Math 16A (Fall 2005)
Kouba
Exam 2

Please PRINT your name here : ____KEY____

Your Exam ID Number ________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. COPYING ANSWERS FROM A CLASSMATE's EXAM IS A VIOLATION OF THE UNIVERSITY HONOR CODE.

4. No notes, books, or handouts may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

5. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive LITTLE or NO credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

6. Make sure that you have 7 pages, including the cover page.

7. You may NOT use the Chain Rule on this exam.

8. You will be graded on proper use of limit and derivative notation.

9. Include units on answers where units are appropriate.

10. You have until 10:50 a.m. sharp to finish the exam.

1.) (7 pts. each) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = 3x^2 + x^{-7} - \frac{5}{11}$

$$y' = 6x - 7x^{-8} - 0$$

b.) $f(x) = x^{100}\sin x$

$$f'(x) = x^{100} \cdot \cos x + 100x^{99} \cdot \sin x$$

c.) $f(x) = \dfrac{x-2}{3-x^5}$

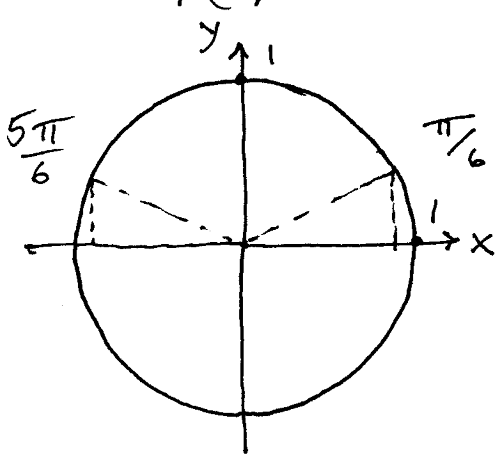
$$f'(x) = \frac{(3-x^5)(1) - (x-2)(-5x^4)}{(3-x^5)^2}$$

d.) $f(x) = (\tan x)^3$ $= \tan x \cdot \tan x \cdot \tan x$ (use TPR)

$$f'(x) = \sec^2 x \cdot \tan x \cdot \tan x + \tan x \cdot \sec^2 x \cdot \tan x + \tan x \cdot \tan x \cdot \sec^2 x$$

2.) (8 pts.) Solve $f'(x) = 0$ for x, $0 \leq x \leq 2\pi$, where $f(x) = x + 2\cos x$.

$$f'(x) = 1 - 2\sin x = 0 \rightarrow \sin x = \frac{1}{2}$$



$\rightarrow$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

3.) (8 pts.) Solve $f'(x) = 0$ for x, where $f(x) = \dfrac{x^3}{x^2 - 12}$.

$$f'(x) = \frac{(x^2-12)\cdot 3x^2 - x^3(2x)}{(x^2-12)^2}$$

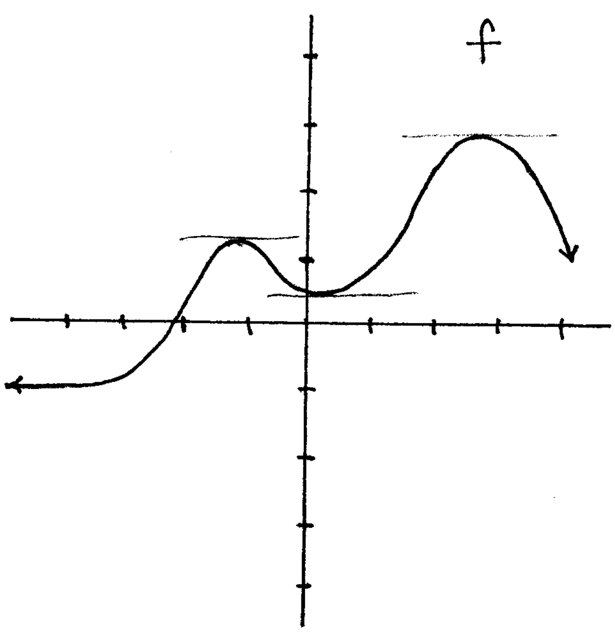
$$= \frac{x^2\left[3(x^2-12) - 2x^2\right]}{(x^2-12)^2}$$

