Math 16A (Summer 2010)
Kouba
Quiz 6

PRINT Name : ____KEY____

Exam ID # : ________

1.) (10 pts. each) Assume that y is a function of x . Find $y' = \frac{dy}{dx}$.

a.) $x^2y^3 + 5x = y$

$\xrightarrow{D} x^2 \cdot 3y^2 y' + (2x)y^3 + 5 = y'$

$\rightarrow 3x^2 y^2 y' - y' = -2xy^3 - 5$

$\rightarrow y'(3x^2y^2 - 1) = -2xy^3 - 5$

$\rightarrow y' = \dfrac{-2xy^3 - 5}{3x^2y^2 - 1}$

b.) $\sin(xy) = y \cdot \tan x$

$\xrightarrow{D} \cos(xy) \cdot (xy' + (1)y) = y \cdot \sec^2 x + y' \cdot \tan x$

$\rightarrow x\cos(xy) \cdot y' + y\cos(xy) = y\sec^2 x + \tan x \cdot y'$

$\rightarrow x\cos(xy) \cdot y' - \tan x \cdot y' = y\sec^2 x - y\cos(xy)$

$\rightarrow y'[x\cos(xy) - \tan x] = y\sec^2 x - y\cos(xy)$

$\rightarrow y' = \dfrac{y\sec^2 x - y\cos(xy)}{x\cos(xy) - \tan x}$

2.) Assume that $xy + y^2 = x + 4$.

a.) (6 pts.) Find the slope of the graph of this equation at the point $x = 0$, $y = 2$.

$$\xrightarrow{D} xy' + (1)y + 2yy' = 1$$

$$\rightarrow xy' + 2yy' = 1 - y$$

$$\rightarrow y'(x+2y) = 1 - y \rightarrow \boxed{y' = \frac{1-y}{x+2y}} \text{ so}$$

$x=0,\ y=2 \rightarrow$

SLOPE $m = y' = \dfrac{1-2}{0+4} = \dfrac{-1}{4}$ and y is $(\downarrow)$.

b.) (6 pts.) Determine if the graph of this equation is concave up or concave down at the point $x = 0$, $y = 2$.

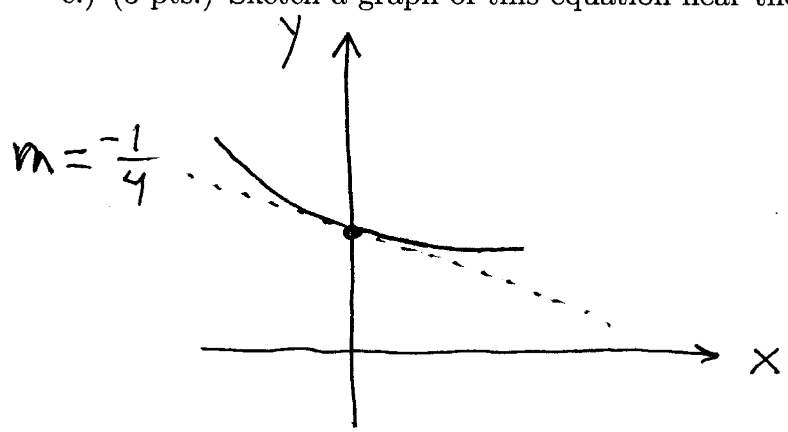
$$\xrightarrow{D} y'' = \frac{(x+2y)(-y') - (1-y)(1+2y')}{(x+2y)^2} \text{ so}$$

$x=0,\ y=2,\ y' = -1/4 \longrightarrow$

$$y'' = \frac{(4)(\frac{1}{4}) - (-1)(1 - \frac{1}{2})}{(4)^2} = \frac{1 + 1/2}{\frac{16}{1}} = \frac{3}{2} \cdot \frac{1}{16}$$

$= 3/32$ so y is $(\cup)$

c.) (3 pts.) Sketch a graph of this equation near the point $x = 0$, $y = 2$.

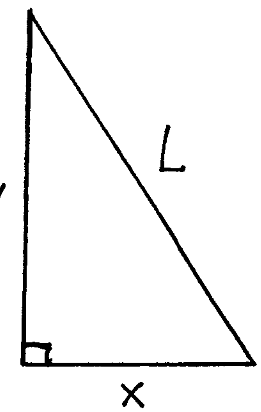


3.) (10 pts. each) Consider the given right triangle. If x is increasing at the rate of 2 ft./min. and y is decreasing at the rate of 1 ft./min., then find the following related rates when $x = 4$ ft. and $y = 3$ ft.

a.) The rate at which its AREA is changing

$$\frac{dx}{dt} = 2\,\frac{ft.}{min.}$$

$$\frac{dy}{dt} = -1\,\frac{ft.}{min}$$



$$A = \frac{1}{2}x \cdot y \xrightarrow{D}$$

$$\frac{dA}{dt} = \frac{1}{2}x \cdot \frac{dy}{dt} + \frac{1}{2} \cdot \frac{dx}{dt} \cdot y$$

$$= \frac{1}{2}(4)(-1) + \frac{1}{2}(2)(3)$$

$$= -2 + 3$$

$$= 1\ ft.^2/min.$$

b.) The rate at which its HYPOTENUSE is changing

$$x^2 + y^2 = L^2 \longrightarrow L = \sqrt{x^2+y^2} \xrightarrow{D}$$

$$\frac{dL}{dt} = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2} \cdot \left[\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y \cdot \frac{dy}{dt}\right]$$

$$= (16+9)^{-1/2} \cdot [(4)(2) + (3)(-1)]$$

$$= \frac{1}{5} \cdot [5] = 1\ ft./min.$$