Math 16A
Section 8.4

Trig Derivatives

Rules: 1.) $D(\sin x) = \cos x$

2.) $D(\cos x) = -\sin x$

3.) $D(\sec x) = \sec x \tan x$

4.) $D(\tan x) = \sec^2 x$

5.) $D(\cot x) = -\csc^2 x$

6.) $D(\csc x) = -\csc x \cot x$

Proof: 3.) $D(\sec x) = D\left(\frac{1}{\cos x}\right)$

$$= \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \cdot \tan x \quad .$$

4.) $D(\tan x) = D\left(\frac{\sin x}{\cos x}\right)$

$$= \frac{\cos x \cdot (\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \left(\frac{1}{\cos x}\right)^2 = (\sec x)^2 = \sec^2 x .$$

Example: Differentiate each function. Do NOT simplify answers.

1.) $y = x^3 \tan x \xrightarrow{D}$

$$y' = x^3 \cdot (\sec^2 x) + (3x^2) \cdot \tan x$$

2.) $y = \sqrt{x} \cdot \sin x \cdot \cos x \xrightarrow{D}$

$$y' = \left(\tfrac{1}{2} x^{-1/2}\right) \sin x \cos x + \sqrt{x}\,(\cos x) \cos x + \sqrt{x} \cdot \sin x \cdot (-\sin x)$$

3.) $y = \dfrac{2x - 3\cos x}{1 + \cot x} \xrightarrow{D}$

$$y' = \frac{(1+\cot x)\{2+3\sin x\} - (2x - 3\cos x)\{-\csc^2 x\}}{(1+\cot x)^2}$$

<u>Example</u>: Find an equation of the line perpendicular to the graph of $y = 3 + 4\cos x$ at $x = \frac{\pi}{3}$.

$$x = \frac{\pi}{3} \to y = 3 + 4\cos\left(\frac{\pi}{3}\right)$$
$$= 3 + 4\left(\tfrac{1}{2}\right) = 5 \; ; \; \xrightarrow{D}$$

$$y' = 0 + 4(-\sin x) \text{ and } x = \frac{\pi}{3} \to$$
$$y' = -4\sin\left(\frac{\pi}{3}\right)$$
$$= -4 \cdot \frac{\sqrt{3}}{2} = -2\sqrt{3} \text{ , so}$$

SLOPE of $(\perp)$ line is

$$m = \frac{-1}{-2\sqrt{3}} = \frac{1}{2\sqrt{3}} \text{ and line is}$$

$$y - 5 = \frac{1}{2\sqrt{3}}\left(x - \frac{\pi}{3}\right) .$$

<u>Example</u>: Solve $f'(\theta) = 0$ for $0 \le \theta \le 2\pi$:

$$f(\theta) = \sin\theta + \sin^2\theta \to$$

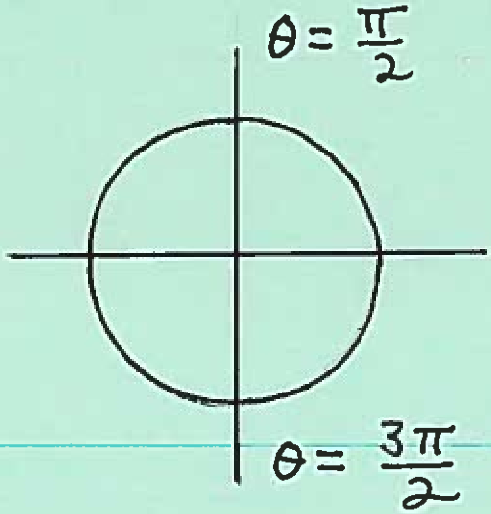
$$f(\theta) = \sin\theta + \sin\theta \cdot \sin\theta \xrightarrow{D}$$

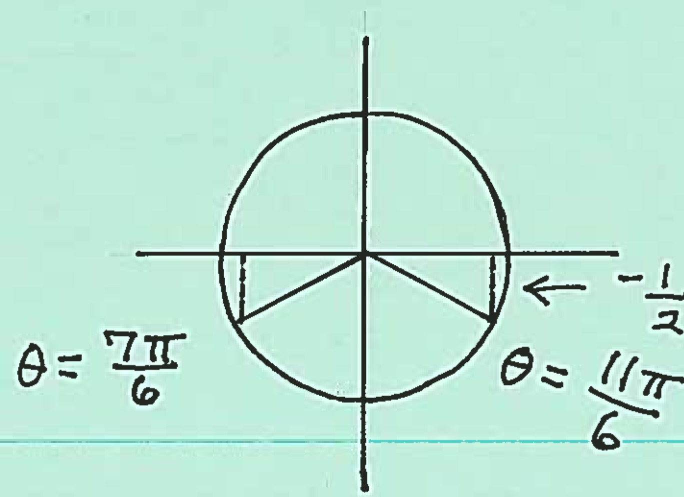
$$f'(\theta) = \cos\theta + \sin\theta \cdot \cos\theta + \cos\theta \cdot \sin\theta$$

