Math 16A (Fall 2020)
Kouba
Exam 3

Printing and signing your name below is a verification that no other person assisted you in the completion of this Exam.

PRINT your name ______________ SIGN your name KEY

Show clear, organized supporting work for your answers. Correct answers without supporting work may not receive full credit. Use of unapproved shortcuts may not receive full credit. There are 11 pages. You must submit exactly 11 pages to Gradescope.

1.) (8 pts. each) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = (x - x^2)(x^3 + 1)^4$ $\xrightarrow{D}$

$$y' = (x - x^2) \cdot 4(x^3+1)^3 \cdot 3x^2 + (1 - 2x) \cdot (x^3+1)^4$$

b.) $y = \tan^4(\cos(\sec x))$

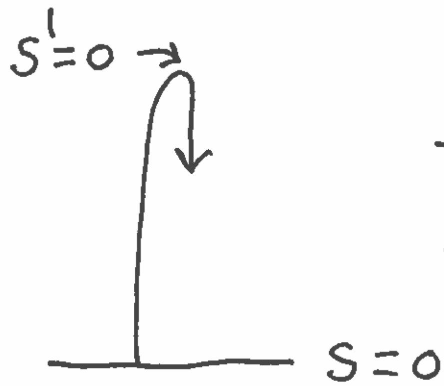
(layers: $(\ )^4$, tan, cos, sec) $\xrightarrow{D}$

$$y' = 4\tan^3(\cos(\sec x)) \cdot \sec^2(\cos(\sec x)) \cdot -\sin(\sec x) \cdot \sec x \tan x$$

2.) (14 pts.) A Fuji apple was projected straight up from ground level and reached a highest point of 144 feet. (Gravity Problem– Start with the Gravity Equation and derive the necessary equations. You may not receive credit for using shortcuts from physics.)

a.) For how many seconds was the apple in the air before striking the ground ?

b.) What was the apple's initial velocity ?



$$s = -16t^2 + v_0 t + s_0$$

$s_0 = 0$

$\rightarrow s = \boxed{-16t^2 + v_0 t = 144}$

$\rightarrow s' = -32t + v_0 = 0 \rightarrow$

$-32t + v_0 = 0 \rightarrow$

$\boxed{32t = v_0} \rightarrow$

$-16t^2 + (32t)t = 144 \rightarrow$

$-16t^2 + 32t^2 = 144 \rightarrow$

$16t^2 = 144 \rightarrow t^2 = 9 \rightarrow t = 3$ sec.

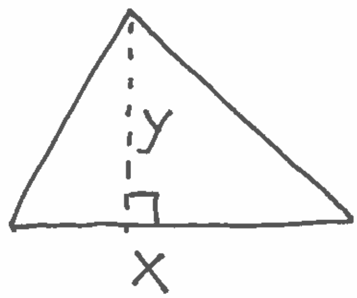
$\rightarrow \boxed{v_0 = 96 \text{ ft./sec.}}$; then

$s = -16t^2 + 96t = 0$ (strike ground)

$\rightarrow -16t(t-6) = 0$

$\rightarrow \boxed{t = 6 \text{ sec.}}$

3.) (12 pts.) The base x of a triangle is decreasing at the rate of 5 in./sec. and the height y of the triangle is increasing at the rate of 4 in./sec. At what rate is the area A of the triangle changing when $x = 4$ inches and $y = 3$ inches ?



Given: $\frac{dx}{dt} = -5$ in./sec.,

$\frac{dy}{dt} = 4$ in./sec.

