Math 16A (Summer 2008)
Kouba
Quiz 6

PRINT Name : KEY

Exam ID # : __________

1.) (10 pts. each) Assume that y is a function of x . Find $y' = \frac{dy}{dx}$.

a.) $x^2y^3 + 5x = y$

$\xrightarrow{D} x^2 \cdot 3y^2 y' + 2x \cdot y^3 + 5 = y'$

$\longrightarrow 3x^2y^2y' - y' = -5 - 2xy^3$

$\longrightarrow y'(3x^2y^2 - 1) = -5 - 2xy^3$

$$\longrightarrow \quad y' = \frac{-5 - 2xy^3}{3x^2y^2 - 1}$$

b.) $\sin(xy) = \tan x + y^4$

$\xrightarrow{D} \cos(xy) \cdot (xy' + (1)y) = \sec^2 x + 4y^3y'$

$\longrightarrow \cos(xy) \cdot xy' + \cos(xy) \cdot y = \sec^2 x + 4y^3y'$

$\longrightarrow x\cos(xy) \cdot y' - 4y^3 y' = \sec^2 x - y\cos(xy)$

$\longrightarrow y'(x\cos(xy) - 4y^3) = \sec^2 x - y\cos(xy)$

$$\longrightarrow \quad y' = \frac{\sec^2 x - y\cos(xy)}{x\cos(xy) - 4y^3}$$

2.) Assume that $xy + y^2 = x^2 + 4$.

a.) (6 pts.) Find the slope of the graph of this equation at the point $x = 0$, $y = 2$.

$$\xrightarrow{D} xy' + (1)y + 2yy' = 2x$$

$$\rightarrow (x+2y)y' = 2x - y$$

$$\rightarrow y' = \frac{2x-y}{x+2y} \quad \text{and } x=0,\ y=2 \longrightarrow$$

$$\text{SLOPE} \quad m = y' = \frac{2(0)-2}{0+2(2)} = \frac{-1}{2}$$

b.) (6 pts.) Determine if the graph of this equation is concave up or concave down at the point $x = 0$, $y = 2$.

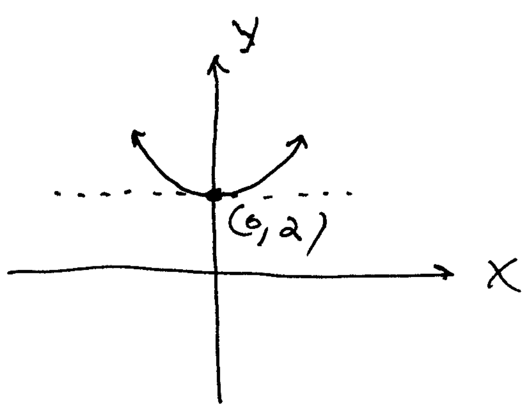
$$\xrightarrow{D} y'' = \frac{(x+2y)(2-y') - (2x-y)(1+2y')}{(x+2y)^2}$$

$$\text{and } x=0,\ y=2,\ y' = -1/2 \longrightarrow$$

$$y'' = \frac{(4)(2+\frac{1}{2}) - (-2)(0)}{(4)^2} = \frac{4 \cdot \frac{5}{2}}{16} = \frac{10}{16} = \frac{5}{8}$$

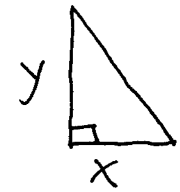
so graph is $(\cup)$.

c.) (3 pts.) Sketch a graph of this equation near the point $x = 0$, $y = 2$.



3.) (10 pts. each) Consider the given right triangle. If x is increasing at the rate of 2 ft./min. and y is decreasing at the rate of 1 ft./min., then find the following related rates when $x = 4$ ft. and $y = 3$ ft.

a.) The rate at which its AREA is changing



__Given__: $\frac{dx}{dt} = 2$ ft/min., $\frac{dy}{dt} = -1$ ft/min.

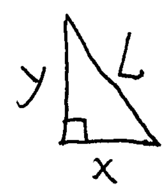
__Find__: $\frac{dA}{dt}$ when $x = 4$, $y = 3$ $\rightarrow$

$$A = \frac{1}{2}xy \xrightarrow{D} \frac{dA}{dt} = \frac{1}{2}\underset{4}{x} \cdot \underset{-1}{\frac{dy}{dt}} + \frac{1}{2}\underset{2}{\frac{dx}{dt}} \cdot \underset{3}{y}$$

$$= \frac{1}{2}(4)(-1) + \frac{1}{2}(2)(3) = -2 + 3$$

$$= 1 \text{ ft}^2/\text{min.}$$

b.) The rate at which its HYPOTENUSE is changing



$$L^2 = x^2 + y^2 \rightarrow L = \sqrt{x^2 + y^2}$$

__Find__: $\frac{dL}{dt}$ when $x = 4$, $y = 3$ $\rightarrow$

$$\frac{dL}{dt} = \frac{1}{2}(x^2 + y^2)^{-1/2} \cdot \left[2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt}\right]$$

$$= \frac{1}{2}(16 + 9)^{-1/2} \cdot [2(4)(2) + 2(3)(-1)]$$

$$= \frac{1}{2} \cdot \frac{1}{5} \cdot [16 - 6] = \frac{10}{10} = 1 \text{ ft./min.}$$