## Math 16A
Sections 8.1, 8.2, 8.3

Trigonometry Review

1. $\sin^2\theta + \cos^2\theta = 1$

2. $1 + \tan^2\theta = \sec^2\theta$

3. $1 + \cot^2\theta = \csc^2\theta$

4. $\sin 2\theta = 2\cos\theta\sin\theta$

5. $\cos 2\theta = 2\cos^2\theta - 1$

$$= 1 - 2\sin^2\theta$$

$$= \cos^2\theta - \sin^2\theta$$

$\cos\theta = x/r$

$\sin\theta = y/r$

$\tan\theta = y/x$

$\sec\theta = r/x$

$\csc\theta = r/y$

$\cot\theta = x/y$

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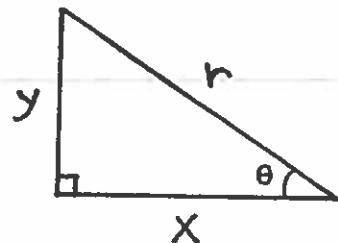
degrees	radians	$\cos\theta$	$\sin\theta$	$\tan\theta$
0°	0	1	0	0
30°	$\pi/6$	$\sqrt{3}/2$	1/2	$1/\sqrt{3}$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	1/2	$\sqrt{3}/2$	$\sqrt{3}$
90°	$\pi/2$	0	1	undefined

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Cosine and Sine Values:

$\theta = \frac{\pi}{2} = 90°$ $(0, 1)$

$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ $\theta = \frac{3\pi}{4} = 135°$

$\theta = \frac{\pi}{4} = 45°$ $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$(-1, 0)$ $\theta = \pi = 180°$

$\theta = 0 = 2\pi = 360°$ $(1, 0)$

$\theta = \frac{7\pi}{4} = 315° = -\frac{\pi}{4} = -45°$
$(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

$\theta = \frac{5\pi}{4} = 225°$
$(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$
$= \frac{-3\pi}{4} = -135°$

$\theta = \frac{3\pi}{2} = 270°$
$= -\frac{\pi}{2} = -90°$
$(0, -1)$

$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ $\theta = \frac{2\pi}{3} = 120°$

$\theta = \frac{\pi}{3} = 60°$ $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$ $\theta = \frac{5\pi}{6} = 150°$

$\theta = \frac{\pi}{6} = 30°$ $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

$\theta = \frac{7\pi}{6} = 210°$
$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$
$= -\frac{5\pi}{6} = -150°$

$\theta = \frac{11\pi}{6} = 330° = -\frac{\pi}{6} = -30°$
$(\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$\theta = \frac{4\pi}{3} = 240°$

$\theta = \frac{5\pi}{3} = 300° = -\frac{\pi}{3} = -60°$

$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$ $= -\frac{2\pi}{3} = -120°$

