Math 16A
Section 2.3

Average Rate of Change, Instantaneous Rate of Change

Definition: Let $y = f(x)$ be a function.

I.) The **Average Rate of Change** of f on interval $[a,b]$ is

$$ARC = \frac{f(b) - f(a)}{b - a}.$$

II.) The **Instantaneous Rate of Change** of f at $x = c$ is

$$IRC = f'(c).$$

Example: Consider the function

$f(x) = x^3 - 2x^2$

1.) Find the ARC on interval

a.) $[-1,3]$ b.) $[0,1]$ c.) $[0,2]$

2.) Find the IRC at

a.) $x=0$ b.) $x=-2$ c.) $x=1$

1.) a.) $ARC = \frac{f(3)-f(-1)}{3-(-1)} = \frac{9-(-3)}{4} = \frac{12}{4} = 3$

b.) $ARC = \frac{f(1)-f(0)}{1-0} = \frac{-1-0}{1} = -1$

c.) $ARC = \frac{f(2)-f(0)}{2-0} = \frac{0-0}{2} = 0$

2.) $f(x) = x^3 - 2x^2 \xrightarrow{D} f'(x) = 3x^2 - 4x$

a.) $IRC = f'(0) = 0-0 = 0$

b.) $IRC = f'(-2) = 12-(-8) = 20$

c.) $IRC = f'(1) = 3-4 = -1$

NOTE: I.) ARC is the SLOPE of a line between two points.

II.) IRC is the SLOPE of a tangent line at a point $x=c$.

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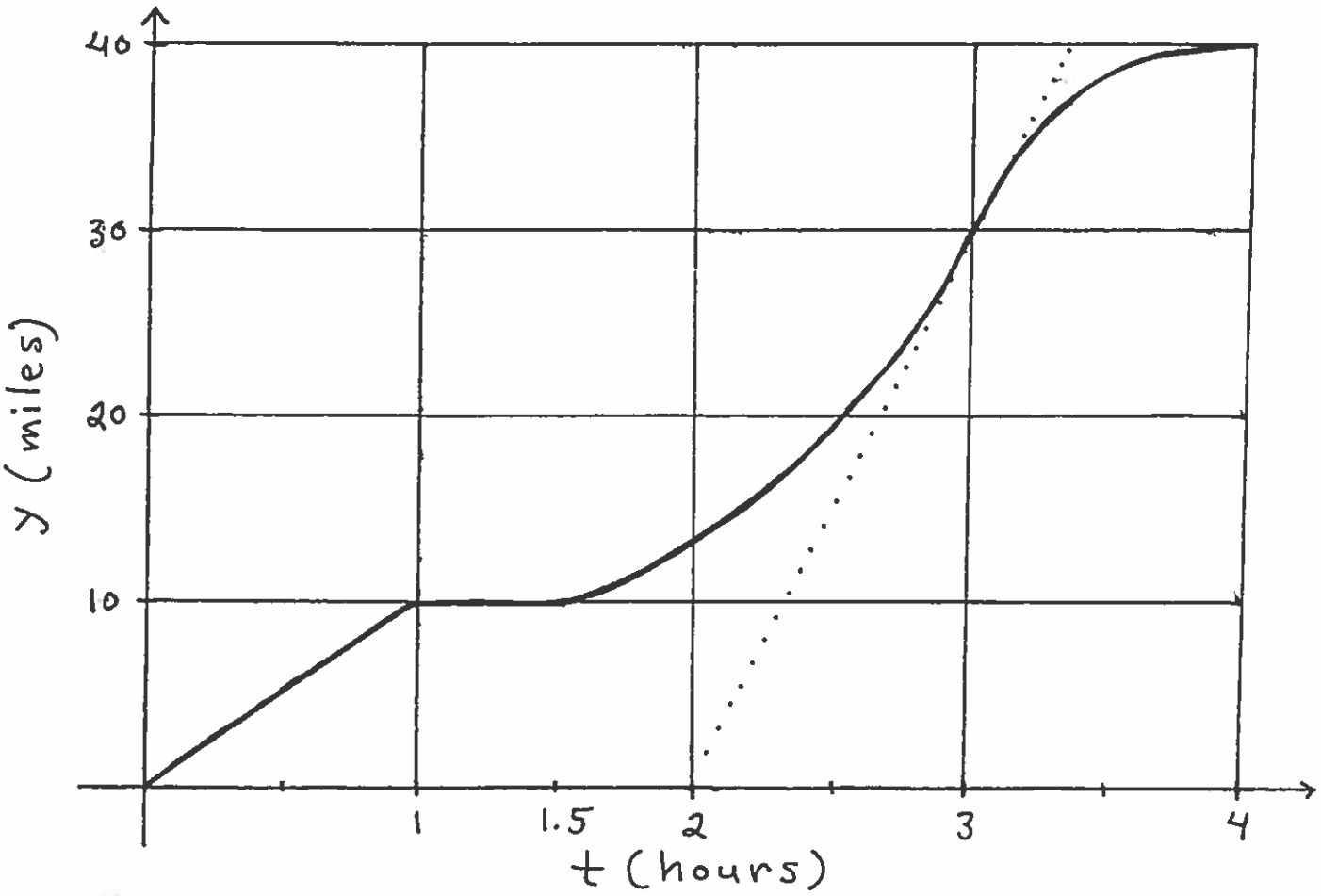
an Example of ARC and IRC

Example: Assume that the following graph represents the number of miles y traveled by a bicycle after t hours.



1.) What is the bike's distance traveled for $t = \frac{1}{2}$ hr.? $t = 1$ hr.? $t = 1.5$ hr.? $t = 3$ hr.?

2.) What is the bike's average velocity (ARC) on the interval $[0,1]$? $[0,2]$? $[1,2.5]$? $[0,4]$?

