

Math 16A

Kouba

Graphing Using First and Second Derivatives

1. If f' is +, then f is increasing (\uparrow).
2. If f' is -, then f is decreasing (\downarrow).
3. If f'' is + (means f' is \uparrow), then f is concave up (\cup).
4. If f'' is - (means f' is \downarrow), then f is concave down (\cap).

$$\begin{array}{c} + \quad 0 \quad - \\ \hline x=a \\ \underbrace{\hspace{2cm}} \end{array} f'$$

relative (or absolute)
maximum

$$\begin{array}{c} - \quad 0 \quad + \\ \hline x=a \\ \underbrace{\hspace{2cm}} \end{array} f'$$

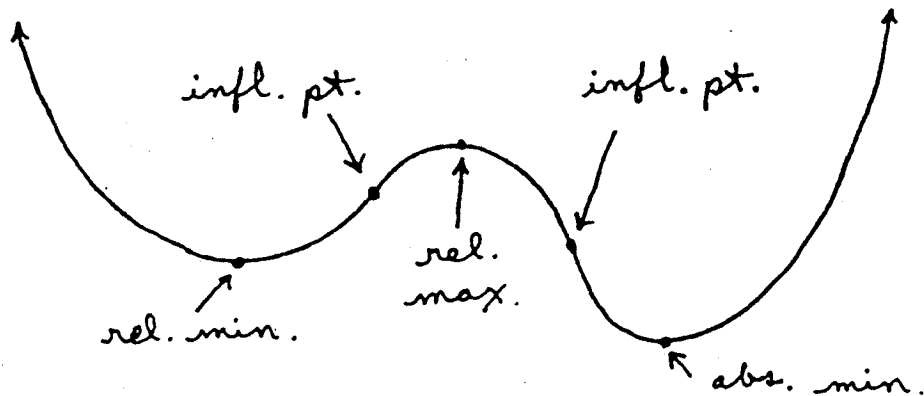
relative (or absolute)
minimum

$$\begin{array}{c} + \quad 0 \quad - \\ \hline x=a \\ \underbrace{\hspace{2cm}} \end{array} f''$$

inflection point

$$\begin{array}{c} - \quad 0 \quad + \\ \hline x=a \\ \underbrace{\hspace{2cm}} \end{array} f''$$

inflection point



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Graphing

For each of the following functions begin by finding the domain of the function. Determine all relative and absolute maximum and minimum values and inflection points. State clearly the intervals on which the function is increasing (\uparrow), decreasing (\downarrow), concave up (\cup), and concave down (\cap). Determine all vertical and horizontal asymptotes (when appropriate) and x- and y-intercepts. Neatly sketch the graph.

Example 1: $f(x) = (x-1)^3(x-5)$
 $f'(x) = (x-1)^3 \cdot (1) + 3(x-1)^2 \cdot (x-5)$
 $= (x-1)^2 [(x-1) + 3(x-5)]$
 $= (x-1)^2 [4x-16] = 0$

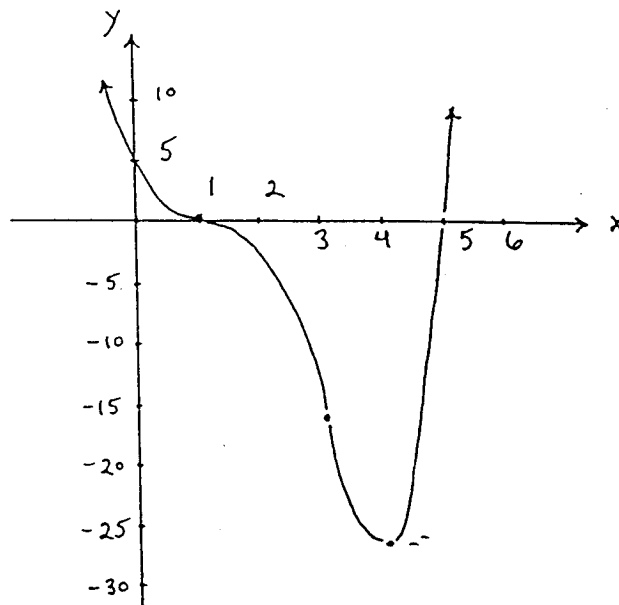
domain: all x-values

	-	0	-	0	+	Y'
?		$x=1$		$x=4$		abs.
		$y=0$		$y=-27$		min.