$(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

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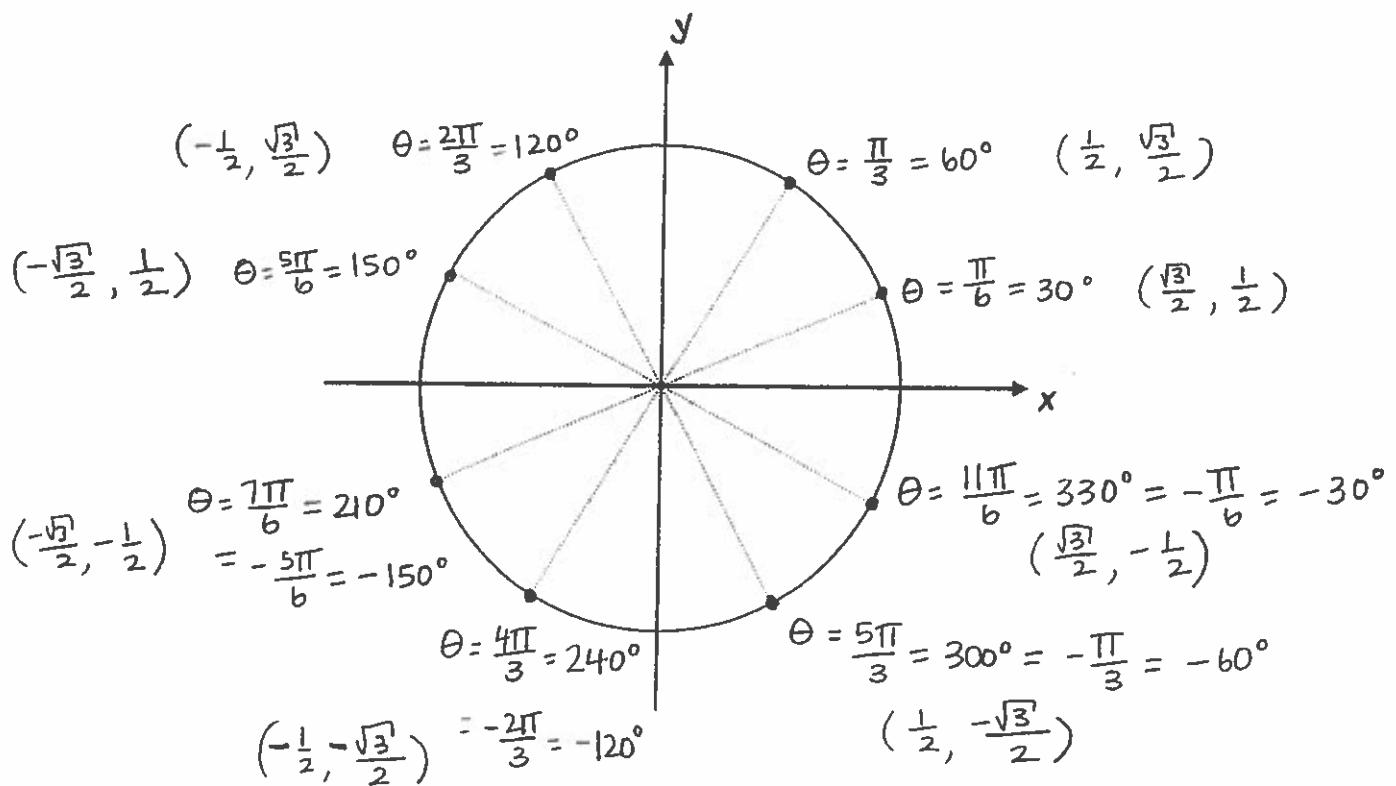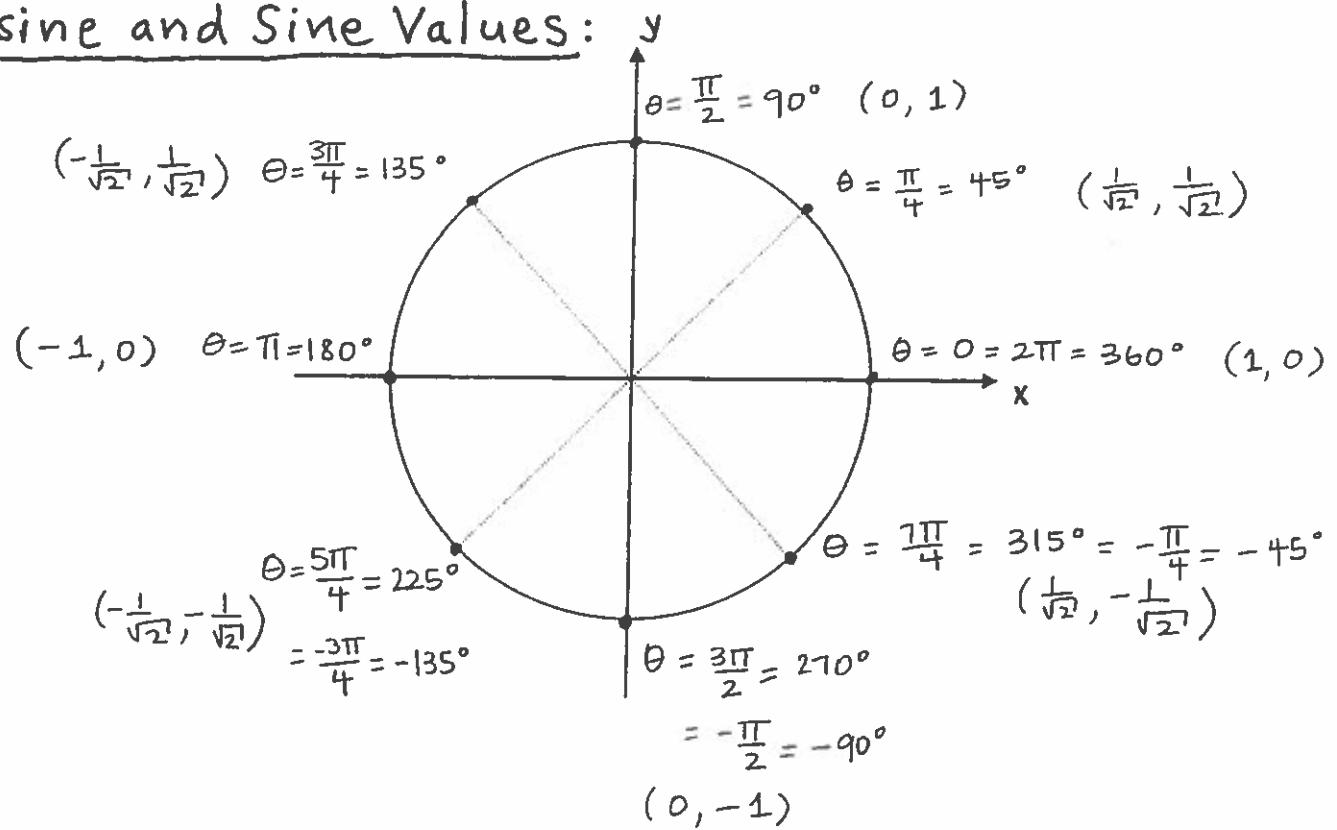
FACT : $360° = 2\pi$ radians

