

Math 16A (Summer 2010)  
Kouba  
Quiz 4

PRINT Name : KEY-----

Exam ID # : -----

1.) (5 pts. each) Use shortcut rules to find the derivatives of each function. Do not simplify answers.

a.)  $y = x \tan x$

$$\frac{D}{\rightarrow} y' = x \cdot \sec^2 x + (1) \cdot \tan x$$

b.)  $f(x) = (7x - 4)^{3/2}$

$$\frac{D}{\rightarrow} f'(x) = \frac{3}{2} (7x - 4)^{1/2} \cdot 7$$

c.)  $y = \frac{2 + \cos x}{3 - \sin x}$

$$\frac{D}{\rightarrow} y' = \frac{(3 - \sin x)(-\cos x) - (2 + \cos x)(-\cos x)}{(3 - \sin x)^2}$$

d.)  $y = \left(\frac{x-4}{5-2x}\right)^{20}$

$$\frac{D}{\rightarrow} y' = 20 \left(\frac{x-4}{5-2x}\right)^{19} \cdot \frac{(5-2x)(1) - (x-4)(-2)}{(5-2x)^2}$$

e.)  $g(x) = 5 + 4 \cos(x^2)$

$$\frac{D}{\rightarrow} g'(x) = 4 \cdot -\sin(x^2) \cdot 2x$$

f.)  $g(x) = \tan^3(\sec(x^3))$  (layers:  $( )^3, \tan, \sec, x^3$ )

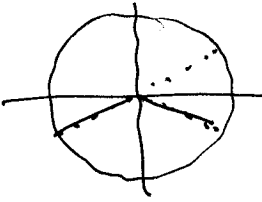
$$\frac{D}{\rightarrow} g'(x) = 3 \tan^2(\sec(x^3)) \cdot \sec^2(\sec(x^3)) \cdot \rightarrow$$

$$\rightarrow \sec(x^3) \tan(x^3) \cdot 3x^2$$

3.) (5 pts. each) a.) Assume that  $f(x) = x^3(x-5)^2$ . Solve  $f'(x) = 0$  for  $x$ .

$$\begin{aligned} \xrightarrow{D} f'(x) &= x^3 \cdot 2(x-5) + 3x^2(x-5)^2 \\ &= x^2(x-5) \cdot [2x + 3(x-5)] = x^2(x-5)[5x-15] = 0 \\ &\rightarrow x=0, x=5, x=3 \end{aligned}$$

b.) Assume that  $f(x) = 4 \sin x - x^2$ . Solve  $f''(x) = 0$  for  $x$ ,  $0 \leq x \leq 2\pi$ .

$$\begin{aligned} \xrightarrow{D} f'(x) &= 4 \cos x - 2x \quad \xrightarrow{D} f''(x) = -4 \sin x - 2 = 0 \\ &\rightarrow \sin x = -\frac{1}{2} \end{aligned}$$


$$\begin{aligned} x &= 210^\circ, 330^\circ \\ &\text{OR} \\ x &= \frac{7\pi}{6}, \frac{11\pi}{6} \end{aligned}$$

4.) Assume that a calculus textbook is projected straight up at 96 feet per second from the top of a building 128 feet high.

a.) (3 pts.) Write an equation for the height  $s(t)$  (feet) of the textbook above the ground at time  $t$  seconds.

$$s(t) = -16t^2 + 96t + 128$$

b.) (2 pts.) What is the height of the textbook when  $t = 2$  seconds?  $t = 6$  seconds?

$$s(2) = -16(4) + 96(2) + 128 = 256 \text{ ft.}$$

$$s(6) = -16(36) + 96(6) + 128 = 128 \text{ ft.}$$

c.) (5 pts.) When does the textbook reach its highest point and how high is it?

$$\leftarrow s'(t) = 0 \quad \xrightarrow{D} s'(t) = -32t + 96 = 0 \rightarrow$$

$$t = 3 \text{ sec. and}$$

$$s(3) = -16(9) + 96(3) + 128 = 272 \text{ ft.}$$