$f''(x) = (x-1)^2(4) + 2(x-1)[4x-16]$
 $= 4(x-1)[(x-1) + 2(x-4)]$
 $= 4(x-1)[3x-9] = 0$

	+	0	-	0	+	Y''
inf. pt.		$x=1$		$x=3$		inf. pt.
		$y=0$		$y=-16$		

f is \uparrow for $x > 4$,
 f is \downarrow for $x < 4$,
 f is \cup for $x < 1, x > 3$,
 f is \cap for $1 < x < 3$,
 $Y=0: x=1, x=5$
 $X=0: Y=5$



Example 2: $Y = 3x^{2/3} - 2x$

domain: all x-values

$Y' = 3 \cdot \frac{2}{3} x^{-1/3} - 2 = 2x^{-1/3} - 2$
 $= 2 \left(\frac{1}{x^{1/3}} - 1 \right) = 2 \left(\frac{1-x^{1/3}}{x^{1/3}} \right) = 0$

	-	+	0	-	Y'
rel. min.		$x=0$		$x=1$	rel. max.
		$y=0$		$y=1$	

$Y'' = 2 \cdot \frac{-1}{3} x^{-4/3} = \frac{-2}{3x^{4/3}}$

	-	-	-	-	Y''
		$x=0$			
		$y=0$			

Y is \uparrow for $0 < x < 1$,
 Y is \downarrow for $x < 0, x > 1$,
 Y is \cap for $x < 0, x > 0$

$$x=0: Y=0$$

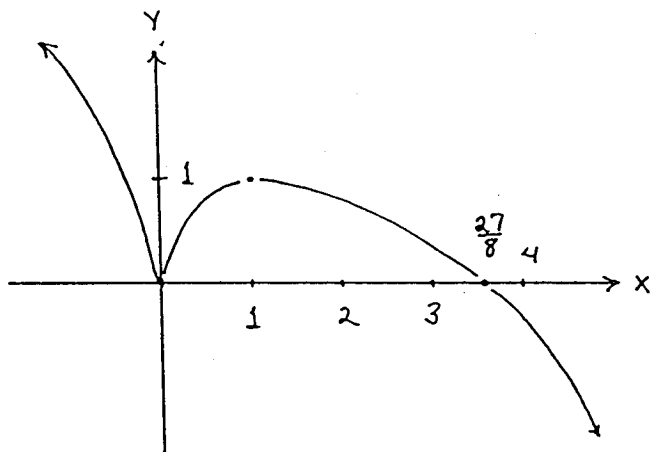
$$Y=0: 3x^{2/3} - 2x = 0$$

$$\rightarrow x^{2/3} (3 - 2x^{1/3}) = 0$$

\downarrow

\downarrow

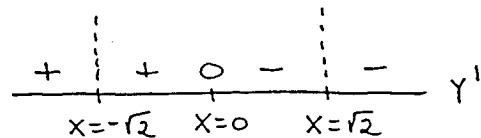
$$x=0 \quad x = \frac{27}{8}$$



Example 3: $Y = \frac{x^2+1}{x^2-2}$

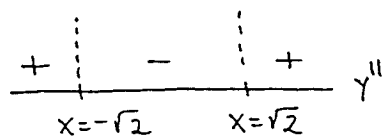
domain: all $x \neq \pm\sqrt{2}$

$$Y' = \frac{(x^2-2)(2x) - (x^2+1)(2x)}{(x^2-2)^2} = \frac{-6x}{(x^2-2)^2} = 0$$



$$Y'' = \frac{(x^2-2)^2(-6) - (-6x) \cdot 2(x^2-2) \cdot 2x}{(x^2-2)^4}$$

$$= \frac{-6(x^2-2)[(x^2-2) - 4x^2]}{(x^2-2)^4} = \frac{6(2+3x^2)}{(x^2-2)^3} = 0$$



Y is \uparrow for $x < -\sqrt{2}, -\sqrt{2} < x < 0$,

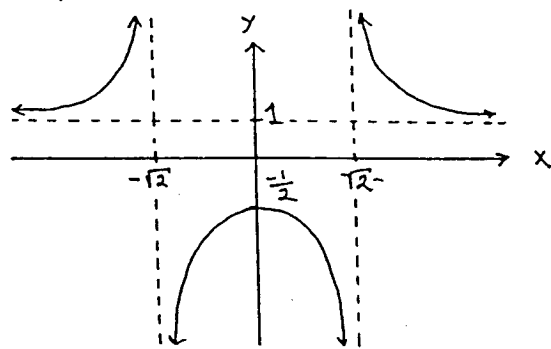
Y is \downarrow for $0 < x < \sqrt{2}, x > \sqrt{2}$,

Y is \cup for $x < -\sqrt{2}, x > \sqrt{2}$,

Y is \cap for $-\sqrt{2} < x < \sqrt{2}$

$$x=0: Y = -\frac{1}{2}$$

$Y=0$: none



$$\lim_{x \rightarrow +\sqrt{2}^+} \frac{x^2+1}{x^2-2} = +\infty, \quad \lim_{x \rightarrow +\sqrt{2}^-} \frac{x^2+1}{x^2-2} = -\infty$$

$$\lim_{x \rightarrow -\sqrt{2}^+} \frac{x^2+1}{x^2-2} = -\infty, \quad \lim_{x \rightarrow -\sqrt{2}^-} \frac{x^2+1}{x^2-2} = +\infty$$

vertical asymptotes
 $x = \pm\sqrt{2}$

$$\lim_{x \rightarrow \pm\infty} \frac{x^2+1}{x^2-2} = \lim_{x \rightarrow \pm\infty} \frac{1 + \frac{1}{x^2}}{1 - \frac{2}{x^2}} = 1 : \text{horizontal asymptote } Y=1$$