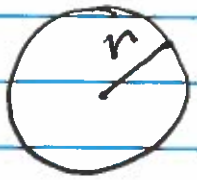
1.) 

Given: $\frac{dr}{dt} = 5 \text{ cm./min.}$

a.) $C = 2\pi r$, Find: $\frac{dC}{dt}$ when $r = 2$;

$\xrightarrow{D} \frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi(5) = 10\pi \text{ cm./min.}$

b.) $A = \pi r^2$, Find: $\frac{dA}{dt}$ when $r = 2$;

$\xrightarrow{D} \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(2)(5) = 20\pi \frac{\text{cm.}^2}{\text{min.}}$

2.) 

Given: $\frac{dx}{dt} = 5 \text{ in./min.}$

and $\frac{dy}{dt} = -4 \text{ in./min.}$

a.) $P = 2x + 2y$, Find: $\frac{dP}{dt}$ when

$x = 3, y = 2; \xrightarrow{D} \frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$

$= 2(5) + 2(-4) = 2 \text{ in/min.}$

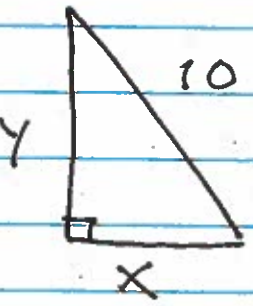
b) $A = xy$, Find: $\frac{dA}{dt}$ when

$x = 3, y = 2; \xrightarrow{D}$

$$\frac{dA}{dt} = x \cdot \frac{dy}{dt} + \frac{dx}{dt} \cdot y$$

$$= (3)(-4) + (5)(2) = -2 \text{ in}^2/\text{min}.$$

3.)

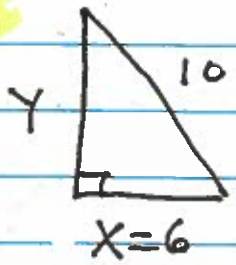


Given: $\frac{dx}{dt} = -2$ ft./min.

Find: $\frac{dy}{dt}$;

$x^2 + y^2 = 10^2 = 100 \longrightarrow$

a.)



$\rightarrow 6^2 + y^2 = 10^2$

$\rightarrow y^2 = 64 \rightarrow y = 8$

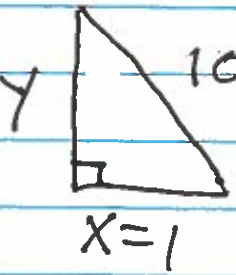
$\xrightarrow{D} 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\rightarrow \boxed{x\frac{dx}{dt} + y\frac{dy}{dt} = 0}$

$\rightarrow (6)(-2) + (8)\frac{dy}{dt} = 0$

$\rightarrow 8\frac{dy}{dt} = 12 \rightarrow \frac{dy}{dt} = \frac{12}{8} = \frac{3}{2}$ ft./min

b.)



$\rightarrow 1^2 + y^2 = 10^2$

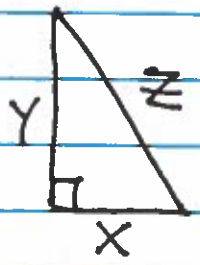
$\rightarrow y^2 = 99 \rightarrow$

$y = \sqrt{99}$

$\rightarrow (1)(-2) + (\sqrt{99})\frac{dY}{dt} = 0$

$\rightarrow \sqrt{99}\frac{dY}{dt} = 2 \rightarrow \frac{dY}{dt} = \frac{2}{\sqrt{99}}$ ft./min

≈ 0.2 ft./min.

4) 

Given: $\frac{dY}{dt} = 2$ in./min.

and $\frac{dX}{dt} = -3$ in./min.

a.) $A = \frac{1}{2}XY$; Find: $\frac{dA}{dt}$ when $X=3$, $Y=4$. $\xrightarrow{D}$

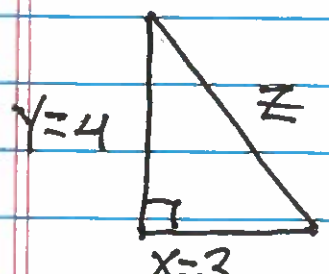
$\frac{dA}{dt} = \frac{1}{2}X\cdot\frac{dY}{dt} + \frac{1}{2}\frac{dX}{dt}\cdot Y$

