Math 16A
Section 2.6

Gravity Problems

FACT: The height $s(t)$ (feet) of an object at time t (seconds) above the ground, with initial height s_0 and initial velocity v_0 is given by

$$\boxed{s(t) = -16t^2 + v_0 t + s_0}$$

I.) The motion is always vertical (up and/or down).

II.) Ground level is $s(t) = 0$.

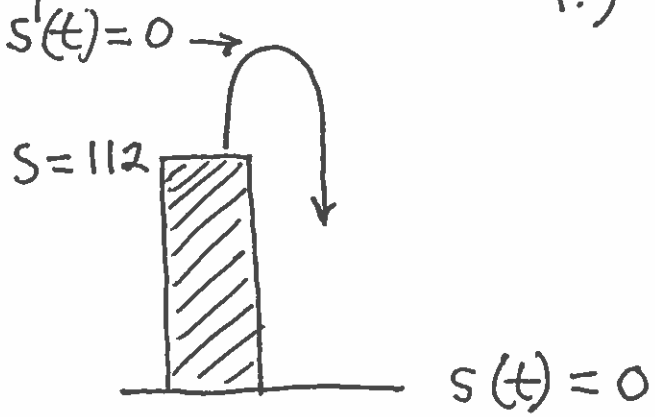
Recall: If $s(t)$ is distance at time t, then

$s'(t) = v(t)$ is velocity at time t

and

$s''(t) = v'(t) = a(t)$ is acceleration at time t.

Example: a ball is projected upward at 96 ft./sec. from the top of a 112 ft. high building.



1.) How high does the ball go?

2.) How long is the ball in the air?

3.) What is the ball's velocity when
   a.) $t=1$ sec.?
   b.) $t=5$ sec.?
   c.) the ball strikes the ground?

4.) What is the ball's acceleration when
   a.) $t=0$ sec.?
   b.) $t=4$ sec.?

Begin with

height:

$s(t) = -16t^2 + 96t + 112$

$\xrightarrow{D}$

velocity:

$s'(t) = -32t + 96$

$\xrightarrow{D}$

acceleration:

$s''(t) = -32$

1.) highest point ($s'(t) = 0$):

$s'(t) = -32t + 96 = 0 \rightarrow t = 3$ sec.

$\rightarrow s(3) = -16(3)^2 + 96(3) + 112 = 256$ ft.

2.) strike ground ($s(t) = 0$):

$s(t) = -16t^2 + 96t + 112$

$= -16(t^2 - 6t - 7)$

$= -16(t-7)(t+1) = 0 \rightarrow$

~~$t \neq -1$~~ (NO) or $t = 7$ sec.

3.) a.) $s'(1) = 64$ ft./sec. ($\uparrow$ up)

b.) $s'(5) = -64$ ft./sec. ($\downarrow$ down)

c.) $s'(7) = -128$ ft./sec.

4.) a.) $s''(0) = -32$ ft./sec.^{2}

b.) $s''(4) = -32$ ft./sec.^{2}

Example: An eagle egg falls from its nest, which is 1600 ft. above the ground.

1.) In how many seconds will the egg strike the ground?

2.) What is its velocity as the egg strikes the ground?

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assume $s(t) = -16t^2 + v_0 t + s_0$;

egg "falls" means $v_0 = 0$ $\longrightarrow$

$$s(t) = -16t^2 + (0)t + 112 \longrightarrow$$

$$\boxed{\begin{aligned} s(t) &= -16t^2 + 1600 \\ s'(t) &= -32t \end{aligned}} \xrightarrow{D}$$

1.) strike ground ($s(t) = 0$):

$$s(t) = -16t^2 + 1600 = 0 \rightarrow t^2 = 100$$

$$\longrightarrow t = 10 \text{ sec.}$$

2.) $s'(10) = -320$ ft./sec.