PREREQUISITE REVIEW 1.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, simplify each expression.

1. $\sqrt{(3-6)^2+[1-(-5)]^2}$

2. $\sqrt{(-2-0)^2+[-7-(-3)]^2}$

3. $\dfrac{5+(-4)}{2}$

4. $\dfrac{-3+(-1)}{2}$

5. $\sqrt{27}+\sqrt{12}$

6. $\sqrt{8}-\sqrt{18}$

In Exercises 7–10, solve for x or y.

7. $\sqrt{(3-x)^2+(7-4)^2}=\sqrt{45}$

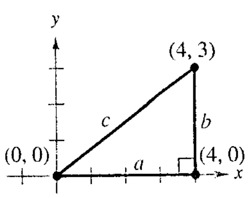
8. $\sqrt{(6-2)^2+(-2-y)^2}=\sqrt{52}$

9. $\dfrac{x+(-5)}{2}=7$

10. $\dfrac{-7+y}{2}=-3$

EXERCISES 1.1

In Exercises 1–6, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.

*1.


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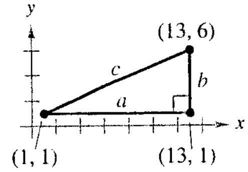

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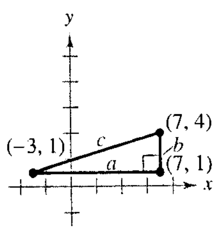

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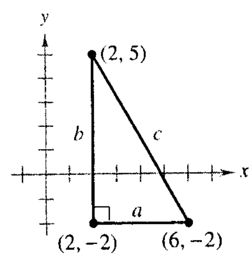

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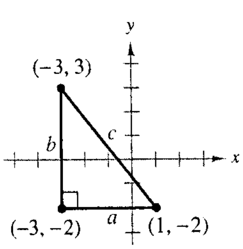
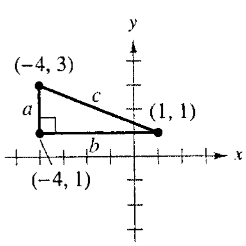


6.


* The answers to the odd-numbered and selected even exercises are given in the back of the text. Worked-out solutions to the odd-numbered exercises are given in the *Student Solutions Guide*.

In Exercises 7–14, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

7. $(3, 1), (5, 5)$

8. $(-3, 2), (3, -2)$

9. $\left(\frac{1}{2}, 1\right), \left(-\frac{3}{2}, -5\right)$

10. $\left(\frac{2}{3}, -\frac{1}{3}\right), \left(\frac{5}{6}, 1\right)$

11. $(2, 2), (4, 14)$

12. $(-3, 7), (1, -1)$

13. $\left(1, \sqrt{3}\right), (-1, 1)$

14. $(-2, 0), \left(0, \sqrt{2}\right)$

In Exercises 15–18, show that the points form the vertices of the given figure. (A rhombus is a quadrilateral whose sides have the same length.)

	Vertices	*Figure*
15.	$(0, 1), (3, 7), (4, -1)$	Right triangle
16.	$(1, -3), (3, 2), (-2, 4)$	Isosceles triangle
17.	$(0, 0), (1, 2), (2, 1), (3, 3)$	Rhombus
18.	$(0, 1), (3, 7), (4, 4), (1, -2)$	Parallelogram

In Exercises 19–22, use the Distance Formula to determine whether the points are collinear (lie on the same line).

19. $(0, -4), (2, 0), (3, 2)$

20. $(0, 4), (7, -6), (-5, 11)$

21. $(-2, -6), (1, -3), (5, 2)$

22. $(-1, 1), (3, 3), (5, 5)$

In Exercises 23 and 24, find x such that the distance between the points is 5.

23. $(1, 0), (x, -4)$

24. $(2, -1), (x, 2)$

In Exercises 25 and 26, find y such that the distance between the points is 8.