Find $\frac{dA}{dt}$ when $x = 4$, $y = 3$:

$$A = \frac{1}{2}xy \xrightarrow{D}$$

$$\frac{dA}{dt} = \frac{1}{2}x \cdot \frac{dy}{dt} + \frac{1}{2}\frac{dx}{dt} \cdot y$$

$$= \frac{1}{2}(4)(4) + \frac{1}{2}(-5)(3)$$

$$= \frac{16}{2} - \frac{15}{2}$$

$$= \frac{1}{2} \text{ in.}^2/\text{sec}.$$

4.) Assume that y is a function of x and $y^2 - y = 2 - x^2$.

a.) (6 pts.) Determine the slope of the graph at the point $(0, 2)$.

$$\xrightarrow{D} 2YY' - Y' = -2X \rightarrow Y'(2Y-1) = -2X$$

$$\rightarrow \boxed{Y' = \frac{-2X}{2Y-1}} \text{ and } X=0, Y=2 \rightarrow$$

$$\text{SLOPE } Y' = \frac{-2(0)}{2(2)-1} = \frac{0}{3} = 0$$

so --------

b.) (6 pts.) Determine the concavity of the graph at the point $(0, 2)$.

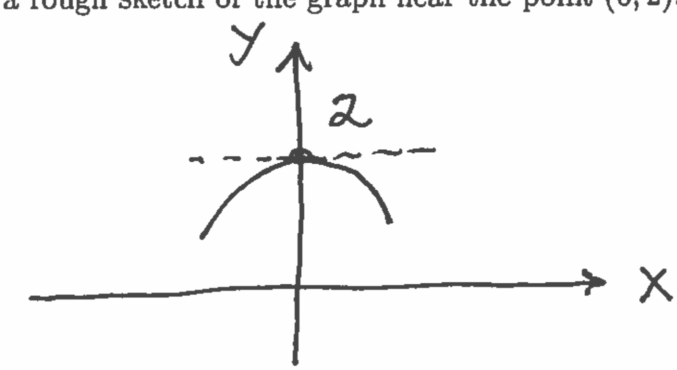
$$\xrightarrow{D} Y'' = \frac{(2Y-1)(-2)-(-2X)(2Y')}{(2Y-1)^2}$$

and $X=0,\ Y=2,\ Y'=0 \longrightarrow$

$$Y'' = \frac{(3)(-2)-(0)(0)}{(3)^2} = \frac{-6}{9} = \frac{-2}{3}$$

so $(\cap)$

c.) (4 pts.) Draw a rough sketch of the graph near the point $(0, 2)$.



5.) (11 pts.) Assume that y is a function of x. Determine $y' = \dfrac{dy}{dx}$.

$(2x + y)^{10} = 5x^2 - y^3 \xrightarrow{D}$

$$10(2x+y)^9 \cdot (2+y') = 10x - 3y^2y' \longrightarrow$$

$$20(2x+y)^9 + 10(2x+y)^9 y' = 10x - 3y^2y' \longrightarrow$$

$$3y^2y' + 10(2x+y)^9 y' = 10x - 20(2x+y)^9 \longrightarrow$$

$$y'\left[3y^2 + 10(2x+y)^9\right] = 10x - 20(2x+y)^9 \longrightarrow$$

$$y' = \frac{10x - 20(2x+y)^9}{3y^2 + 10(2x+y)^9}$$

6.) (10 pts.) The surface S of a sphere is *increasing* at the rate of 64π $cm.^2/min.$ At what rate is the the volume V changing when the radius $r = 4$ $cm.$? Assume that the Surface Area of a sphere is $S = 4\pi r^2$ and the Volume of a sphere is $V = (4/3)\pi r^3$.

Given: $\frac{dS}{dt} = 64\pi$ $cm.^2/min.$

Find: $\frac{dV}{dt}$ when $r = 4$ cm.

$$S = 4\pi r^2 \xrightarrow{D}$$

$$\frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8\pi(4)\frac{dr}{dt} = 32\pi \frac{dr}{dt} = 64\pi$$

