.}}$$

c.) (3 pts. each) i.) Estimate the specific time t at which the rate of weight change is largest. ii.) Estimate the value of this rate.

i.) $t = 3$ wks.

ii.) $IRC = W'(3) \approx \frac{\text{rise}}{\text{run}} = \frac{102}{1.5} = 68 \ \frac{\text{lbs.}}{\text{wk.}}$

10.) (9 pts.) Solve $y' = 0$ for θ, $0 \le \theta \le 2\pi$.

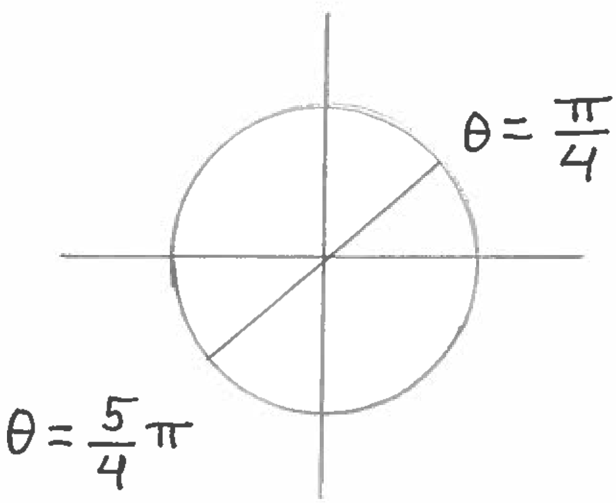
$y = \sin\theta\cos\theta$

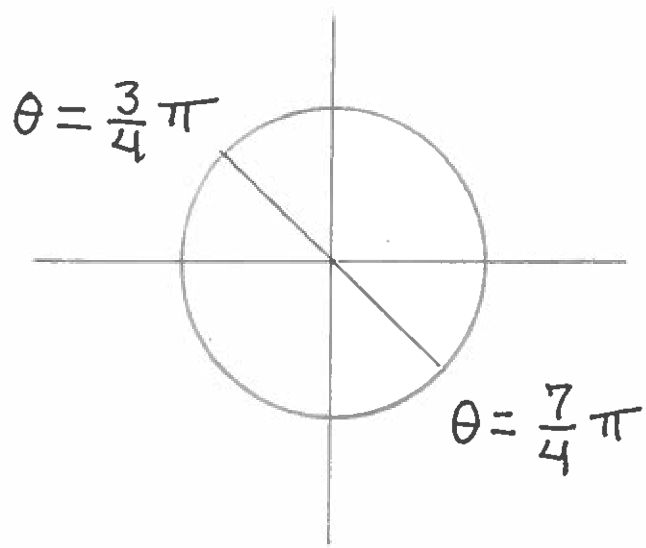
$$\xrightarrow{D} \quad y' = \sin\theta\cdot(-\sin\theta) + (\cos\theta)\cdot\cos\theta$$

$$= \cos^2\theta - \sin^2\theta$$

$$= (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$$

$$\cos\theta = \sin\theta \qquad\qquad \cos\theta = -\sin\theta$$





$$\theta = \frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$$

11.) (9 pts.) Find equation(s) for all line(s) which are simultaneously perpendicular to the graphs of $y = (-1/4)x + 3$ and $y = \frac{x}{x-1}$, or explain why this is impossible.

Slope of $y = \frac{-1}{4}x + 3$ is $m = \frac{-1}{4}$,

so $\perp$ slope is $\boxed{m = 4}$; and

$$y = \frac{x}{x-1} \xrightarrow{D} y' = \frac{(x-1)(1) - x(1)}{(x-1)^2}$$

$$= \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2} \text{ is}$$

slope of tangent line $\rightarrow$

$$\boxed{\frac{-1}{(x-1)^2} = \frac{-1}{4}} \longrightarrow (x-1)^2 = 4 \rightarrow$$

$$x = 3,\ y = \frac{3}{2} \xrightarrow{\perp \text{ line}} y - \frac{3}{2} = 4(x-3)$$

or

$$x = -1,\ y = \frac{1}{2} \xrightarrow{\perp \text{ line}} y - \frac{1}{2} = 4(x+1)$$