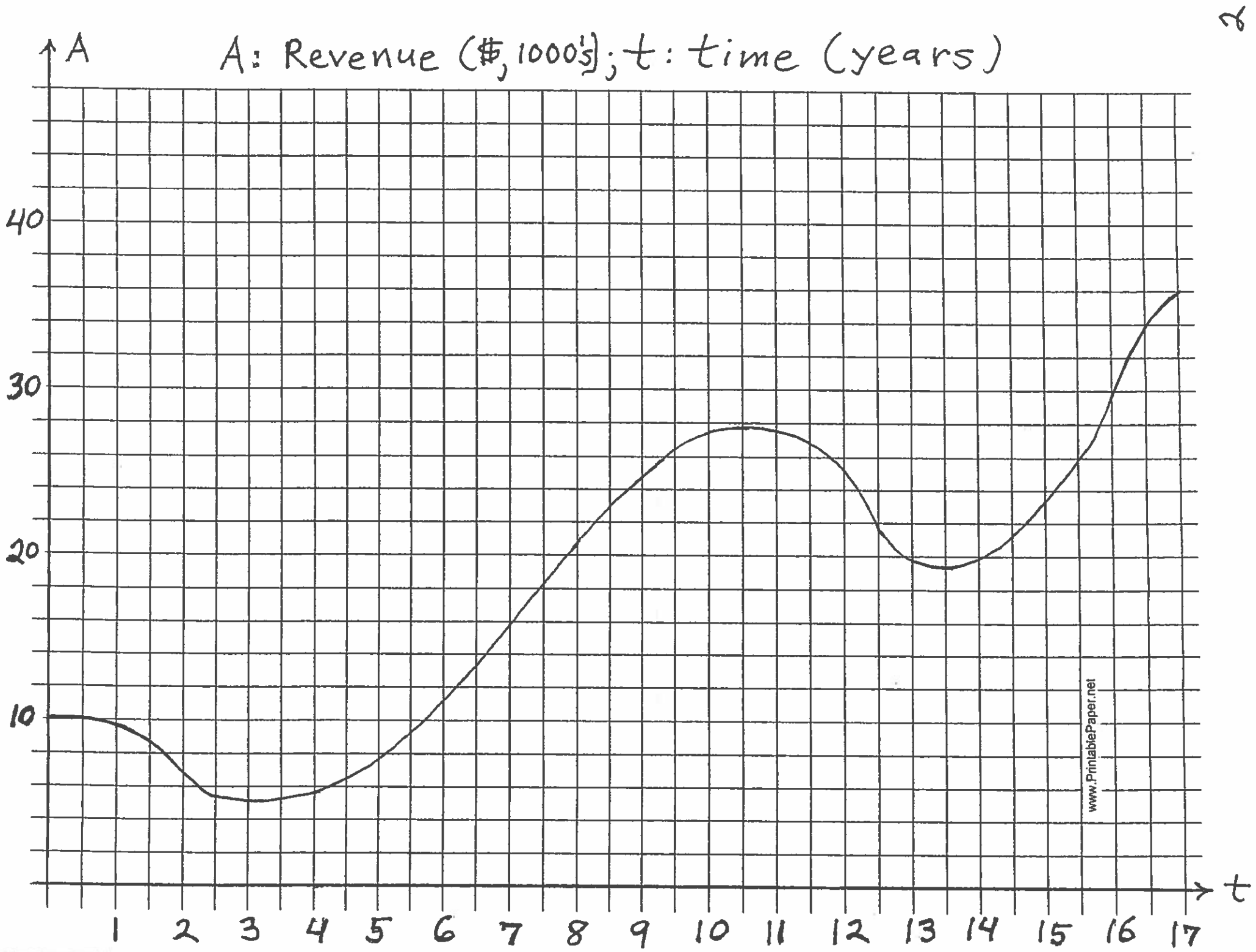
Math 16A
Section 2.1

Derivatives and Rates of Change

FACT: Assume that $y = f(x)$ is a function, where x has units "A" and y has units "B". Then the units for the derivative $y' = f'(x)$ are "$\frac{B}{A}$":

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \left(\text{units: } \frac{B}{A}\right)$$

Example: Consider the following graph which plots Revenue A ($, 1000's) vs. time t (years).


A
A: Revenue ($, 1000's); t: time (years)
40
30
20
10
t
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
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Example: Estimate the SLOPE of the TANGENT line at

1.) $t = 1.5$

2.) $t = 5$

3.) $t = 8$

4.) $t = 10.5$

5.) $t = 12$

6.) $t = 15$

0.) Revenue at

a.) $t = 2$

b.) $t = 7$

c.) $t = 12.5$

d.) $t = 16.5$

For what value of t is the SLOPE

7.) largest? (What is the SLOPE?)