$\rightarrow$ $\frac{dr}{dt} = 2$ cm./min; then

$$V = \frac{4}{3}\pi r^3 \xrightarrow{D} \frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \cdot \frac{dr}{dt}$$

$$= 4\pi(4)^2(2) = 128\pi \quad cm.^3/min.$$

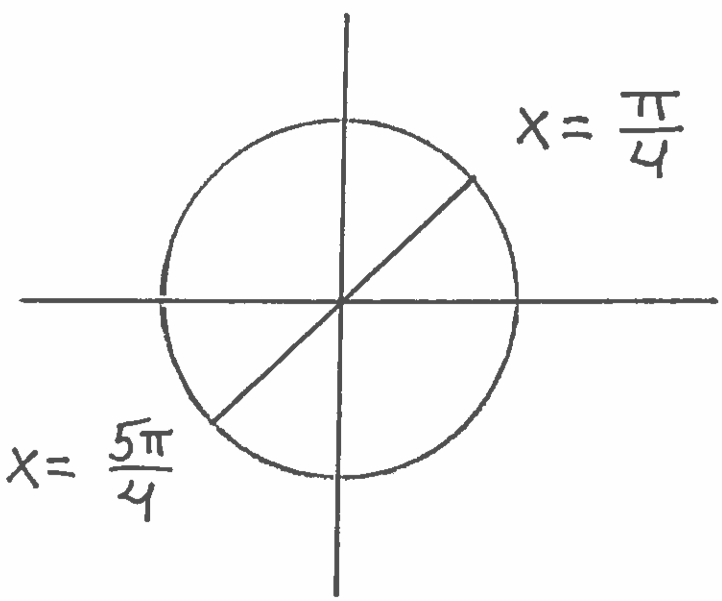
7.) (10 pts.) Consider the function $y = \cos x - \sin x$ on the interval $[0, 2\pi]$. Determine all inflection points (x, y) for the graph of this function. You need not graph the function.

$$\xrightarrow{D} \quad y' = -\sin x - \cos x$$

$$\xrightarrow{D} \quad y'' = -\cos x - (-\sin x)$$

$$= \sin x - \cos x = 0 \longrightarrow$$

$$\sin x = \cos x$$



$-$ 0 $+$ 0 $-$ y''

$x = \frac{\pi}{4}$, $y = 0$ $x = \frac{5\pi}{4}$, $y = 0$

Infl. Pts.

8.) (10 pts.) Consider function $y = \dfrac{4x}{x^2+3}$. Determine all relative and absolute maxima and minima (x, y) for the graph of this function. You need not graph the function.

$$y = \frac{4x}{x^2+3} \xrightarrow{D}$$

$$y' = \frac{(x^2+3)(4) - 4x(2x)}{(x^2+3)^2}$$

$$= \frac{4x^2 + 12 - 8x^2}{(x^2+3)^2}$$

$$= \frac{12 - 4x^2}{(x^2+3)^2} = \frac{4(3-x^2)}{(x^2+3)^2} = 0$$

$$\begin{array}{ccccc} - & 0 & + & 0 & - \quad y' \\ & x = -\sqrt{3} & & x = \sqrt{3} & \end{array}$$

$$\underbrace{y = \frac{-2}{3}\sqrt{3}}_{\text{ABS. MIN.}} \qquad \underbrace{Y = \frac{2}{3}\sqrt{3}}_{\text{ABS. MAX.}}$$

since $y > 0$ for $x > 0$,
$y < 0$ for $x < 0$

9.) (16 pts.) Consider the following function f. Determine the domain, all absolute and relative maximum and minimum values, inflection points, and x- and y-intercepts. State clearly the x-values for which f is increasing (↑), decreasing (↓), concave up (⋃), and concave down (⋂). Determine all possible asymptotes. Neatly sketch the graph of f. More work space and a set of axes is continued on the next page.

$$f(x) = \frac{x^2}{x-2}$$

$$f'(x) = \frac{x^2-4x}{(x-2)^2}$$

$$f''(x) = \frac{8}{(x-2)^3}$$

Domain : all $X \neq 2$

$$f'(X) = \frac{X(X-4)}{(X-2)^2} = 0$$

+	0	−	NO	−	0	+	f'
	$X=0$, $Y=0$		$X=2$		$X=4$, $Y=8$		

REL. MAX. $\left\{ \begin{array}{l} X=0 \\ Y=0 \end{array} \right.$

$X=2$

$\left. \begin{array}{l} X=4 \\ Y=8 \end{array} \right\}$ REL. MIN

−	NO	+	f''
	$X=2$		

$X=0$: $Y=0$,

$Y=0$: $\dfrac{X^2}{X-2} = 0 \rightarrow X=0$;

$$\lim_{X \to \pm\infty} \frac{X^2}{X-2} \cdot \frac{1/X}{1/X} = \lim_{X \to \pm\infty} \frac{X}{1-\frac{2}{X}} = \frac{\pm\infty}{1-0} = \pm\infty$$

so NO H.A. ;

