Math 16A (Fall 2005)
Kouba
Exam 3

Please PRINT your name here : KEY

Your Exam ID Number ____________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. COPYING ANSWERS FROM A CLASSMATE's EXAM IS A VIOLATION OF THE UNIVERSITY HONOR CODE.

4. No notes, books, or handouts may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

5. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive LITTLE or NO credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

6. Make sure that you have 6 pages, including the cover page.

7. You will be graded on proper use of derivative notation.

8. Include units on answers where units are appropriate.

9. You have until 10:50 a.m. sharp to finish the exam.

1.) (5 pts. each) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = x^2(3x+4)^5$

$D \rightarrow \; y' = x^2 \cdot 5(3x+4)^4 \cdot (3) + 2x\,(3x+4)^5$

b.) $f(x) = \left(\dfrac{x+2}{1-x}\right)^{-3}$

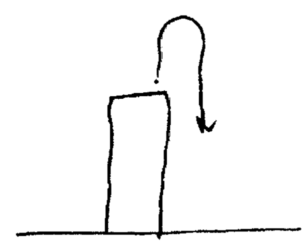
$D \rightarrow \; f'(x) = -3\left(\dfrac{x+2}{1-x}\right)^{-4} \cdot \dfrac{(1-x)(1)-(x+2)(-1)}{(1-x)^2}$

c.) $y = \sin^3(2X)$

$D \rightarrow \; y' = 3\sin^2(2X) \cdot \cos(2X) \cdot 2$

2.) (5 pts. each) A ball is thrown straight up at 64 ft./sec. from the top of a building which is 80 ft. high. It's height h above the ground after t seconds is given by $h(t) = -16t^2 + 64t + 80$.

a.) How high does the ball go ?



$h'(t) = -32t + 64 = 0 \rightarrow t = 2$ sec.

and $h(2) = -16(4) + 64(2) + 80$

$= 144$ ft.

b.) How long is the ball in the air ?

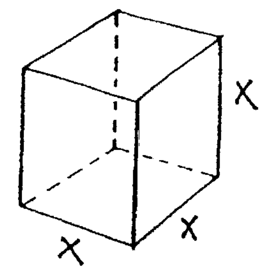
hit ground : $h(t) = 0 \rightarrow 0 = -16t^2 + 64t + 80 \rightarrow$

$-16(t^2 - 4t - 5) = -16(t-5)(t+1) \rightarrow$ $t = 5$ sec. ;

c.) What is the ball's velocity as it strikes the ground ?

$h'(5) = -32(5) + 64 = -160 + 64 = -96$ ft./sec.

3.) (10 pts.) The volume of a cube is increasing at the rate of 3 $in.^3$/sec. How fast is the surface area of the cube changing when each edge is 2 inches long?



$\frac{dV}{dt} = 3$ in./sec. ; Find $\frac{dS}{dt}$ when $\boxed{X = 2 \text{ in.}}$; $V = X^3 \xrightarrow{D} \frac{dV}{dt} = 3X^2 \frac{dX}{dt}$

$\rightarrow 3 = 3(2)^2 \cdot \frac{dX}{dt} \rightarrow \boxed{\frac{dX}{dt} = \frac{1}{4} \text{ in./sec.}}$

$S = 6X^2 \xrightarrow{D} \frac{dS}{dt} = 12X \cdot \frac{dX}{dt} = 12(2)\left(\frac{1}{4}\right) \rightarrow$

$\boxed{\frac{dS}{dt} = 6 \text{ in.}^2/\text{sec.}}$

4.) (10 pts.) Assume that y is a function of x and $y^3 + xy = 8$. Find an equation of the line perpendicular to the graph of this equation at $x = 0$.

$y^3 + xy = 8 \xrightarrow{D} 3y^2y' + xy' + (1) \cdot y = 0 \rightarrow$

$(3y^2 + x)y' = -y \rightarrow \underbrace{y' = \frac{-y}{3y^2 + x}}$;

$X = 0 \rightarrow y = 2$ so tangent slope is

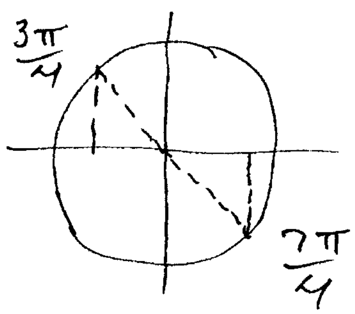
$m = \frac{-2}{12+0} = \frac{-1}{6}$ so $\perp$ slope $m = 6$ and

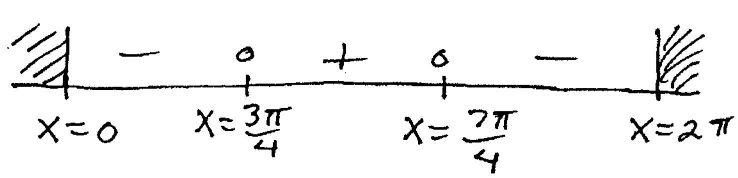
line is $y - 2 = 6(x - 0) \rightarrow \boxed{y = 6X + 2}$

5.) (10 pts.) Determine all inflection points (x, y) for the graph of $f(x) = \sin x + \cos x$ on the interval $[0, 2\pi]$. DO NOT GRAPH THE FUNCTION.