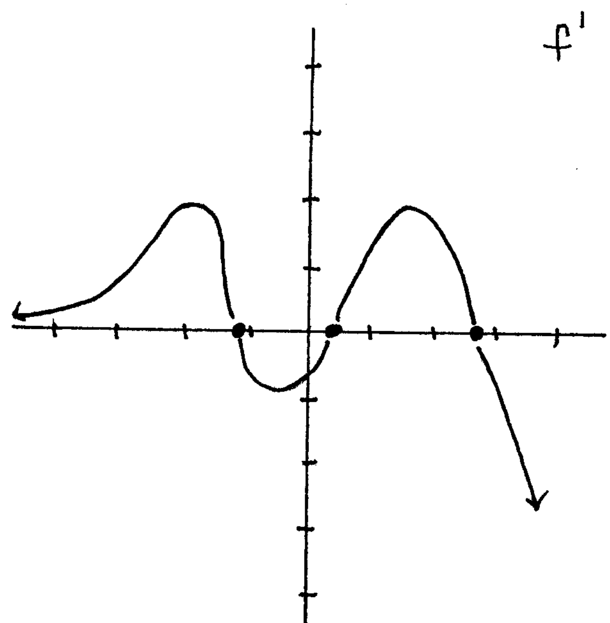
$$= \frac{x^2\left[3x^2 - 36 - 2x^2\right]}{(x^2-12)^2}$$

$$= \frac{x^2\left[x^2 - 36\right]}{(x^2-12)^2}$$

$$= \frac{x^2(x-6)(x+6)}{(x^2-12)^2} = 0 \longrightarrow \quad x=0,\ x=6,\ x=-6$$

4.) (8 pts.) Sketch a graph of the derivative f' using the given graph of f.





5.) (8 pts.) Use $f'(x) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ to differentiate $f(x) = 2x^2 - 3x + 7$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 3(x+\Delta x)+7 - (2x^2 - 3x+7)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x^2 + 2x\cdot\Delta x + (\Delta x)^2) - 3x - 3\cdot\Delta x + 7 - 2x^2 + 3x - 7}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x\cdot\Delta x + 2(\Delta x)^2 - \cancel{3x} - 3\cdot\Delta x + \cancel{7} - \cancel{2x^2} + \cancel{3x} - \cancel{7}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\cdot(4x + 2\cdot\Delta x - 3)}{\cancel{\Delta x}}$$

$$= 4x - 3$$

6.) (8 pts.) Find all points (x, y) on the graph of $f(x) = (2/3)x^{3/2} - x + 1$ with tangent lines to the graph of f *parallel* to the line $3y - 6x = 7$.

$3y - 6x = 7 \rightarrow 3y = 6x + 7 \rightarrow y = 2x + 7/3 \rightarrow$

slope $m = 2$; then

$f'(x) = \frac{2}{3}\cdot\frac{3}{2}x^{1/2} - 1 = \sqrt{x} - 1$

$\rightarrow \sqrt{x} - 1 = 2$

$\rightarrow \sqrt{x} = 3$

$\rightarrow x = 9,\ y = \frac{2}{3}(9)^{3/2} - 9 + 1 = \frac{2}{3}(27) - 8 = 10$

$\rightarrow (x, y) = (9, 10)$

7.) (8 pts.) Determine an equation of the line *perpendicular* to the graph (i.e., perpendicular to the tangent line) of $f(x) = 2x - \sqrt{x}$ at $x = 4$.

$$f'(x) = 2 - \frac{1}{2}x^{-1/2} = 2 - \frac{1}{2\sqrt{x}} \text{ at } x=4 \rightarrow$$

$$f'(4) = 2 - \frac{1}{2\sqrt{4}} = 2 - \frac{1}{4} = \frac{7}{4} \text{ so } \perp \text{ slope}$$