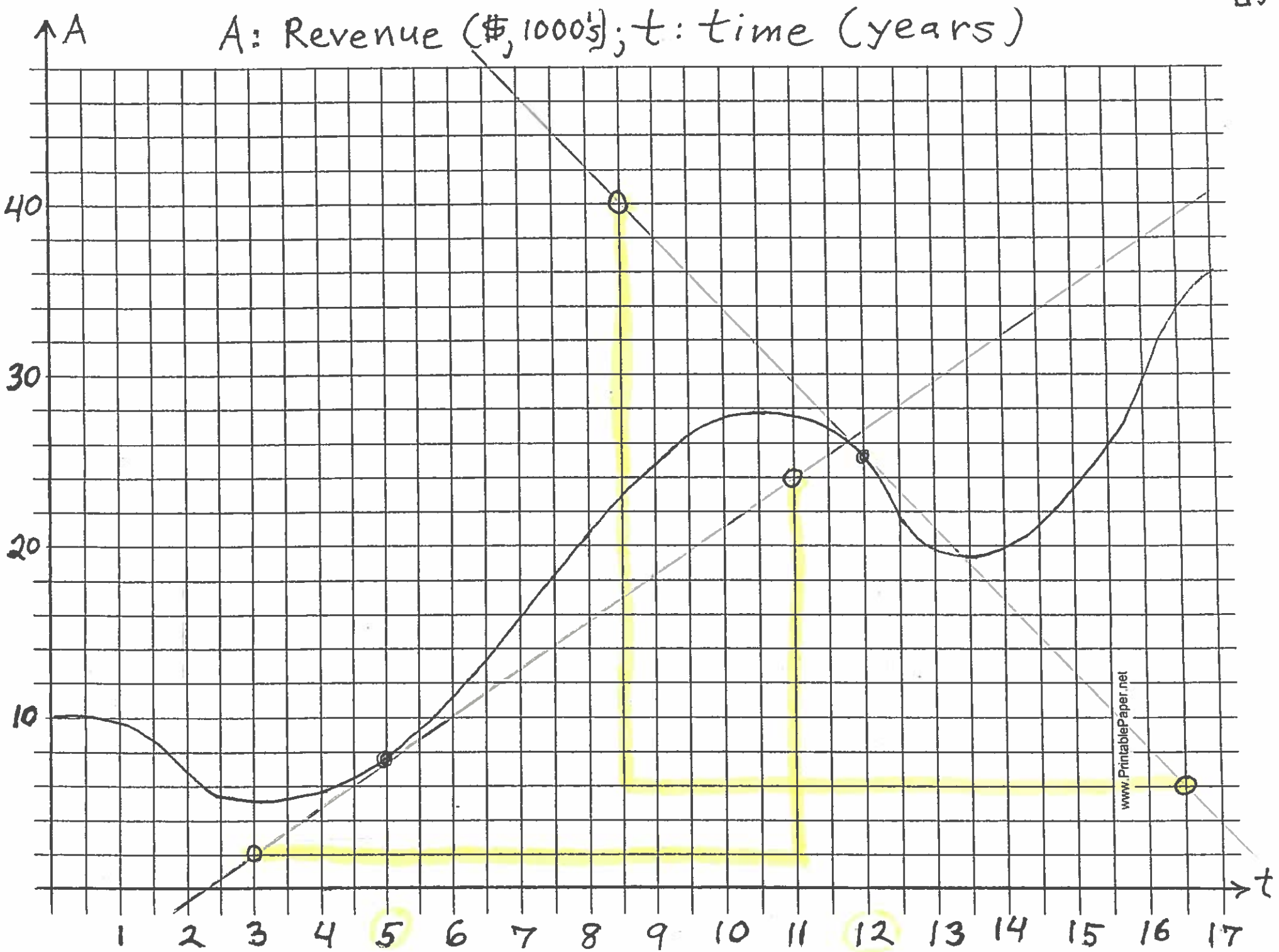
8.) smallest? (What is the SLOPE?)

