

Math 16A (Summer 2008)
Kouba
Quiz 5

KEY

PRINT Name : -----

Exam ID # : -----

1.) (20 pts.) For the following function f determine all absolute and relative maximum and minimum values, inflection points, and x - and y -intercepts. State clearly the open intervals for which f is increasing (\uparrow), decreasing (\downarrow), concave up (\cup), and concave down (\cap). Neatly sketch the graph of f .

$$f(x) = x^3 - 3x^2 \quad \text{on the interval } [-1, 4]$$

$$\frac{D}{\rightarrow} f'(x) = 3x^2 - 6x = 3x(x-2) = 0$$

//	+	0	-	0	+	//	f'
$x = -1$		$x = 0$		$x = 2$		$x = 4$	
$y = -4$		$y = 0$		$y = -4$		$y = 16$	
<u>abs.</u>		<u>rel.</u>		<u>abs.</u>		<u>abs.</u>	
min.		max.		min.		max.	

$$\frac{D}{\rightarrow} f''(x) = 6x - 6 = 6(x-1) = 0$$

//	-	0	+	//	f''
$x = -1$		$x = 1$		$x = 4$	
		$y = -2$			
		<u>infl. pt.</u>			

f is \uparrow for $-1 < x < 0, 2 < x < 4$,

f is \downarrow for $0 < x < 2$,

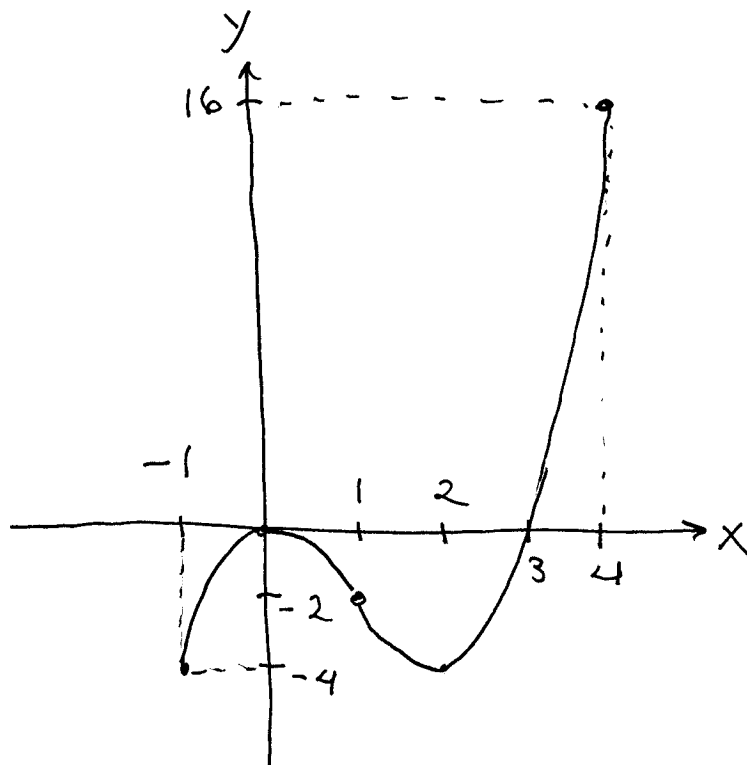
f is \cup for $1 < x < 4$,

f is \cap for $-1 < x < 1$;

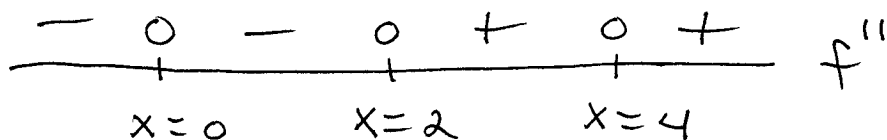
$$x = 0: y = 0$$

$$y = 0: x^3 - 3x^2 = x^2(x-3) = 0$$

\downarrow \downarrow
 $x = 0$ $x = 3$



2.) (10 pts.) If $f''(x) = x^2(x-2)^3(x-4)^2$, then determine all of the x -values corresponding to inflection points for the graph of f .



infl. pt. at $x=2$

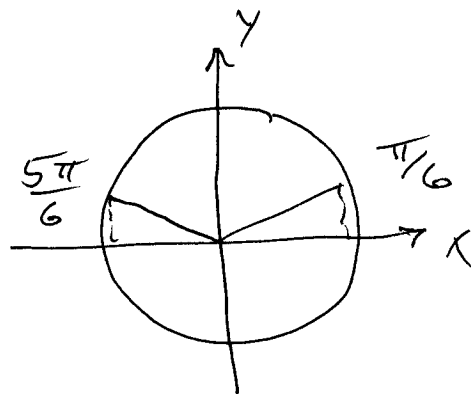
3.) (10 pts.) Let $f(x) = x^2 + 4 \sin x$. Solve $f''(x) = 0$ for x , $0 \leq x \leq 2\pi$.

$$\xrightarrow{D} f'(x) = 2x + 4 \cos x$$

$$\xrightarrow{D} f''(x) = 2 - 4 \sin x = 0$$

$$\rightarrow \sin x = \frac{1}{2}$$

$$\rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$



4.) (10 pts.) Let $f(x) = \frac{x^2}{x-2}$. Set up a sign chart for the first derivative, f' .

$$\xrightarrow{D} f'(x) = \frac{(x-2) \cdot 2x - x^2(1)}{(x-2)^2} = \frac{2x^2 - 4x - x^2}{(x-2)^2}$$

$$= \frac{x^2 - 4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2} = 0 \rightarrow x(x-4) = 0 \rightarrow$$

$$x=0, x=4$$