3.) What is the bike's instantaneous velocity (IRC) when $t = \frac{1}{2}$ hr.? $t = 1.25$ hr.? $t = 3$ hr.? $t = 3.5$ hr.?

4.) Describe the bike's behavior for a.) $0 \le t \le 1$ b.) $1 \le t \le 1.5$ c.) $1.5 \le t \le 3$ d.) $3 \le t \le 4$

5.) When is the bike traveling fastest?

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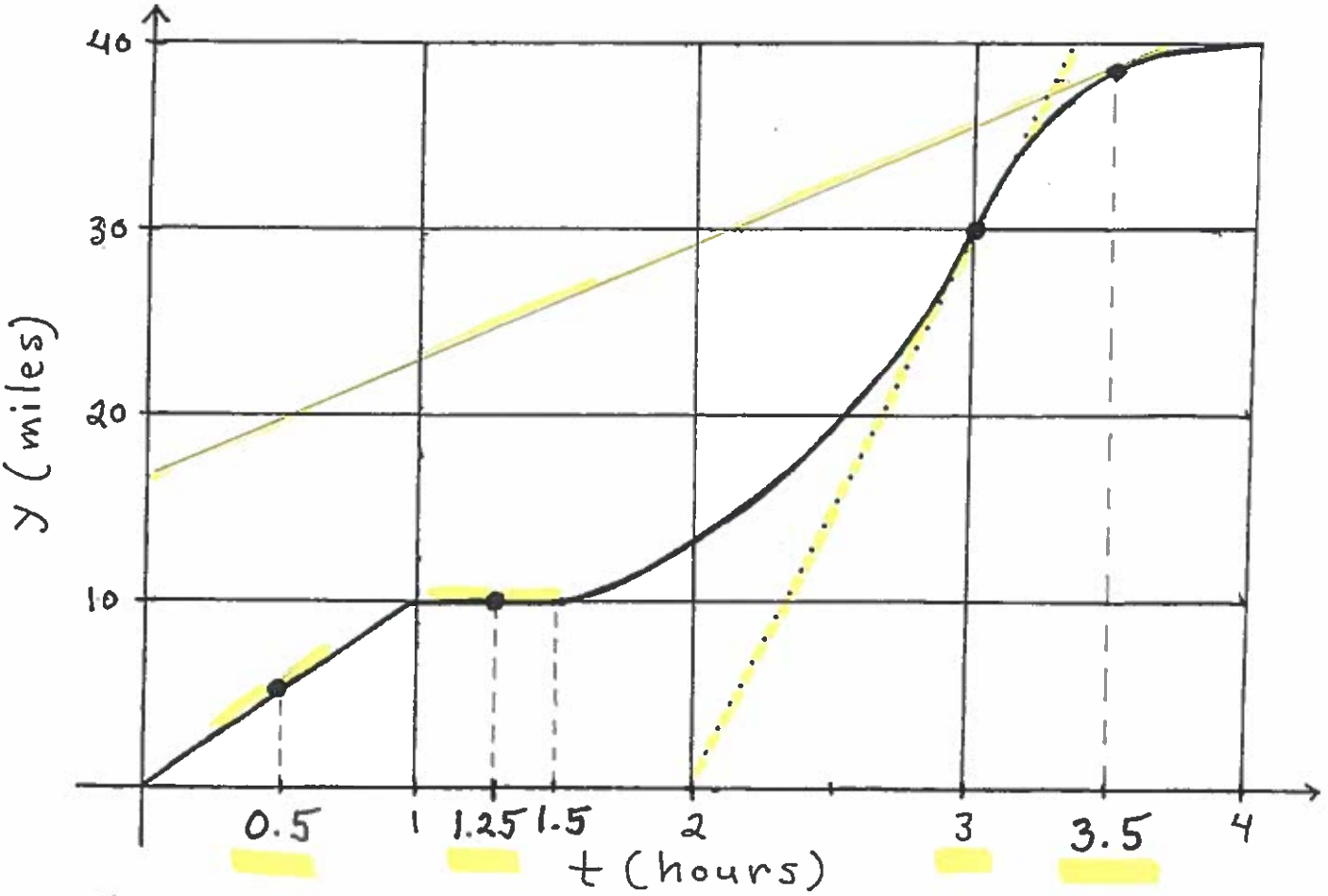

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ANSWERS (Estimates)

1.) a.) $t = \frac{1}{2}$: $y = 5$ mi.

b.) $t = 1$: $y = 10$ mi.

c.) $t = 1.5$: $y = 10$ mi.

d.) $t = 3$: $y = 30$ mi.

2.) a.) $ARC = \frac{10-0}{1-0} = 10$ mph.

b.) $ARC = \frac{13-0}{2-0} = 6.5$ mph.

c.) $ARC = \frac{20-10}{2.5-1} = \frac{20}{3} \approx 6.67$ mph.

d.) $ARC = \frac{40-0}{4-0} = 10$ mph.

3.) (Draw Tangent Lines and estimate their SLOPES.)

a.) $IRC = \frac{\text{rise}}{\text{run}} = \frac{10-0}{1-0} = 10$ mph.

b.) $IRC = \frac{\text{rise}}{\text{run}} = \frac{10-10}{1.5-1} = 0$ mph.

c.) $IRC = \frac{rise}{run} = \frac{30-0}{3-2} = 30$ mph.

d.) $IRC = \frac{rise}{run} = \frac{38-20}{3.5-0.5} = 6$ mph.

4.) a.) constant velocity

b.) stopped

c.) increasing velocity (accelerating)

d.) decreasing velocity (decelerating)

5.) @ $t = 3$ hr. $\rightarrow$

speed = 30 mph.