$= \frac{1}{2}(3)(2) + \frac{1}{2}(-3)(4)$

$= 3 - 6 = -3$ in.^{2}/min.

b.) $X^2 + Y^2 = Z^2$; Find: $\frac{dZ}{dt}$ when $X=3$, $Y=4$; $\xrightarrow{D}$

$2X\frac{dX}{dt} + 2Y\frac{dY}{dt} = 2Z\frac{dZ}{dt}$;



$3^2 + 4^2 = Z^2 \rightarrow Z^2 = 25$

$\rightarrow Z = 5$ then

$(3)(-3) + (4)(2) = (5)\frac{dz}{dt} \longrightarrow$

$\frac{dz}{dt} = \frac{-1}{5}$ in./min.

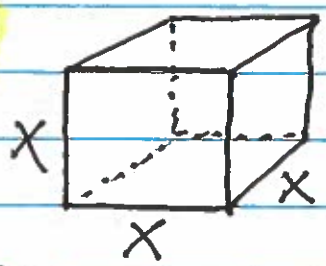
c.) $P = x + y + z$, Find: $\frac{dP}{dt}$ when $x = 3, y = 4$; $\xrightarrow{D}$

$$\frac{dP}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt}$$

$$= (-3) + (2) + \left(-\tfrac{1}{5}\right)$$

$$= -1 - \frac{1}{5} = \frac{-6}{5} \text{ in./min.}$$

5.)



Given: $\frac{dx}{dt} = 4$ in./min.

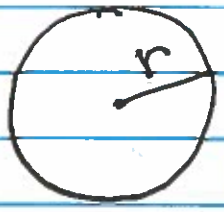
a.) $S = 6x^2$, Find: $\frac{dS}{dt}$ when $x = 20$; $\xrightarrow{D}$

$$\frac{dS}{dt} = 12x \frac{dx}{dt} = 12(20)(4)$$

$$= 960 \text{ in.}^2/\text{min.}$$

b.) $V = x^3$, Find: $\frac{dV}{dt}$ when $x = 20$; $\xrightarrow{D}$ $\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

$= 3(20)^2(4) = 4800$ in.^{3}/min.

6.) 

Given: $\frac{dr}{dt} = 5$ cm./hr.

a.) $z = 2r$, Find: $\frac{dz}{dt}$ when $r = 10$; $\xrightarrow{D}$ $\frac{dz}{dt} = 2\frac{dr}{dt}$

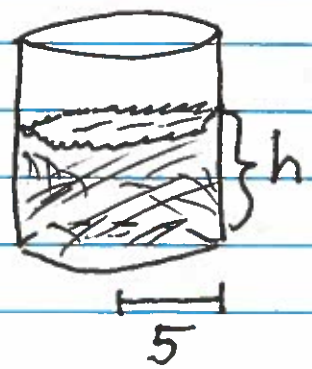
$= 2(5) = 10$ cm./hr.

b.) $S = 4\pi r^2$, Find: $\frac{dS}{dt}$ when $r = 10$; $\xrightarrow{D}$ $\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$= 8\pi(10)(5) = 400\pi$ cm.^{2}/hr.

c.) $V = \frac{4}{3}\pi r^3$, Find: $\frac{dV}{dt}$ when $r = 10$; $\xrightarrow{D}$ $\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$= 4\pi(10)^2(5) = 2000\pi$ cm.^{3}/hr.

7.) 

$V = \pi r^2 h = \pi(5)^2 h$

$\rightarrow$ $\boxed{V = 25\pi h}$

Given: $\frac{dV}{dt} = 5\pi$ ft.^{3}/min.

Find $\frac{dh}{dt}$ when $h = 15$; $\xrightarrow{D}$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt} \rightarrow 5\pi = 25\pi \frac{dh}{dt}$$

$$\rightarrow \frac{dh}{dt} = \frac{5\pi}{25\pi} = \frac{1}{5} \text{ ft./min.}$$

8.)

