1.) (12 pts.) Use a Three-Step Process to determine if the following function is continuous at $x = 1$.

$$f(x) = \begin{cases} x^2 + 2x\ , & \text{if } x \le 1 \\ 3 - x^2\ , & \text{if } x > 1 \end{cases}$$

i.) $f(1) = (1)^2 + 2(1) = 3$

ii.) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2 + 2x) = 1 + 2 = \underline{\underline{3}}$

and $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3 - x^2) = 3 - 1 = \underline{\underline{2}}$

so $\lim_{x \to 1} f(x)$ DNE and f is NOT continuous at $x = 1$.

2.) Consider the function $f(x) = \dfrac{x^2}{x^2 - 9}$

a.) (4 pts.) Use limits to determine all horizontal asymptotes (H.A.).

$$\lim_{x \to \pm\infty} \frac{x^2}{x^2 - 9} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \to \pm\infty} \frac{1}{1 - 9/x^2} = \frac{1}{1 - 0} = 1$$

so $\boxed{y = 1}$ is H.A.

b.) (8 pts.) Use limits to determine all vertical asymptotes (V.A.).

$\lim_{x \to 3^+} f(x) = \text{"}\frac{9}{0^+}\text{"} = +\infty$

$\lim_{x \to 3^-} f(x) = \text{"}\frac{9}{0^-}\text{"} = -\infty$

2.9 3.1

$x = 3$

$\boxed{x = 3}$ is V.A.

$\lim_{x \to -3^+} f(x) = \text{"}\frac{9}{0^-}\text{"} = -\infty$

$\lim_{x \to -3^-} f(x) = \text{"}\frac{9}{0^+}\text{"} = +\infty$

−3.1 −2.9

$x = -3$

$\boxed{x = -3}$ is V.A.

c.) (6 pts.) Use x- and y-intercepts and the above information to sketch a graph of f.

$x = 0$: $y = 0$

$y = 0$: $\dfrac{x^2}{x^2 - 9} = 0$

$\rightarrow x^2 = 0 \rightarrow x = 0$