$$= \cos\theta + 2\sin\theta\cos\theta$$

$$= \cos\theta\,(1 + 2\sin\theta) = 0 \rightarrow$$

$$\cos\theta = 0 \quad \text{OR} \quad \sin\theta = \frac{-1}{2}$$



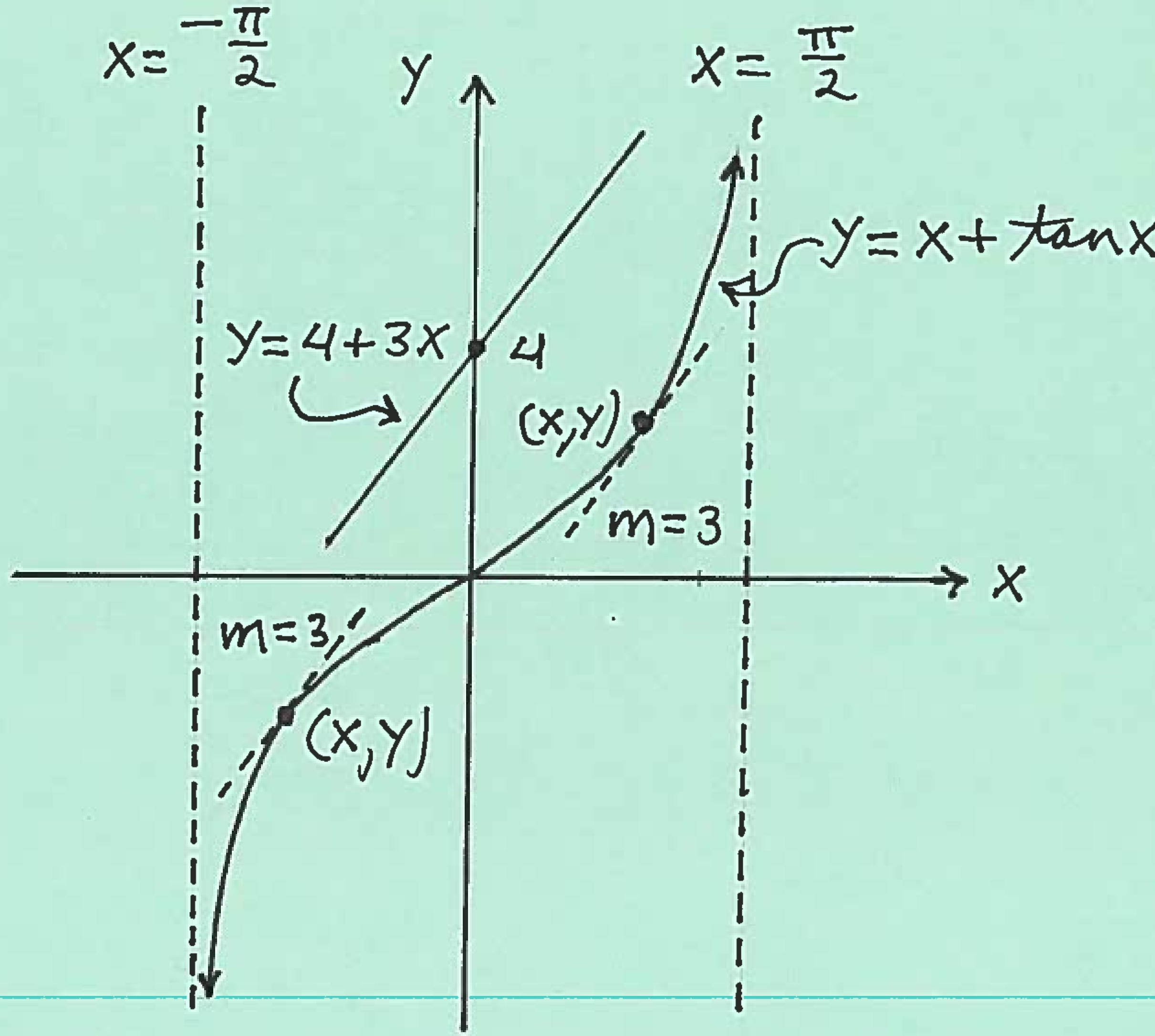


$$\rightarrow \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

<u>Example</u>: Find all points x, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, with Tangent Lines to the graph of

$$y = x + \tan x$$

parallel to the line $y = 4 + 3x$.



$$y = x + \tan x \xrightarrow{D} y' = \underline{\underline{1 + \sec^2 x = 3}} \rightarrow$$

$$\sec^2 x = 2 \rightarrow \sec x = \pm\sqrt{2} \rightarrow$$

$$\frac{1}{\cos x} = \pm\sqrt{2} \rightarrow \cos x = \frac{\pm 1}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2},$$

but $-\frac{\pi}{2} < x < \frac{\pi}{2}$, so $\cos x = \frac{+\sqrt{2}}{2} \rightarrow$

$$x = \frac{\pi}{4}, \quad x = -\frac{\pi}{4}$$