Using a Differential to Estimate the Value of a Number

Example: Use a Differential to estimate the value of $\sqrt{29}$.

We need to start with:

1. a function
2. two x-values, "close" together
3. one of the x-values must be "convenient", and it will be the "starting" x-value;

Let $f(x) = \sqrt{x}$ and use $x: 25 \to 29$;

then $f'(x) = \frac{1}{2\sqrt{x}}$ and $\Delta x = 29 - 25 = 4$;

$\Delta y = f(29) - f(25) = \sqrt{29} - \sqrt{25} = \sqrt{29} - 5$;

$dy = f'(25)\,\Delta x = \frac{1}{2\sqrt{25}}(4) = 0.4$; and

$\Delta y \approx dy \to \sqrt{29} - 5 \approx 0.4 \to$

$\boxed{\sqrt{29} \approx 5.4}$. Is it a good estimate?

Calculator: $\sqrt{29} \approx 5.385$, so YES!

Example: Use a Differential to estimate the value of $(61)^{1/3}$.

Let $f(x) = x^{1/3}$ and use $x: 64 \to 61$;

then $f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$ and

$\Delta x = 61 - 64 = -3$;

$\Delta y = f(61) - f(64) = (61)^{1/3} - (64)^{1/3} = (61)^{1/3} - 4$;

$dy = f'(64) \cdot \Delta x = \frac{1}{3(64)^{2/3}}(-3) = \frac{1}{3(16)}(-3)$

$= \frac{-1}{16} = -0.0625$; then

$\Delta y \approx dy \longrightarrow (61)^{1/3} - 4 \approx -0.0625 \to$

$\boxed{(61)^{1/3} \approx 3.9375}$.

Is it a good estimate?

Calculator: $(61)^{1/3} \approx 3.9365$,

so YES!

Example: Use a Differential to estimate the value of $(30)^{3/2}$, using

1.) $x = 25$ as a starting x-value.

2.) $x = 36$ as a starting x-value.

Let $f(x) = x^{3/2} \xrightarrow{D} f'(x) = \frac{3}{2}x^{1/2}$;

1.) Use $x: 25 \to 30$; then

$$\Delta x = 30 - 25 = 5 \text{ ;}$$

$$\Delta y = f(30) - f(25) = (30)^{3/2} - (25)^{3/2}$$

$$= (30)^{3/2} - (25)^{1/2 \cdot 3} = (30)^{3/2} - 125 \text{ ;}$$

$$dy = f'(25) \cdot \Delta x = \frac{3}{2}\sqrt{25} \cdot (5)$$

$$= \frac{75}{2} = 37.5 \text{ ; then}$$

$$\Delta y \approx dy \to (30)^{3/2} - 125 \approx 37.5 \to$$

$$\boxed{(30)^{3/2} \approx 162.5} \text{ ;}$$

Calculator: $(30)^{3/2} \approx 164.32$

2.) Use $X: 36 \to 30$; then

$$\Delta X = 30 - 36 = -6 \ ;$$

$$\Delta Y = f(30) - f(36) = (30)^{3/2} - (36)^{3/2}$$

$$= (30)^{3/2} - (36)^{\frac{1}{2}\cdot 3} = (30)^{3/2} - 216 \ ;$$

$$dy = f'(36) \cdot \Delta X = \frac{3}{2}\sqrt{36} \cdot (-6)$$

$$= \frac{3}{2}(-36) = -54 \ ; \text{ then}$$

$$\Delta Y \approx dy \to (30)^{3/2} - 216 \approx -54 \to$$

$$\boxed{(30)^{3/2} \approx 162} \ ;$$

Calculator: $(30)^{3/2} \approx 164.32$