Given: $\frac{dS}{dt} = 48\pi \frac{\text{cm.}^2}{\text{hr.}}$

a.) $S = 4\pi r^2$, Find: $\frac{dr}{dt}$

when $r = 30$; $\xrightarrow{D} \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$$\rightarrow \cancel{48}^{6}\pi = \cancel{8}\pi(30)\frac{dr}{dt} \rightarrow$$

$$\frac{dr}{dt} = \frac{6}{30} = \frac{1}{5} \text{ cm./hr.}$$

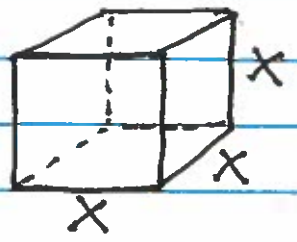
b.) $V = \frac{4}{3}\pi r^3$, Find: $\frac{dV}{dt}$

when $r = 30$; $\xrightarrow{D}$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

$$= 4\pi(30)^2\left(\frac{1}{5}\right) = 720\pi \frac{\text{cm.}^3}{\text{hr.}}$$

9.)



$V = x^3$, $S = 6x^2$;

Given: $\frac{dV}{dt} = -60 \text{ ft.}^3/\text{min.}$;

Find: $\frac{dS}{dt}$ when $x = 20$; $\xrightarrow{D}$

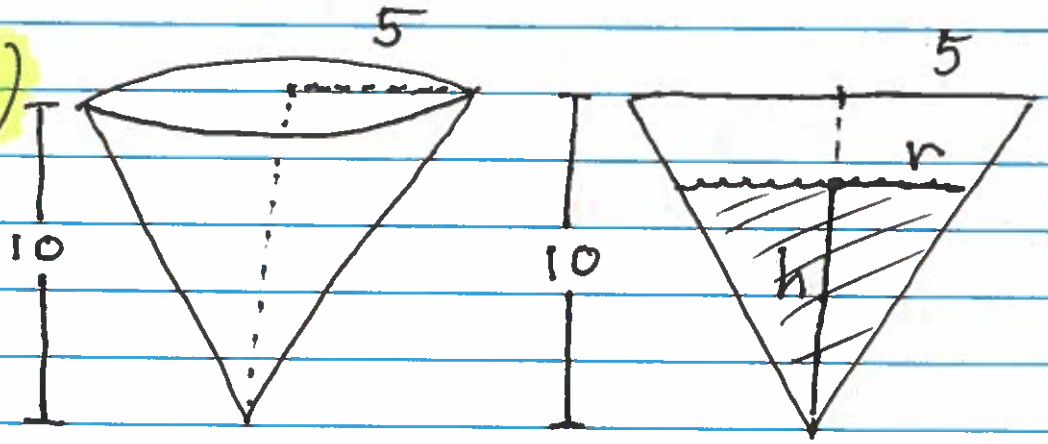
$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt} \rightarrow -60 = 3(20)^2 \frac{dx}{dt}$$

$$\rightarrow \frac{dx}{dt} = \frac{-60}{1200} = \frac{-1}{20} \text{ ft./min.; then}$$

$$\xrightarrow{D} \frac{dS}{dt} = 12x \frac{dx}{dt} = 12(20)\left(\frac{-1}{20}\right)$$

$$= -12 \text{ ft.}^2/\text{min}$$

10.)



By similar Δ's:

$$\frac{5}{10} = \frac{r}{h} \rightarrow \boxed{r = \frac{1}{2}h};$$

Given: $\frac{dV}{dt} = 2\pi$ ft.^{3}/hr.;

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h \rightarrow$

$\boxed{V = \frac{1}{12}\pi h^3}$, Find: $\frac{dh}{dt}$; $\xrightarrow{D}$

$\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \frac{dh}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

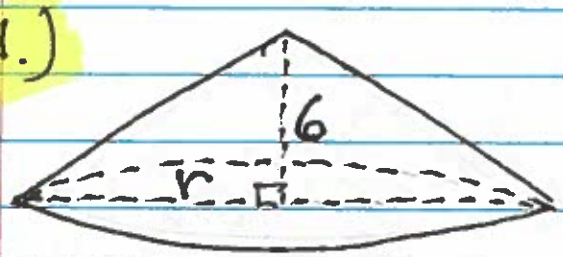
a.) $h = 1$: $2\pi = \frac{1}{4}\pi (1)^2 \frac{dh}{dt} \rightarrow$

$\frac{dh}{dt} = 8$ ft./hr.

b.) $h = 9$: $2\pi = \frac{1}{4}\pi (9)^2 \frac{dh}{dt} \rightarrow$

$\frac{dh}{dt} = \frac{8}{81}$ ft./hr.