Example : Solve for θ, $0 \leq \theta \leq 2\pi$.

1.) $\sin \theta = \frac{1}{2}$

$\rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

(circle diagram with $\frac{5\pi}{6}$, $\frac{\pi}{6}$, $1/2$, labels 1, -1)

2.) $\cos \theta = \frac{-\sqrt{2}}{2}$

$\rightarrow \theta = \frac{3}{4}\pi, \frac{5}{4}\pi$

(circle diagram with $\frac{3}{4}\pi$, $\frac{5\pi}{4}$, $-\frac{\sqrt{2}}{2}$, labels 1, -1)
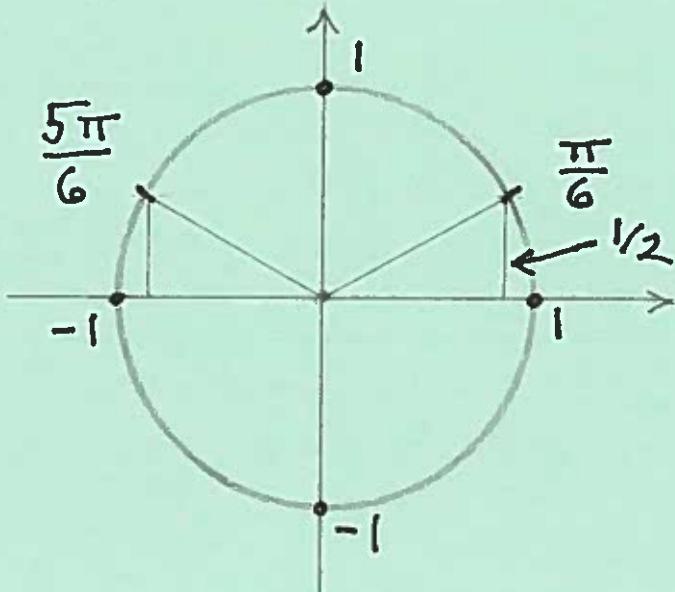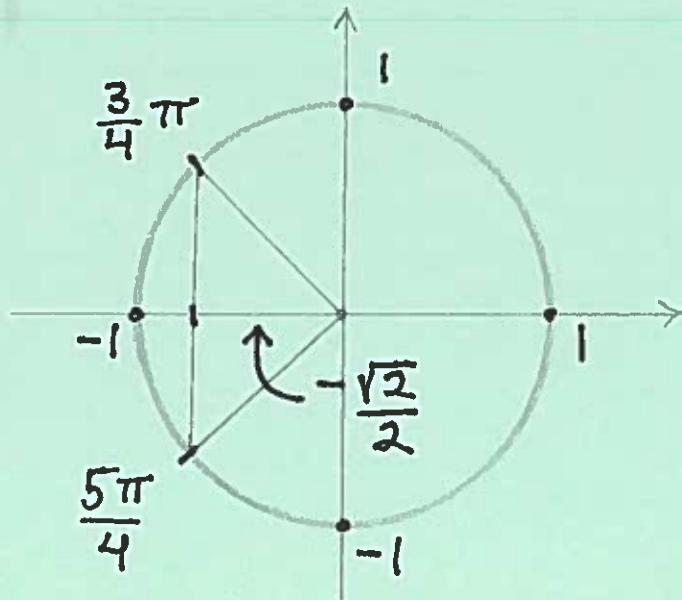
3.) $\cos 2\theta + \cos \theta = 0 \rightarrow$
$(2\cos^2\theta - 1) + \cos \theta = 0 \rightarrow$
$2(\cos \theta)^2 + (\cos \theta) - 1 = 0 \rightarrow$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0 \longrightarrow$$

