Math 16A
Section 2.6

Higher-Order Derivatives

Example: $f(x) = x^3 + x^4 \xrightarrow{D}$

$f'(x) = 3x^2 + 4x^3 \xrightarrow{D}$

$f''(x) = 6x + 12x^2 \xrightarrow{D}$

$f'''(x) = 6 + 24x \xrightarrow{D}$

(NOTATION CHANGE)

$f^{(4)}(x) = 24 \xrightarrow{D}$

$f^{(5)}(x) = 0$

Example: $f(x) = x \sin x \xrightarrow{D}$

$f'(x) = x\cos x + \sin x \xrightarrow{D}$

$f''(x) = -x\sin x + \cos x + \cos x$

$= 2\cos x - x\sin x \xrightarrow{D}$

$f'''(x) = -2\sin x - (x\cos x + \sin x)$

$= -3\sin x - x\cos x$

Example: Solve $f'(x)=0$ and $f''(x)=0$ for x.

1.) $f(x) = 4x^3 - 3x^2 + 7 \xrightarrow{D}$

$$f'(x) = 12x^2 - 6x$$

$$= 6x(2x-1) = 0 \longrightarrow$$

$$\boxed{x=0,\ x=\frac{1}{2}} \quad ; \quad \xrightarrow{D}$$

$$f''(x) = 24x - 6 = 0 \longrightarrow$$

$$\boxed{x = \frac{1}{4}}$$

2.) $f(x) = x(3x-5)^3 \xrightarrow{D}$

$$f'(x) = x \cdot 3(3x-5)^2(3) + (1)(3x-5)^3$$

$$= (3x-5)^2[9x + (3x-5)]$$

$$= (3x-5)^2[12x-5] = 0 \longrightarrow$$

$$\boxed{x = \frac{5}{3},\ x = \frac{5}{12}} \quad \xrightarrow{D}$$

$$f''(x) = (3x-5)^2(12) + 2(3x-5)(3)[12x-5]$$

$$= 6(3x-5)[2(3x-5) + (12x-5)]$$

$$= 6(3x-5)[6x-10+12x-5]$$

$$= 6(3x-5)[18x-15] = 0 \longrightarrow$$

$$\boxed{x = \frac{5}{3}, \quad x = \frac{15}{18} = \frac{5}{6}}$$

3.) $f(x) = \dfrac{x}{x^2+16} \xrightarrow{D}$

$$f'(x) = \frac{(x^2+16)(1) - x(2x)}{(x^2+16)^2}$$

$$= \frac{x^2+16-2x^2}{(x^2+16)^2} = \frac{16-x^2}{(x^2+16)^2}$$

$$= \frac{(4-x)(4+x)}{(x^2+16)^2} = 0 \longrightarrow \boxed{x=4, x=-4};$$

$$\rightarrow f''(x) = \frac{(x^2+16)^2(-2x) - (16-x^2)\cdot 2(x^2+16)(2x)}{(x^2+16)^4}$$

$$= \frac{2x\,\cancel{(x^2+16)}\,[-(x^2+16) - 2(16-x^2)]}{(x^2+16)^{\cancel{4}\,3}}$$

$$= \frac{2x[-x^2-16-32+2x^2]}{(x^2+16)^3}$$

$$= \frac{2x\,[x^2 - 48]}{(x^2+16)^3}$$

$$= \frac{2x\,(x-\sqrt{48})(x+\sqrt{48})}{(x^2+16)^3} = 0 \rightarrow$$

$$\boxed{x = 0, \quad x = \pm\sqrt{48} = \pm 4\sqrt{3}}$$

4.) $f(x) = \sqrt{3}\sin x - \cos x, \quad 0 \le x \le 2\pi;$

$\xrightarrow{D} f'(x) = \sqrt{3}\cos x - (-\sin x) = 0 \rightarrow$

$$\sin x = -\sqrt{3}\cos x \longrightarrow$$

$$\frac{\sin x}{\cos x} = -\sqrt{3} = \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} \longrightarrow$$

$$\boxed{x = \frac{5\pi}{3}, \quad x = \frac{2\pi}{3}} \quad ; \xrightarrow{D}$$

$$f''(x) = -\sqrt{3}\sin x + \cos x = 0 \rightarrow$$

$$\sqrt{3}\sin x = \cos x \longrightarrow$$

$$\frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} = \frac{-1/2}{-\sqrt{3}/2} \longrightarrow$$

$$\boxed{x = \frac{\pi}{6}, \quad x = \frac{7\pi}{6}}$$