11.)



$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (6) \rightarrow$

$\boxed{V = 2\pi r^2}$; Given: $\frac{dV}{dt} = 4\pi$ m.^{3}/min.,

a.) Find: $\frac{dr}{dt}$ when $V = 200\pi$

$\xrightarrow{D} \frac{dV}{dt} = 4\pi r \frac{dr}{dt}$; $V = 200\pi$

$\rightarrow 200\pi = 2\pi r^2 \rightarrow r^2 = 100 \rightarrow$

$\boxed{r = 10} \rightarrow \cancel{4\pi} = \cancel{4\pi}(10)\frac{dr}{dt} \rightarrow$

$\frac{dr}{dt} = \frac{1}{10}$ m./min.

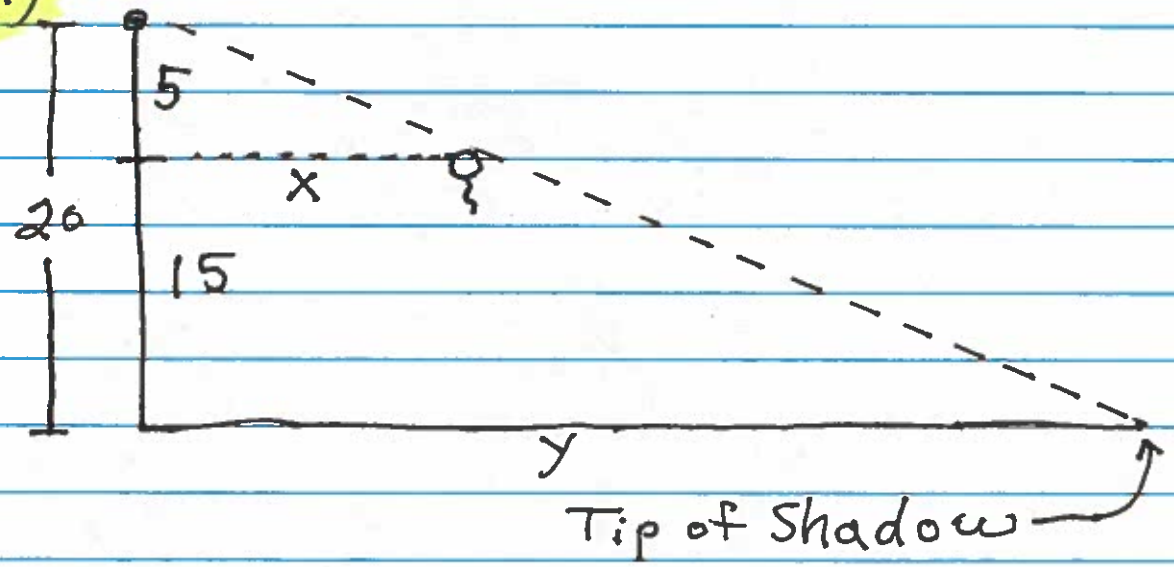
b.) $A = \pi r^2$, Find: $\frac{dA}{dt}$

when $V = 200\pi \rightarrow r = 10$; $\xrightarrow{D}$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(10)\left(\frac{1}{10}\right)$

$= 2\pi$ m.^{2}/min.

12.)



Given: $\frac{dx}{dt} = 2$ ft./sec.,

Find: $\frac{dY}{dt}$ when $X = 30$;