0.) a.) $A \approx \$6800$

b.) $A \approx \$16,000$

c.) $A \approx \$20,800$

d.) $A \approx \$34,000$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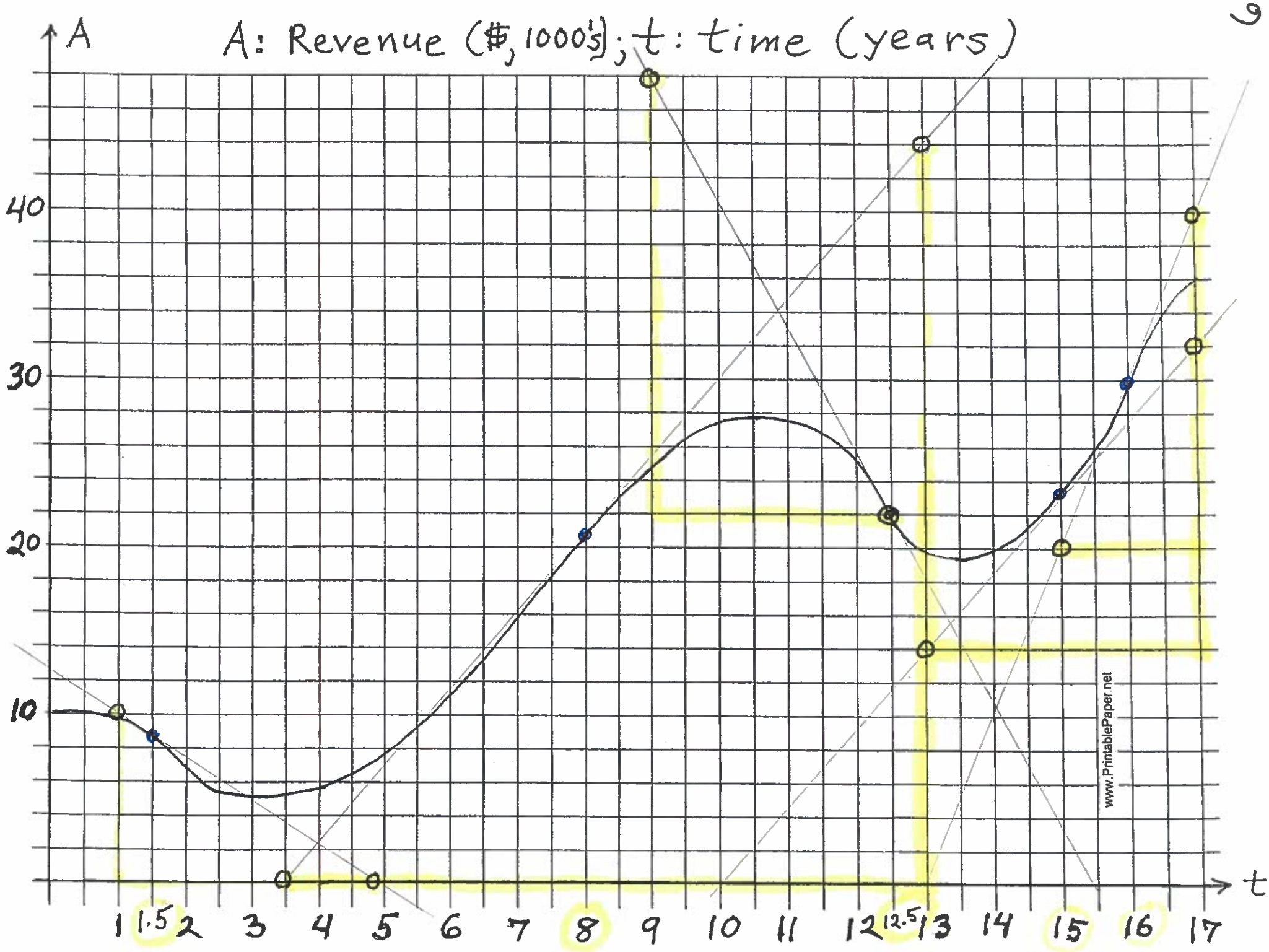
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17
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1.) $m = \frac{-10,000}{3.8} \approx -\$2631/yr.$

2.) $m = \frac{22,000}{8} = \$2750/yr.$

3.) $m = \frac{44,000}{9.5} = \$4632/yr.$

4.) $m = \$0/yr.$

5.) $m = \frac{-34,000}{8} = -\$4250/yr.$

6.) $m = \frac{18,000}{4} = \$4500/yr.$

7.) LARGEST SLOPE @ $t = 16$

$$m = \frac{20{,}000}{2} = \$10{,}000/\text{yr}.$$

8.) SMALLEST SLOPE @ $t = 12.5$

$$m = \frac{-28{,}000}{3.5} = -\$8000/\text{yr}.$$