25. $(0, 0), (3, y)$

26. $(5, 1), (5, y)$

27. Use the Midpoint
points that divide th
into four equal parts

28. Show that $\left(\frac{1}{3}[2x_1 +\right.$
trisection of the line
Then, find the seco
midpoint of the segn

$$\left(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[\right.$$

29. Use Exercise 27 to fi
ment joining the give

(a) $(1, -2), (4, -1)$

(b) $(-2, -3), (0, 0)$

30. Use Exercise 28 to fi
segment joining the g

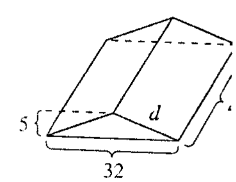
(a) $(1, -2), (4, 1)$

(b) $(-2, -3), (0, 0)$

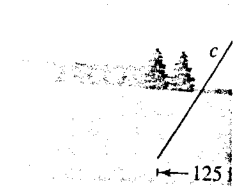
31. ***Building Dimensi***
trusses for the roof
respectively (see figu

(a) Find the distance
roof.

(b) The length of the
part (a) to find the



32. ***Wire Length*** A guy
ing tower at a point 20
125 feet from the base



The symbol indicates an ex
graphing technology or a symbo
tions of other exercises may al
technology.

ed in earlier sections. You will

-3)]^2

= $\sqrt{52}$

the points, (b) find the distai
nd the midpoint of the line segm

8. $(-3, 2), (3, -2)$

10. $\left(\frac{2}{3}, -\frac{1}{3}\right), \left(\frac{5}{6}, 1\right)$

12. $(-3, 7), (1, -1)$

14. $(-2, 0), \left(0, \sqrt{2}\right)$

t the points form the vertices of t
quadrilateral whose sides have t

	Figure
	Right triangle
	Isosceles triangle
3)	Rhombus
−2)	Parallelogram

e Distance Formula to determi
near (lie on the same line).

20. $(0, 4), (7, -6), (-5, 1$

2) **22.** $(-1, 1), (3, 3), (5, 5)$

such that the distance between t

24. $(2, -1), (x, 2)$

such that the distance between t

26. $(5, 1), (5, y)$

. Use the Midpoint Formula repeatedly to find the three points that divide the segment joining (x_1, y_1) and (x_2, y_2) into four equal parts.

. Show that $\left(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[2y_1 + y_2]\right)$ is one of the points of trisection of the line segment joining (x_1, y_1) and (x_2, y_2). Then, find the second point of trisection by finding the midpoint of the segment joining

$$\left(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[2y_1 + y_2]\right) \text{ and } (x_2, y_2).$$

. Use Exercise 27 to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$

(b) $(-2, -3), (0, 0)$

. Use Exercise 28 to find the points of trisection of the line segment joining the given points.

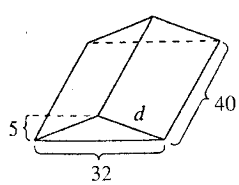
(a) $(1, -2), (4, 1)$

(b) $(-2, -3), (0, 0)$

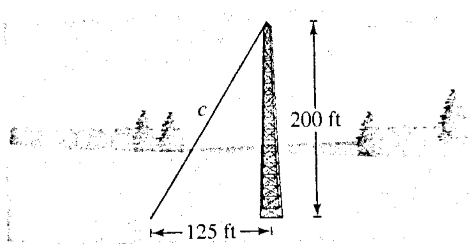
1. ***Building Dimensions*** The base and height of the trusses for the roof of a house are 32 feet and 5 feet, respectively (see figure).

(a) Find the distance d from the eaves to the peak of the roof.

(b) The length of the house is 40 feet. Use the result of part (a) to find the number of square feet of roofing.



2. ***Wire Length*** A guy wire is stretched from a broadcasting tower at a point 200 feet above the ground to an anchor 125 feet from the base (see figure). How long is the wire?



The symbol ⊕ indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

⊕ In Exercises 33 and 34, use a graphing utility to graph a scatter plot, a bar graph, or a line graph to represent the data. Describe any trends that appear.