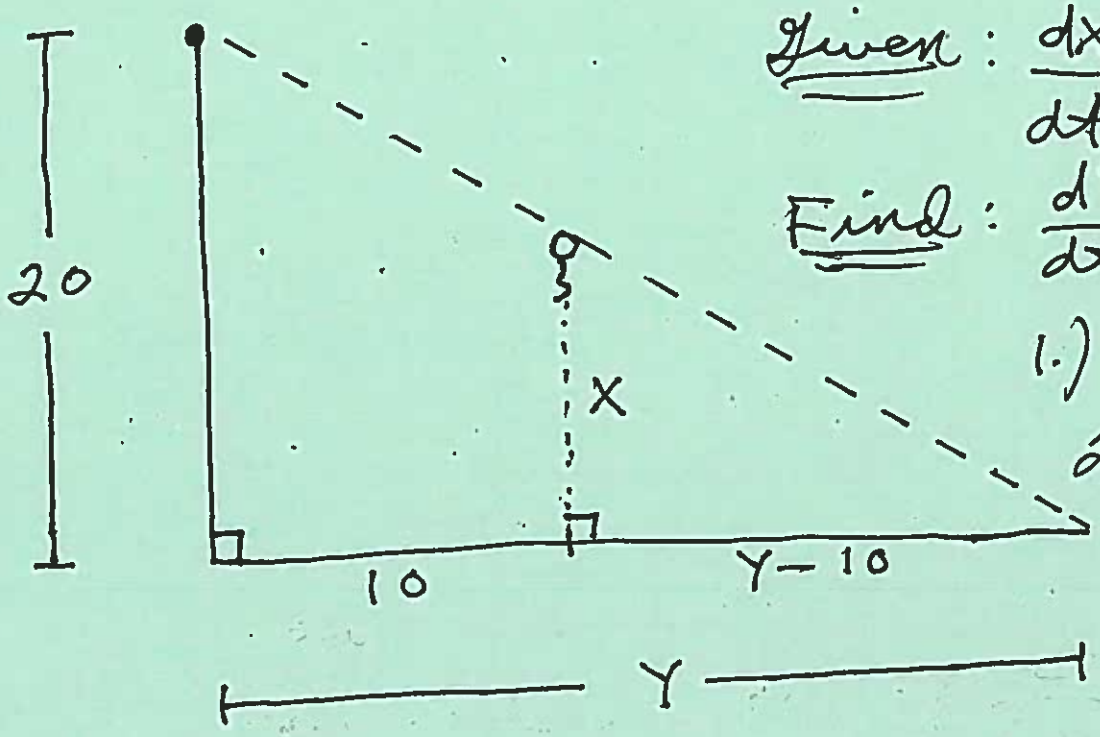
By similar $\triangle$'s:

$$\frac{5}{X} = \frac{20}{Y} \rightarrow 5Y = 20X \rightarrow$$

$$\boxed{Y = 4X} \xrightarrow{D} \frac{dY}{dt} = 4\frac{dX}{dt}$$

$$= 4(2) = 8 \text{ ft./sec.}$$

13.)



Given: $\frac{dx}{dt} = 3$ ft./sec.

Find: $\frac{dY}{dt}$ when

1.) $x = 5$ ft.

2.) $x = 19$ ft.

By similar $\triangle$'s:

$$\frac{20}{Y} = \frac{x}{Y-10} \longrightarrow \boxed{20Y - 200 = xY}$$

D $\longrightarrow$ $\boxed{20\frac{dY}{dt} = x\cdot\frac{dY}{dt} + \frac{dx}{dt}\cdot Y}$

a.) $x = 5 \longrightarrow 20Y - 200 = 5Y \longrightarrow$

$15Y = 200 \longrightarrow Y = \frac{200}{15} = \frac{40}{3}$ ft.;

then

$$20\frac{dY}{dt} = (5)\frac{dY}{dt} + (3)\left(\frac{40}{3}\right) \longrightarrow$$

$$15\frac{dY}{dt} = 40 \rightarrow \frac{dY}{dt} = \frac{40}{15} = \frac{8}{3} \text{ ft./sec.}$$

b.) $X = 19 \rightarrow 20Y - 200 = 19Y \rightarrow$

$Y = 200 \rightarrow$

$$20\frac{dY}{dt} = 19\frac{dY}{dt} + (3)(19) \rightarrow$$

$$\frac{dY}{dt} = 57 \text{ ft./sec.}$$

c.) $X = 19.9 \rightarrow 20Y - 200 = 19.9Y \rightarrow$

$0.1Y = 200 \rightarrow Y = 2000 \rightarrow$

$$20\frac{dY}{dt} = (19.9)\frac{dY}{dt} + (3)(2000) \rightarrow$$

$$0.1\frac{dY}{dt} = 6000 \rightarrow$$

$$\frac{dY}{dt} = 60{,}000 \text{ ft./sec.}$$