$f'(x) = \cos X - \sin X \rightarrow f''(x) = -\sin X - \cos X = 0 \rightarrow$

$-\sin X = \cos X \rightarrow \frac{\sin X}{\cos X} = -1 = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = \frac{\sqrt{2}/2}{-\sqrt{2}/2}$





$y = 0$ $y = 0$

both are infl. Pts.

6.) (20 pts.) For the following function f state the domain and determine all absolute and relative maximum and minimum values, inflection points, and x- and y-intercepts. State clearly the x-values for which f is increasing ($\uparrow$), decreasing ($\downarrow$), concave up ($\bigcup$), and concave down ($\bigcap$). Find equations for all vertical and horizontal asymptotes. Neatly sketch the graph of f.

$$f(x) = \frac{x}{x^2+1}$$

Domain : all x-values

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = 0$$

$$f''(x) = \frac{(x^2+1)^2(-2x) - (1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$$

f': $-$ 0 $+$ 0 $-$

abs. min. $\begin{cases} x=-1 \\ y=-\frac{1}{2} \end{cases}$ $\quad$ $\begin{cases} x=1 \\ y=\frac{1}{2} \end{cases}$ abs. max.

$$= \frac{2x\cancel{(x^2+1)}\left[-(x^2+1)-2(1-x^2)\right]}{(x^2+1)^{\cancel{4}\,3}} = \frac{2x\left[x^2-3\right]}{(x^2+1)^3} = 0$$

f'': $-$ 0 $+$ 0 $-$ 0 $+$

$x=-\sqrt{3}$, $y=\frac{-\sqrt{3}}{4}$ $\quad$ $x=0$, $y=0$ $\quad$ $x=\sqrt{3}$, $y=\frac{\sqrt{3}}{4}$

all are infl. pts.

f is $\uparrow$ for $-1<x<1$,

f is $\downarrow$ for $x<-1$, $x>1$,

f is $\cup$ for $-\sqrt{3}<x<0$, $x>\sqrt{3}$,

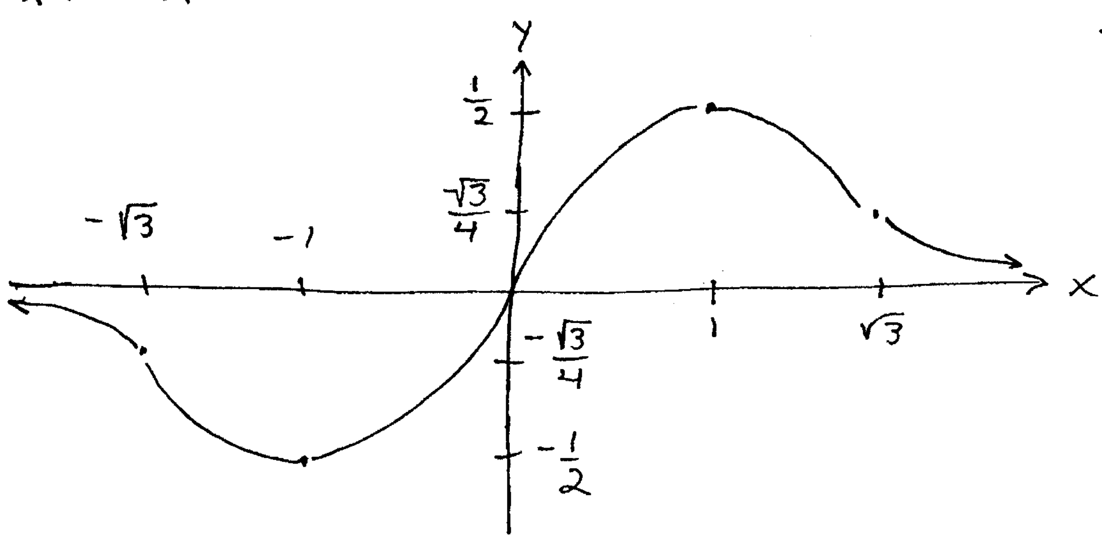
f is $\cap$ for $x<-\sqrt{3}$, $0<x<\sqrt{3}$;

$$\lim_{x\to\infty} \frac{x}{x^2+1} = \lim_{x\to\infty} \frac{1}{x+\frac{1}{x}} = \frac{1}{\infty+0} = 0$$

so H.A. is $\boxed{y=0}$

$x=0$: $y=0$

$y=0$: $x=0$



7.) (10 pts.) Determine the x-values for which the following function is increasing (↑) and decreasing (↓). DO NOT GRAPH THE FUNCTION.

