Math 16A
Section 3.8

## The Differential

The Differential of a function is a formula which we will use to

1.) estimate the values of numbers

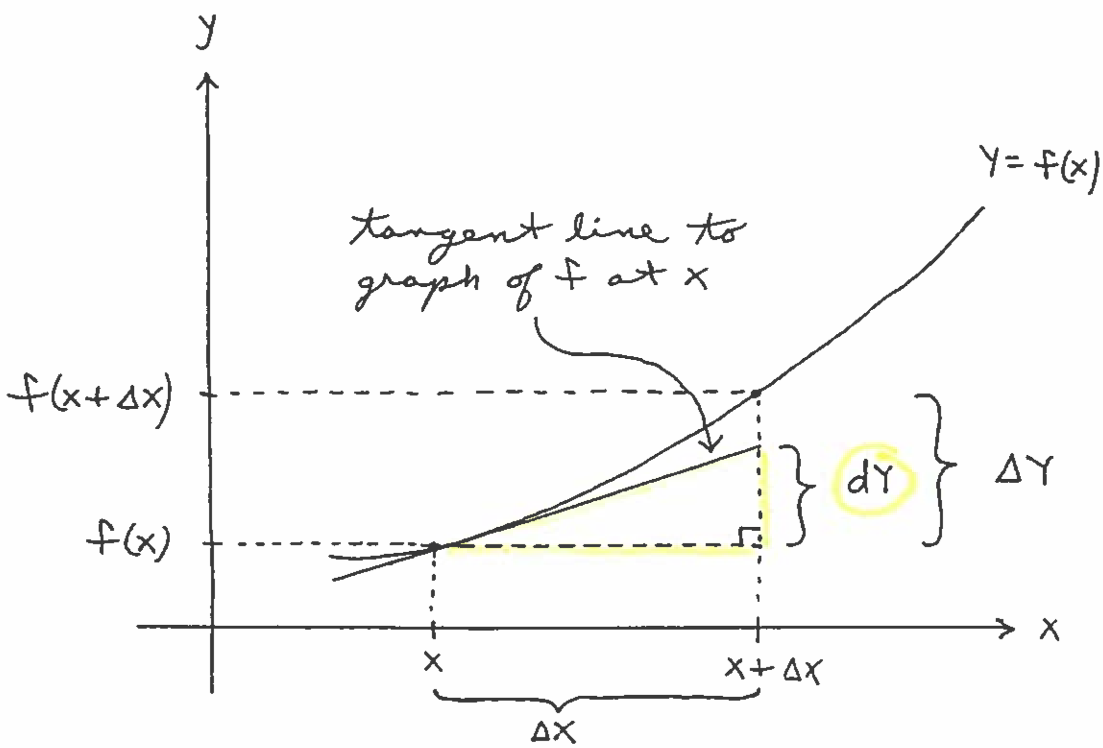
and

2.) estimate propagated absolute percentage errors.

Please study the diagram and derivations on the next page.

The Differential is denoted as $\boxed{dy}$ in the digram. It is the vertical edge of the highlighted right triangle, which is formed by two x-values, $x$ and $x+\Delta x$, and the tangent line at point $(x, f(x))$.

Math 16A
Kouba
The Differential




Define the <u>exact change</u> in y to be

$$\boxed{\Delta Y = f(x+\Delta x) - f(x)} \ .$$

Define dY to be the height of the right triangle in the diagram. Then

$$\boxed{\Delta Y \approx dY} \quad \text{for} \quad \Delta x \quad \text{small}.$$

In addition, the slope of the tangent line to the graph of f at x is

$$f'(x) = \frac{\text{rise}}{\text{run}} = \frac{dY}{\Delta x} \qquad \text{so that}$$

$$\boxed{dY = f'(x) \cdot \Delta x} \ ;$$

dY is called the <u>differential</u> of Y.

Summary: Let $y = f(x)$ be a function.

1.) The Differential requires a function and two x-values, $x$ and $x + \Delta x$.

   a.) $\Delta x$ can be (+) or (−).

   b.) $x$ is considered the "starting" x-value. It is not necessarily the smaller of the two x-values.

2.) The Differential $dy$ is the vertical edge of the right triangle formed by $x$, $x+\Delta x$, and the tangent line at the point $(x, f(x))$. The Differential can be (+), (−), or (0).

3.) The Exact Change of y-values is $\boxed{\Delta y = f(x+\Delta x) - f(x)}$.

4.) The Differential $dy$ is

$$\boxed{dy = f'(x) \cdot \Delta x}$$

↑ "starting" x-value

5.) If $\Delta X$ is a "small" number (either (+) or (−)), then

$$\boxed{\Delta Y \approx dY} .$$

<u>Example</u>: Let $f(x) = x^3 - 2x^2$ .
Find $\Delta Y$ and $dY$ for each of the following pairs of x-values. Compare their values.

1.) $X : 2 \rightarrow 2.8$

2.) $X : 2 \rightarrow 2.1$

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1.) $X : 2 \rightarrow 2.8$, so $\Delta X = 2.8 - 2 = 0.8$;

$y = x^3 - 2x^2 \xrightarrow{D} y' = 3x^2 - 4x$ ;

$$\begin{aligned} \Delta Y &= f(2.8) - f(2) \\ &= (2.8)^3 - 2(2.8)^2 - (8-8) \\ &= \boxed{6.272} \ ; \end{aligned}$$

$$\begin{aligned} dY &= f'(2) \cdot \Delta X \\ &= (12-8) \cdot (0.8) \\ &= \boxed{3.2} \ ; \text{ but } dY \not\approx \Delta Y \end{aligned}$$

since 2.8 is NOT "near" 2.

2.) $x : 2 \to 2.1$, so $\Delta x = 2.1 - 2 = 0.1$ ;

$y = x^3 - 2x^2 \xrightarrow{D} y' = 3x^2 - 4x$ ;

$$\Delta y = f(2.1) - f(2)$$
$$= (2.1)^3 - 2(2.1)^2 - (0)$$
$$= \boxed{0.441} \; ;$$

$$dy = f'(2) \cdot \Delta x$$
$$= (4)(0.1)$$
$$= \boxed{0.400} \; ; \text{ and } dy \approx \Delta y$$

since 2.1 is "near" 2.