$$\cos\theta = \frac{1}{2} \quad \text{or} \quad \cos\theta = -1$$

$$\longrightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3} \qquad\qquad \longrightarrow \theta = \pi$$

4.) $4\sin^2\theta = 3 : \sin^2\theta = \frac{3}{4} \longrightarrow$

$$\sin\theta = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2} \quad ;$$

a.) $\sin\theta = \frac{\sqrt{3}}{2}$

$$\longrightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

b.) $\sin \theta = \dfrac{-\sqrt{3}}{2}$
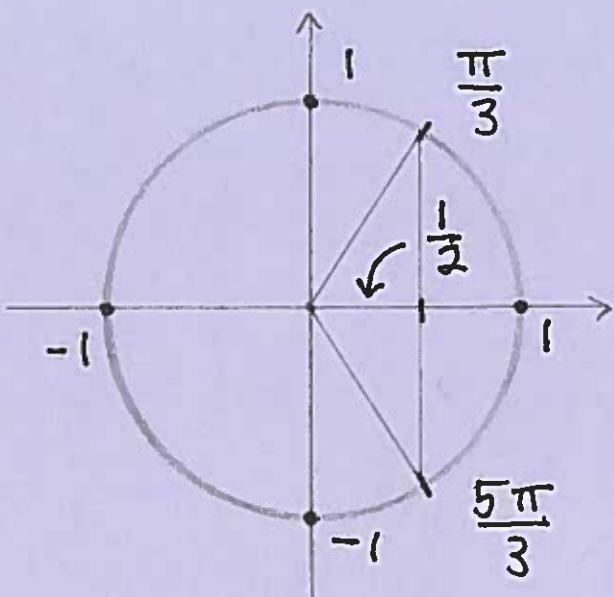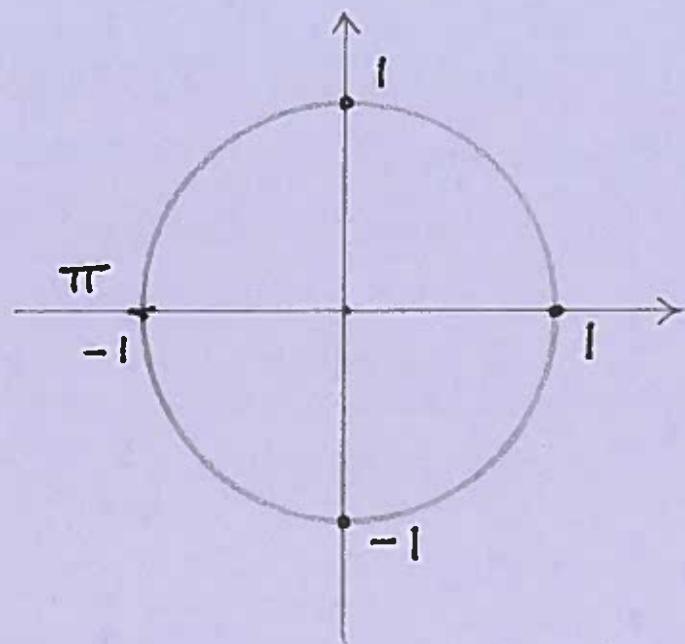
$\rightarrow \quad \theta = \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$
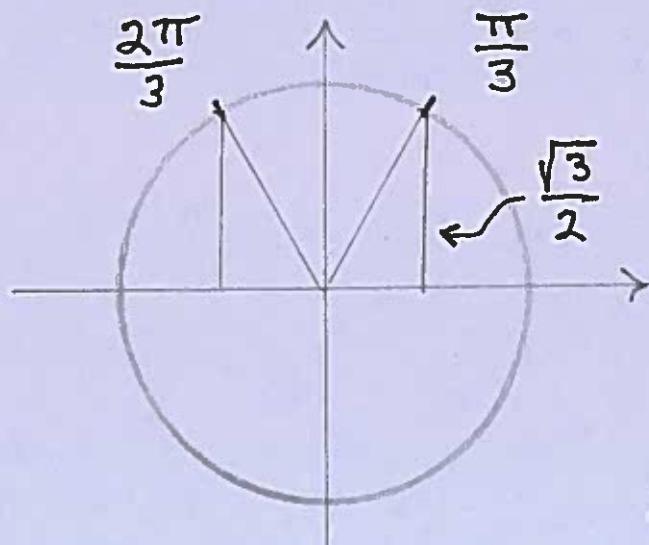
(circle diagram with labels: 1, -1, $-\dfrac{\sqrt{3}}{2}$, $\dfrac{4\pi}{3}$, -1, $\dfrac{5\pi}{3}$)
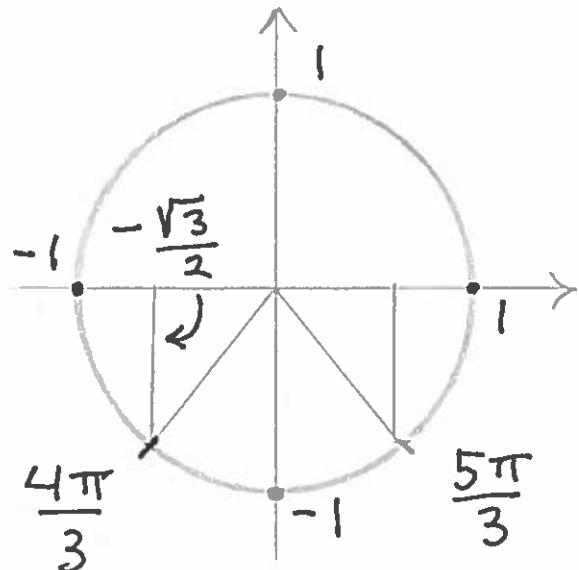
Example : Determine the following limits.

1.) $\displaystyle\lim_{\theta \to \frac{\pi}{2}} \dfrac{\sin 2\theta}{\cos \theta} = \dfrac{\sin \pi}{\cos \frac{\pi}{2}} = "\dfrac{0}{0}"$

$\quad = \displaystyle\lim_{\theta \to \frac{\pi}{2}} \dfrac{2 \sin \theta \cos \theta}{\cos \theta} = 2 \sin \dfrac{\pi}{2} = 2(1) = 2$

2.) $\displaystyle\lim_{\theta \to \frac{\pi}{2}^-} \tan \theta = \displaystyle\lim_{\theta \to \frac{\pi}{2}^-} \dfrac{\sin \theta}{\cos \theta}$

$\quad = \dfrac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \dfrac{"1"}{0^+} = +\infty$

(number line with $89°$ arrow pointing to $x = \dfrac{\pi}{2}$ (90°))

$x = \dfrac{\pi}{2} \ (90°)$

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