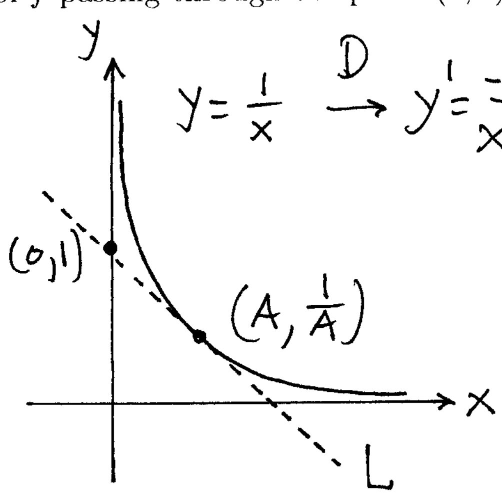
is $m = -4/7$; $x=4 \rightarrow f(4) = 8 - \sqrt{4} = 6$

so line is $y - y_1 = m(x - x_1) \rightarrow$

$$y - 6 = -\frac{4}{7}(x-4)$$

8.) (8 pts.) Find all points (x, y) on the graph of $f(x) = \dfrac{1}{x}$ with tangent lines to the graph of f passing through the point $(0, 1)$.

$$y = \frac{1}{x} \xrightarrow{D} y' = \frac{-1}{x^2}$$



tangent line at $x=A$

SLOPE of line L is

$$\frac{\frac{1}{A} - 1}{A - 0} \text{ or } f'(A) = \frac{-1}{A^2} \rightarrow$$

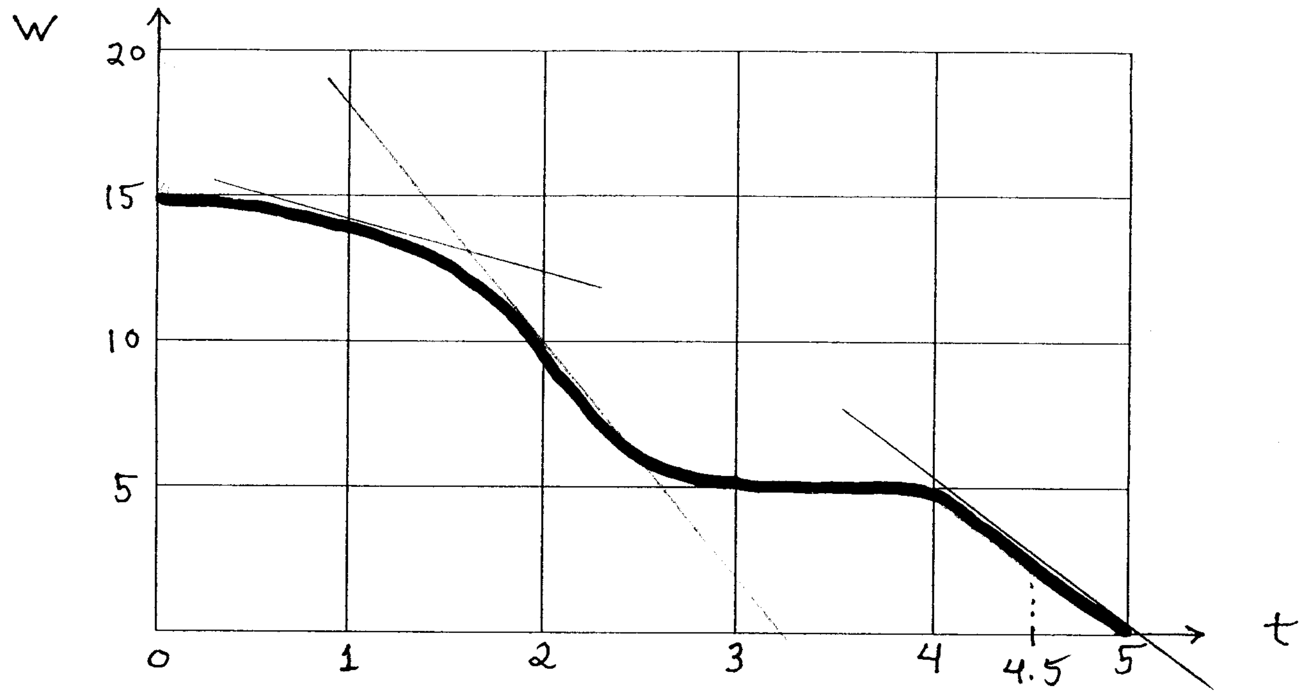
$$\frac{\frac{1}{A} - 1}{A} = \frac{-1}{A^2} \rightarrow$$

$$\frac{1}{A} - 1 = \frac{-1}{A} \rightarrow \frac{2}{A} = 1 \rightarrow$$

$$A = 2 \rightarrow$$

pt. $(x,y) = (2, \frac{1}{2})$

9.) The given graph represents the weight W (lbs.) of a melting block of ice after t hours.



