Math 16A
Section 2.4

Product Rule, Quotient Rule

Rules for Differentiation

1.) $D(c) = 0$

2.) $D(mx+b) = m$

3.) $D(x^n) = n x^{n-1}$

4.) $D(f(x) \pm g(x)) = f'(x) \pm g'(x)$

5.) $D(c f(x)) = c f'(x)$

6.) $D(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$
(Product Rule)

7.) $D(f(x)g(x)h(x)) = f'(x)g(x)h(x)$
$+ f(x)g'(x)h(x) + f(x)g(x)h'(x)$
(Triple Product Rule)

8.) $D\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

(Quotient Rule)

Example: Differentiate. Do not simplify answers.

1.) $y = (x^3 - 7x) \cdot x^{-7}$ $\xrightarrow{D}$

$y' = (x^3 - 7x) \cdot (-7x^{-8}) + (3x^2 - 7) \cdot x^{-7}$

2.) $f(x) = \dfrac{1 + 4x^{1/2}}{x^2 - x}$ $\xrightarrow{D}$

$$f'(x) = \frac{(x^2 - x)(4 \cdot \frac{1}{2} x^{-1/2}) - (1 + 4x^{1/2})(2x - 1)}{(x^2 - x)^2}$$

3.) $g(x) = (5x+7)(x^{10} + x^{-10})(3\sqrt{x})$ $\xrightarrow{D}$

$g'(x) = \{5\} \cdot (x^{10} + x^{-10})(3\sqrt{x})$

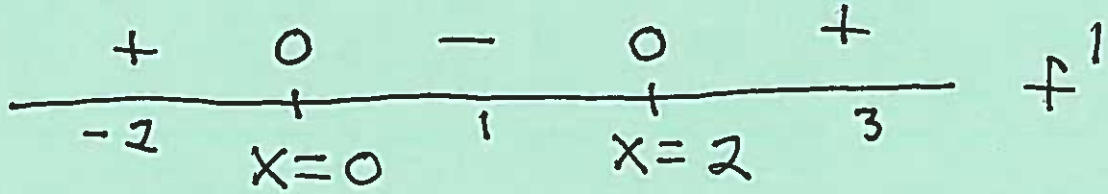
$+ (5x+7) \cdot \{10x^9 - 10x^{-11}\} \cdot (3 \cdot \sqrt{x})$

$+ (5x+7)(x^{10} + x^{-10}) \cdot \{3 \cdot \frac{1}{2} x^{-1/2}\}$

Example: Differentiate each function. Then solve $f'(x) = 0$ for x and set up a Sign Chart for f'.

1.) $f(x) = x^4(2x - 5)$ $\xrightarrow{D}$

$$f'(x) = x^4 \cdot (2) + (4x^3)(2x-5)$$
$$= 2x^4 + 8x^4 - 20x^3$$
$$= 10x^4 - 20x^3$$
$$= 10x^3(x-2) = 0$$

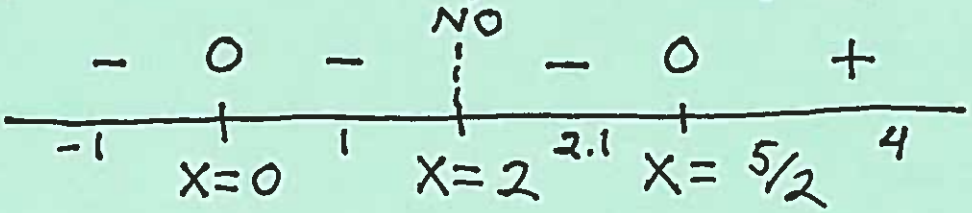


2.) $y = \dfrac{x^5}{2x-4}$ $\xrightarrow{D}$

$$y' = \frac{(2x-4)(5x^4) - x^5(2)}{(2x-4)^2}$$
$$= \frac{10x^5 - 20x^4 - 2x^5}{(2x-4)^2}$$
$$= \frac{8x^5 - 20x^4}{(2x-4)^2} = \frac{4x^4(2x-5)}{(2x-4)^2} = 0$$

$\rightarrow$ $4x^4(2x-5) = 0 \rightarrow x = 0,\ x = 5/2$

and $2x - 4 \neq 0$, so $x \neq 2$ $\rightarrow$



3.) $y = 2x - \sqrt{x}$ $\xrightarrow{D}$

$y' = 2 - \frac{1}{2}x^{-1/2} = \frac{2}{1} - \frac{1}{2\sqrt{x}}$

$= \frac{4\sqrt{x} - 1}{2\sqrt{x}} = 0 \rightarrow$

$4\sqrt{x} - 1 = 0 \rightarrow 4\sqrt{x} = 1 \rightarrow \sqrt{x} = \frac{1}{4}$

$\rightarrow x = \frac{1}{16}$, and $x \neq 0 \rightarrow$