Math 16A
Kouba
Worksheet 1

1. A rectangle is three times as long as it is wide. Find its dimensions if its diagonal is 40 cm.

2. a. Write an equation which has x = 10 as the only solution.
   b. Write an equation which has two different solutions, one of which is x = 10.

3. Consider the polynomial $x^2 + A x + 8$. Solve for A so that (x - 4) is a factor of the polynomial.

4. Solve for x : $(x + 3/x) (2 x - 5) = (x - 3/x) (x + 5)$

5. The sum of a certain integer and its square root is 992. Find the integer.

6. At exactly 12:00 noon the hour hand and minute hand of a clock are together. Exactly when is the next time (hour, minute, second) that the hour hand and minute hand of this clock are together ?

7. Juan leaves from Cactus Corners driving straight east at a constant speed. At the same time, Denise takes the road going straight north and drives 20 miles per hour faster than Juan. After two hours they are 200 miles apart. How fast did Denise drive ?

8. Two concentric circles are given in the diagram at right. The area of the shaded region is 75% of the area of the smaller circle. The radius of the larger circle is 10. Compute the area of the smaller circle.
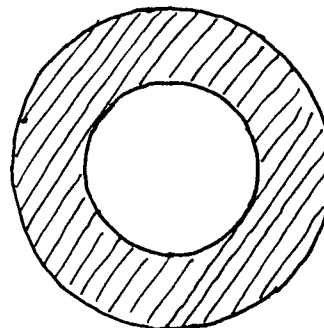
9. Compute the distance between "opposite" corners in a rectangular box with dimensions 3 ft. x 4 ft. x 5 ft.

10. It has been discovered that snow tree crickets have the following peculiar characteristic. All such crickets chirp at the same rate if they are exposed to the same temperature. This means that the chirp rate is a function of temperature, i.e., if we know the temperature, we can determine the chirp rate. Surprisingly, this chirp rate C, in chirps per minute, increases steadily with temperature T, in degrees Fahrenheit, and is closely approximated by the function
$$C = f(T) = 4 T - 160$$
for T between 40° F and 136° F (the highest temperature recorded at a weather station).

   a. What are the domain and range of function f ? Sketch a graph of f.
   b. Find the inverse function f^{-1}, a function for which the number of cricket chirps per minute will determine the temperature.
   c. What is the chirp rate when T = 95° F ?
   d. What is the temperature when the chirp rate is 30 chirps per minute ?

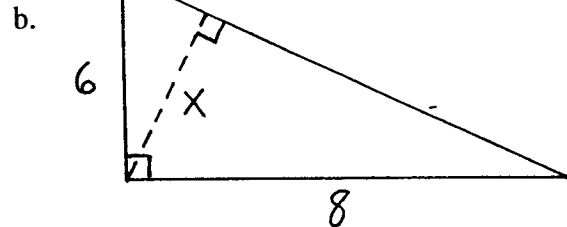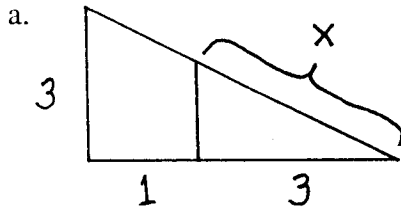11. The following functions have inverses. Find the inverse of each.

