Math 16A (Summer 2010)
Kouba
Quiz 5

PRINT Name : KEY

Exam ID # : ____________

1.) (20 pts.) For the following function f determine all absolute and relative maximum and minimum values, inflection points, and x- and y-intercepts. State clearly the open intervals for which f is increasing (↑), decreasing (↓), concave up (⋃), and concave down (⋂). Neatly sketch the graph of f.

$f(x) = x(x-3)^2$ on the interval $[-1, 5]$

$\xrightarrow{D} f'(x) = x \cdot 2(x-3) + (1)(x-3)^2$

$= (x-3)[2x + (x-3)] = (x-3)[3x-3] = 0$

f': + 0 − 0 +

$x=-1$, $y=-16$: ABS MIN

$x=1$, $y=4$: REL MAX

$x=3$, $y=0$: REL MIN

$x=5$, $y=20$: ABS MAX

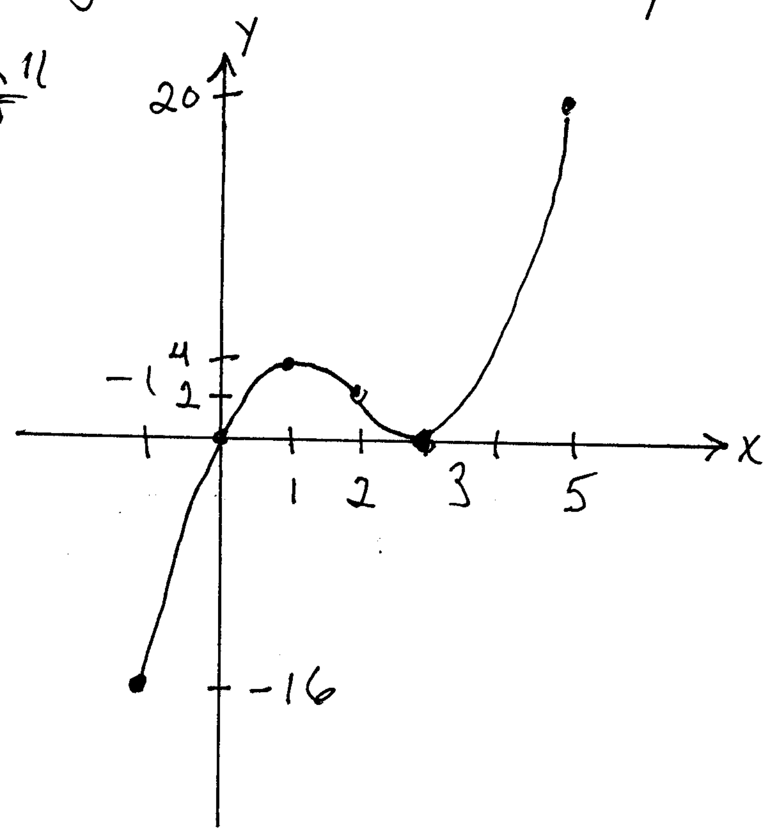
$\xrightarrow{D} f''(x) = (x-3)(3) + (1)[3x-3]$

$= 3x - 9 + 3x - 3 = 6x - 12 = 6(x-2) = 0$

f'': − 0 +

$x=-1$, $x=2$, $x=5$

infl. pt. $\begin{cases} x = 2 \\ y = 2 \end{cases}$



y is ↑ for $-1<x<1$, $3<x<5$,

y is ↓ for $1<x<3$,

y is ⋃ for $2<x<5$,

y is ⋂ for $-1<x<2$;

$x=0$: $y=0$

$y=0$: $x=0$, $x=3$

2.) (5 pts. each) Let $f(x) = \dfrac{2x}{1-x}$

a.) Use limits to determine equation(s) for all horizontal asymptotes.

$$\lim_{x\to\pm\infty} \frac{2x}{1-x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\pm\infty} \frac{2x}{1-x}\cdot\frac{1/x}{1/x} = \lim_{x\to\pm\infty} \frac{2}{\frac{1}{x}-1}$$

$$= \frac{2}{0-1} = -2 \text{ so H.A. is } \boxed{Y=-2}$$

b.) Use limits to determine equation(s) for all vertical asymptotes.

$$\lim_{x\to 1} \frac{2x}{1-x} = \frac{2}{1-1} = \frac{2}{0} = \pm\infty \text{ so}$$

$$\text{V.A. is } \boxed{X=1}$$

3.) (10 pts.) Let $f(x) = \cos x - \sqrt{3}\sin x$. Solve $f''(x) = 0$ for x, $0 \le x \le 2\pi$.

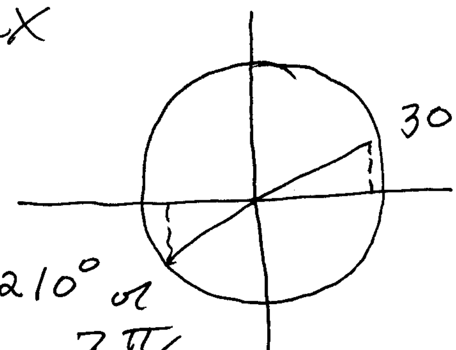
$$\xrightarrow{D} f'(x) = -\sin x - \sqrt{3}\cos x$$

$$\xrightarrow{D} f''(x) = -\cos x - \sqrt{3}\cdot -\sin x$$

$$= -\cos x + \sqrt{3}\sin x = 0 \rightarrow$$

$$\sqrt{3}\sin x = \cos x \rightarrow$$

$$\sqrt{3} = \frac{\cos x}{\sin x} = \frac{\sqrt{3}/2}{1/2} = \frac{-\sqrt{3}/2}{1/2}$$



$30°$ or $\frac{\pi}{6}$

$210°$ or $7\pi/6$

$$\rightarrow x = \frac{\pi}{6}, \frac{7\pi}{6}$$

4.) (10 pts.) Let $f(x) = 6x^{1/3} - x$. Set up a sign chart for the first derivative, f'. Indicate the open intervals on which f is (↑) and (↓) .

Domain: all x-values

$$\xrightarrow{D} f'(x) = 6\cdot\frac{1}{3}x^{-2/3} - 1 = \frac{2}{x^{2/3}} - \frac{x^{2/3}}{x^{2/3}} = \frac{2-x^{2/3}}{x^{2/3}} = 0$$

$$\rightarrow 2 - x^{2/3} = 0$$

$$\rightarrow x^{2/3} = 2 \rightarrow x^2 = 2^3 = 8 \rightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

$-$ 0 $+$ NO $+$ 0 $-$

-8 $x=-\sqrt{8}$ -1 $x=0$ 1 $x=\sqrt{8}$ 8

Y is ↑ for $-\sqrt{8}<x<0$, $0<x<\sqrt{8}$,

Y is ↓ for $x<-\sqrt{8}$, $x>\sqrt{8}$