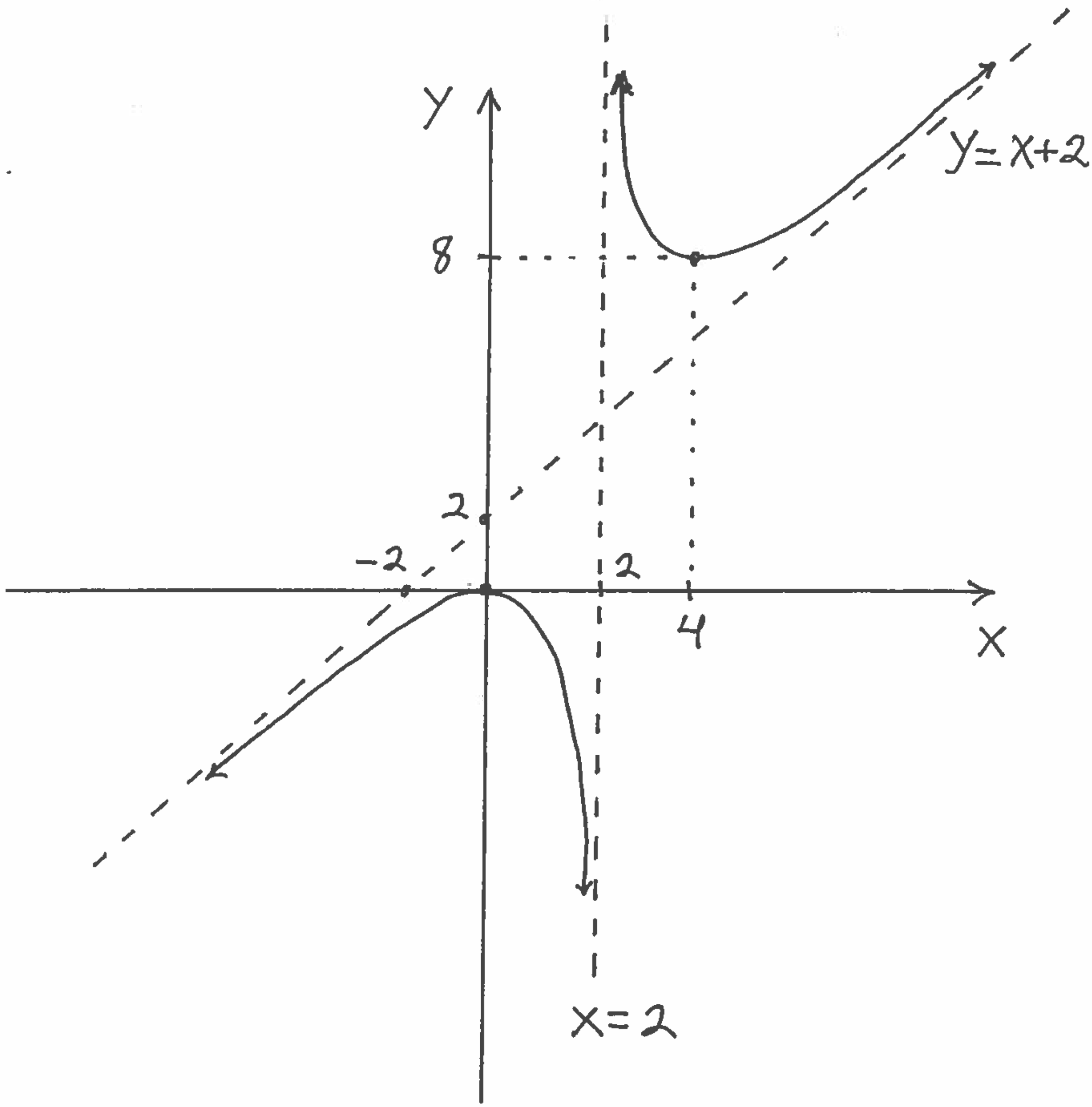
$$\lim_{X \to 2^+} \frac{X^2}{X-2} = \frac{4}{0^+} = +\infty, \quad \lim_{X \to 2^-} \frac{X^2}{X-2} = \frac{4}{0^-} = -\infty,$$