    a. $f(x) = (2 - 3x)/(x - 1)$ for $x \neq 1$ b. $f(x) = x^2 - 4x$ for $x \leq 2$

12. Let $f(x) = x/(x + 5)$. Find a function $g(x)$ so that $(f \circ g)(x) = f(g(x)) = 5 - x^3$.

13. Find all points on the graph of the circle $x^2 + y^2 = 4$ at which tangent lines pass through the point $(3, 0)$. See diagram at right.
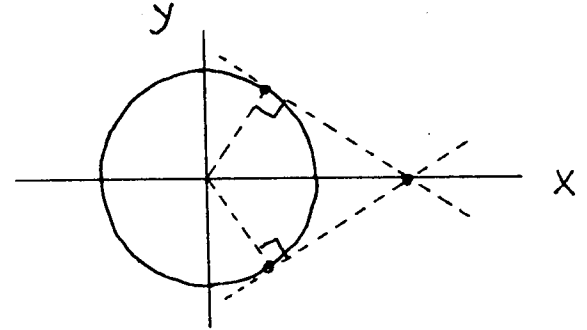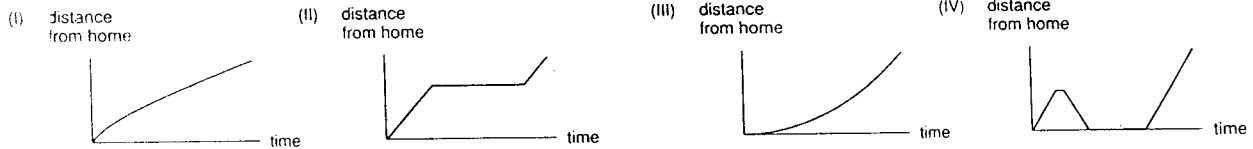
14. Solve for x in each triangle. HINT : You may need to use similar triangles.

    a.   b.

15. Which of the graphs in the figure below best match the following stories ?

    a. I had just left home when I realized I had forgotten my books, and so I went back to pick them up.

    b. Things went fine until I had a flat tire.

    c. I started out calmly but sped up when I realized I was going to be late.

16. Right after a certain drug is administered to a patient with a rapid heart rate, the heart rate plunges dramatically and then slowly rises again as the drug wears off. Sketch a possible graph of the heart rate against time from the moment right before the drug is administered.

17. Describe what the figure at right tells you about an assembly line whose productivity is represented as a function of the number of workers on the line.
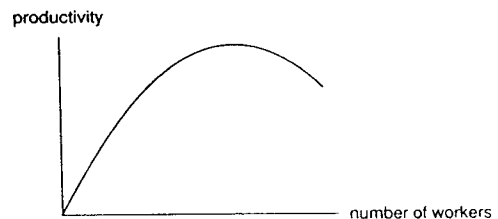

18. In her Guide to Excruciatingly Correct Behavior, Miss Manners states : There are three possible parts to a date of which at least two must be offered : entertainment, food, and affection. It is customary to begin a series of dates with a great deal of entertainment, a moderate amount of food and the merest suggestion of affection. As the amount of affection increases, the entertainment can be reduced proportionately.

When the affection has replaced entertainment, we no longer call it dating. Under no circumstances can the food be omitted.

Based on this statement, sketch a graph showing entertainment as a function of affection, assuming the amount of food to be constant. Mark the point on the graph at which the relationship starts, as well as the point at which the relationship ceases to be called dating.

19. Specify the domain and range of the function $y = f(x)$ given below.
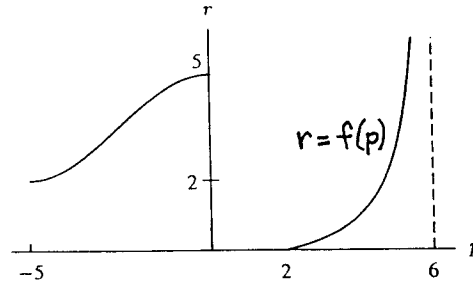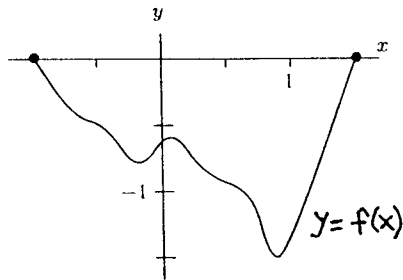
20. The graph of $r = f(p)$ is given above.
    a. What could be the domain of f ?   b. What could be the range of f ?
    c. What values of r could correspoond to exactly one value of p ?