a.) (4 pts.) What is the average melting rate (lbs./hr.) for $t = 0$ to $t = 5$ hrs.?

$$ARC \approx \frac{0-15}{5-0} = -3 \text{ lbs./hr.}$$

b.) (4 pts.) Does the graph reflect time when the block of ice is NOT melting ? If so, when ?

Yes, from $t=3$ to $t=4$ hrs.

c.) (4 pts.) Estimate the instantaneous melting rate of the block of ice for i.) $t = 1$ hr. and ii.) $t = 4.5$ hrs.

i.) at $t = 1$ hr. : $IRC \approx \frac{12.5-15}{2-0.5} = \frac{-2.5}{1.5} = \frac{-5}{3}$ lbs./hr.

ii.) at $t = 4.5$ hrs. : $IRC \approx \frac{0-5}{1} = -5$ lbs./hr.

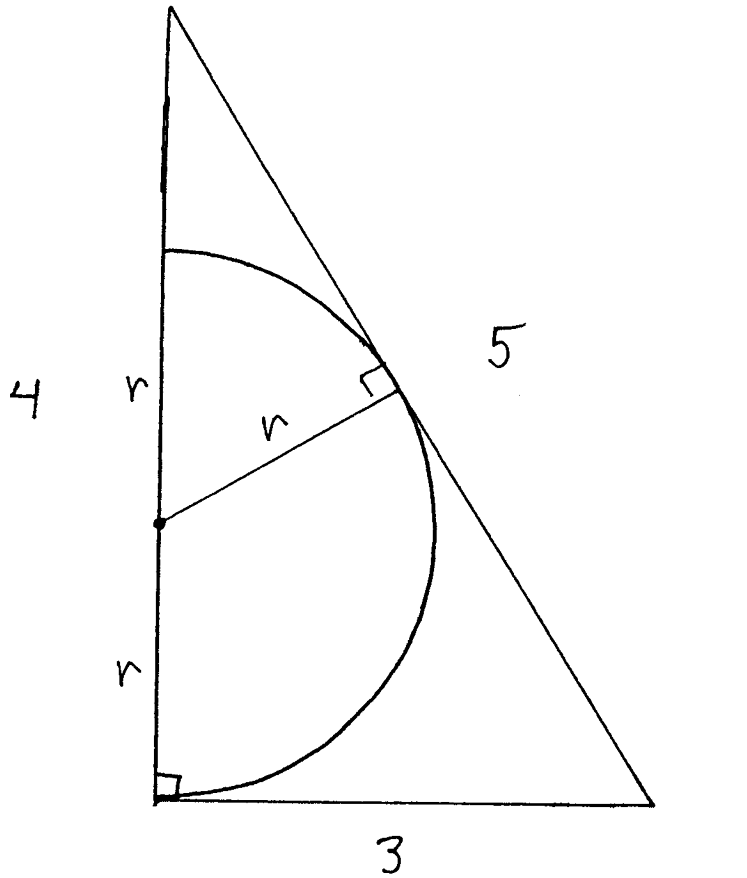
d.) (4 pts.) Estimate the specific time t at which the block of ice is melting most rapidly, and estimate the value of this rate.

approximately $t = 2$ hrs. ;

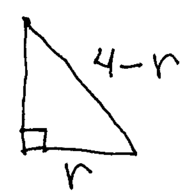
$$IRC \approx \frac{2-18}{3-1} = \frac{-16}{2} = -8 \text{ lbs./hr.}$$

EXTRA CREDIT PROBLEM– The following problem is worth 10 points. This problem is OPTIONAL.

1.) Determine the radius r of the semi-circle which is inscribed in the given right triangle.



By similar triangles :



$$\frac{5}{3} = \frac{4-r}{r} \rightarrow 5r = 12 - 3r$$

$$\rightarrow 8r = 12$$

$$\rightarrow r = 3/2$$