so V.A. is $\boxed{X=2}$;

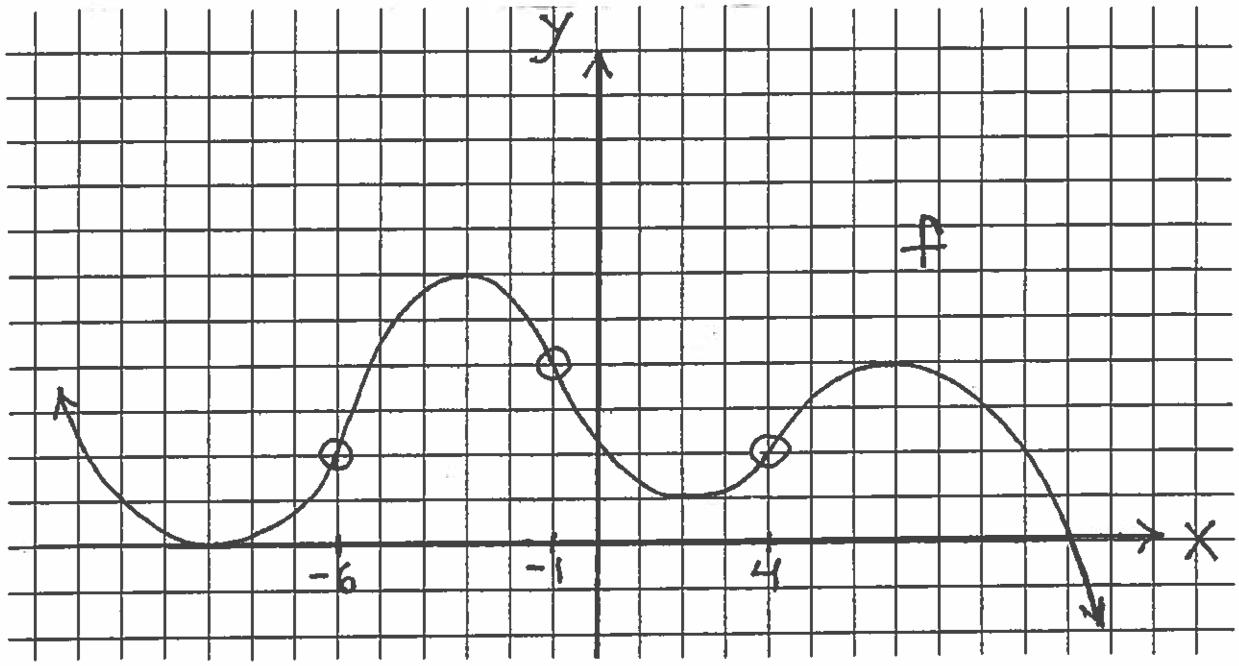
$$\begin{array}{r} x+2 \\ x-2 \overline{) x^2 \qquad} \\ -(x^2-2x) \\ \hline 2x \\ -(2x-4) \\ \hline 4 \end{array}$$

so T.A. is $\boxed{y = x+2}$;

f is (↑) for $x<0$, $x>4$; f is (↓) for $0<x<2$, $2<x<4$;
f is (∪) for $x>2$, f is (∩) for $x<2$;



10.) (10 pts.) The graph of function f is given below. Sketch a possible graph for its second derivative f''.



+ 0 − 0 + 0 − f''

(∪) x=−6 (∩) x=−1 (∪) x=4 (∩)