21. A linear equation was used to generate the values in the table below. Find the equation.

x	5.2	5.3	5.4	5.5	5.6
y	27.8	29.2	30.6	32.0	33.4

22. An equation of a line is $3x + 4y = -12$. Find the length of the portion of the line that lies between its x- and y-intercepts.

23. A car rental company offers cars at $40 per day and 15 cents per mile. Its competitor's cars are $50 per day and 10 cents per mile.
    a. For each company, write a formula giving the cost of renting a car for a day as a function of the distance traveled.
    b. On the same axes, sketch graphs of both functions.
    c. How should you decide which company is cheaper ?

24. Hot peppers have been rated according to Scoville units, with a maximum human tolerance level of 14,000 Scovilles per dish. The West Coast Restaurant, known for spicy dishes, promises a daily special to satisfy the most avid spicy-dish fans. The restaurant imports Indian peppers rated at 1200 Scovilles each and Mexican peppers with a Scoville rating of 900 each.
    a. Determine the Scoville constraint equation relating the maximum number of Indian and Mexican peppers the restaurant should use for their specialty dish.
    b. Solve the equation from part a. to show explicity the number of Indian peppers needed in the hottest dishes as a function of the number of Mexican peppers.

25. When a cold ham is put into a hot oven to bake, the temperature of the ham rises. The rate R (in degrees per minute) at which the temperature of the ham rises is governed by Newton's Law of Heating, which says that the rate is directly proportional to the temperature difference between the ham and the oven. Assume that the oven is at $350°$ F and the temperature of the ham is $H°$ F .
    a. Write a formula giving R as a function of H.

b. Sketch the graph of R against H.

26. The values of a function f are given in the table below, where the domain of f is the set of integers from 1 to 7.
   a. What is the domain of f^{-1} ?   b. Write a table of values for f^{-1}.

x	1	2	3	4	5	6	7
$f(x)$	3	-7	19	4	178	2	1

27. The cost of producing q articles is given by the function $C = f(q) = 100 + 2\,q$.
   a. Find a formula for the inverse function.
   b. Explain in practical terms what the inverse function tells you.

28. The figure at right is the graph of the function f, where $f(t)$ is the number (in millions) of motor vehiclesregistered in the world in the year t. (In 1988, one-third of the registered vehicles in the world were in the United States.)
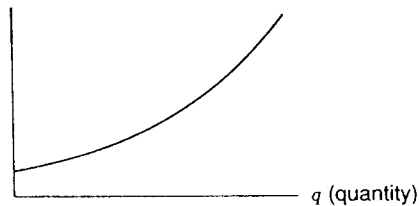   a. Is f invertible ?
   b. Evaluate f^{-1} (400) and determine its meaning in practical terms.
   c. Sketch the graph of f^{-1}.

World population of motor vehicles
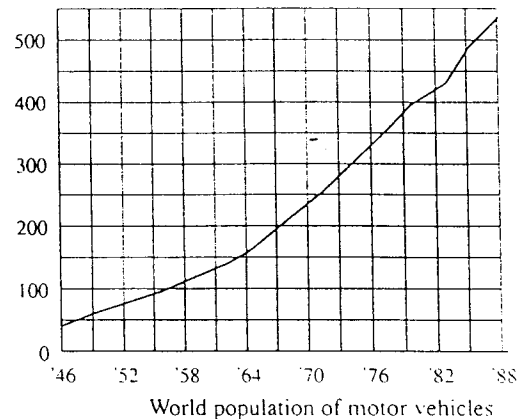
29. Economists are interested in how the quantity, q, of an item which is manufactured and sold depends on its price, p. Since manufacturers and consumers react differently to changes in price, there are two functions relating p and q. The *supply curve* represents how the quantity of an item that manufacturers are willing to supply depends on the price for which the item will be sold. The *demand curve* represents how the quantity of an item demanded by consumers depends on its price. In the diagram below which is the supply curve and which is the demand curve ? Briefly explain your reasoning.

(a)   p (price per unit)   (b)   p (price per unit)

q (quantity)   q (quantity)