

Math 16A (Summer 2010)

Kouba

Quiz 3

KEY

PRINT Name : _____

Exam ID # : _____

1.) (10 pts.) Use $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x) = 3x^2 - x + 1$.

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - (x+h) + 1 - [3x^2 - x + 1]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3(x^2 + 2hx + h^2) - x - h + 1 - 3x^2 + x - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{3x^2} + 6hx + \cancel{3h^2} - x - h + 1 - \cancel{3x^2} + x - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(6x + 3h - 1)}{h} = 6x - 1
 \end{aligned}$$

2.) (5 pts. each) Use shortcut rules to find the derivatives of each function.

a.) $y = (1/2)x - 3$

$$\rightarrow y^1 = \frac{1}{2}$$

$$\text{b.) } f(x) = x^{-3} + x^2 - 8x^{1/4}$$

$$\rightarrow f'(x) = -3x^{-4} + 2x - 8 \cdot \frac{1}{x^4} x^{-3/4}$$

$$\text{c.) } y = \frac{x^3 - x}{x - 3}$$

$$\stackrel{x=3}{\rightarrow} y^1 = \frac{(x-3)(3x^2-1) - (x^3-x)(1)}{(x-3)^2}$$

$$\text{d.) } g(x) = (2 - x)(3x + 7)(4 - \sqrt{x})$$

$$\begin{aligned} \stackrel{\text{D}}{\rightarrow} g'(x) &= (-1)(3x+7)(4-\sqrt{x}) + (2-x)(3)(4-\sqrt{x}) \\ &\quad + (2-x)(3x+7)\left(-\frac{1}{2}x^{-\frac{1}{2}}\right) \end{aligned}$$

3.) (6 pts.) Find an equation of the line tangent to the graph of $f(x) = \frac{x^2}{2-x}$ at the point $x=3$.

$$\Rightarrow f'(x) = \frac{(2-x)(2x) - x^2(-1)}{(2-x)^2} \text{ and } x = 3$$

so SLOPE $m = f'(3) = \frac{-6+9}{1} = 3$ and
 $x=3, y=-9$ so line is

$$y - (-9) = 3(x-3) \rightarrow y = 3x - 18$$

4.) (7 pts. each) Use the graph of f to draw a rough sketch of the graph of its derivative f' .