$f(x) = x^2 - 32\sqrt{x}$

$$f'(x) = 2x - 32 \cdot \frac{1}{2} x^{-1/2} = 2x - \frac{16}{\sqrt{x}}$$

$$= \frac{2x^{3/2} - 16}{\sqrt{x}} = \frac{2(x^{3/2} - 8)}{\sqrt{x}} = 0 \longrightarrow$$

	$-$	0	$+$	f'
$x=0$		$x=4$		

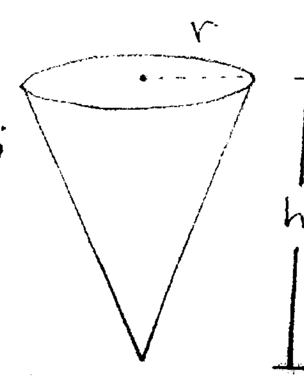
f is ↑ for $x > 4$,

f is ↓ for $0 < x < 4$.

8.) (10 pts.) The radius r of a right circular **cone** is *increasing* at 3 cm./sec. and the height h is *decreasing* at 4 cm./sec. At what rate is the volume of the cone changing when $r = 5$ cm. and $h = 6$ cm. ? Assume that the volume of a **cone** is $V = (1/3)\pi r^2 h$.

$\frac{dr}{dt} = 3$ cm./sec., $\frac{dh}{dt} = -4$ cm./sec.,

Find $\frac{dV}{dt}$ when $r = 5$ cm., $h = 6$ cm.;



$$V = \frac{1}{3}\pi r^2 h \xrightarrow{D}$$

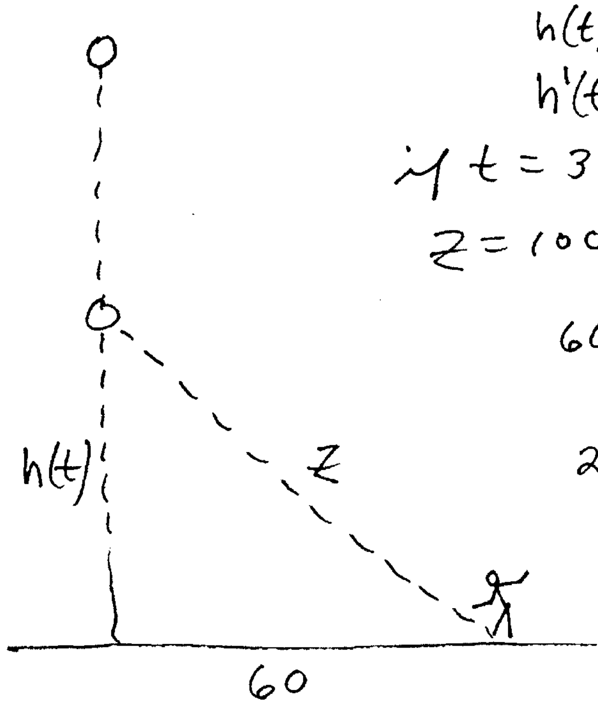
$$\frac{dV}{dt} = \frac{1}{3}\pi \left(r^2 \cdot \frac{dh}{dt} + 2r \frac{dr}{dt} \cdot h\right)$$

$$= \frac{1}{3}\pi \left(5^2 \cdot (-4) + 2(5)(3)(6)\right)$$

$$= \frac{1}{3}\pi (80) = \frac{80}{3}\pi \ \text{cm.}^3/\text{sec.}$$

EXTRA CREDIT PROBLEM– The following problem is worth 10 points. This problem is OPTIONAL.

1.) A watermelon is dropped from a height of 224 feet. It will strike the ground 60 feet from where you are standing. How fast is the distance between you and the watermelon changing after it has fallen for 3 seconds ?



$h(t) = -16t^2 + 224$

$h'(t) = -32t$

if $t = 3$ sec., $h(3) = 80$, and

$Z = 100$; then

$$60^2 + (h(t))^2 = Z^2 \xrightarrow{D}$$

$$2\underset{\overset{\uparrow}{100}}{Z}\,\frac{dZ}{dt} = 2\underset{\overset{\uparrow}{80}}{(h(t))}\cdot \underset{\overset{\uparrow}{-96}}{h'(t)} \longrightarrow$$

$$\frac{dZ}{dt} = \frac{\cancel{2}(80)(-96)}{\cancel{2}(100)} = -76.8 \text{ ft./sec.}$$