33. ***Consumer Trends*** The numbers (in millions) of cable television subscribers in the United States for 1992–2001 are shown in the table. *(Source: Nielsen Media Research)*

Year	1992	1993	1994	1995	1996
Subscribers	57.2	58.8	60.5	63.0	64.6

Year	1997	1998	1999	2000	2001
Subscribers	65.9	67.0	68.5	69.3	73.0

34. ***Consumer Trends*** The numbers (in millions) of cellular telephone subscribers in the United States for 1993–2002 are shown in the table. *(Source: Cellular Telecommunications & Internet Association)*

Year	1993	1994	1995	1996	1997
Subscribers	16.0	24.1	33.8	44.0	55.3

Year	1998	1999	2000	2001	2002
Subscribers	69.2	86.0	109.5	128.4	140.8

Dow Jones Industrial Average In Exercises 35 and 36, use the figure below showing the Dow Jones Industrial Average for common stocks. *(Source: Dow Jones, Inc.)*

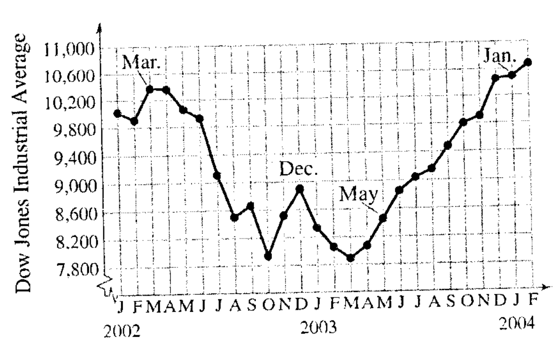
35. Estimate the Dow Jones Industrial Average for each date.

(a) March 2002 (b) December 2002

(c) May 2003 (d) January 2004

36. Estimate the percent increase or decrease in the Dow Jones Industrial Average (a) from April 2002 to November 2002 and (b) from June 2003 to February 2004.

Figure for 35 and 36



Construction In Exercises 37 and 38, use the figure, which shows the median sales prices of existing one-family homes sold (in thousands of dollars) in the United States from 1987 to 2002. *(Source: National Association of Realtors)*

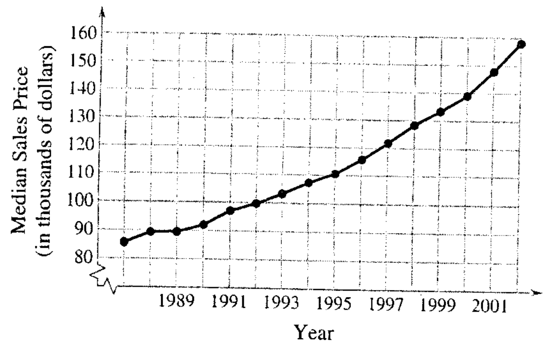
37. Estimate the median sales price of existing one-family homes for each year.

(a) 1987 (b) 1992

(c) 1997 (d) 2002

38. Estimate the percent increases in the value of existing one-family homes (a) from 1993 to 1994 and (b) from 2001 to 2002.

Figure for 37 and 38



Research Project In Exercises 39 and 40, (a) use the Midpoint Formula to estimate the revenue and profit of the company in 2001. (b) Then use your school's library, the Internet, or some other reference source to find the actual revenue and profit for 2001. (c) Did the revenue and profit increase in a linear pattern from 1999 to 2003? Explain your reasoning. (d) What were the company's expenses during each of the given years? (e) How would you rate the company's growth from 1999 to 2003? *(Source: Walgreen Company and The Yankee Candle Company)*

39. Walgreen Company

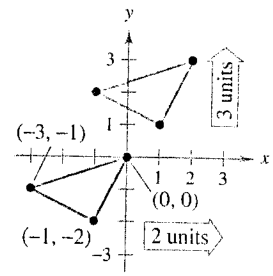
Year	1999	2001	2003
Revenue (millions of $)	17,839		32,505
Profit (millions of $)	624.1		1157.3

40. The Yankee Candle Company

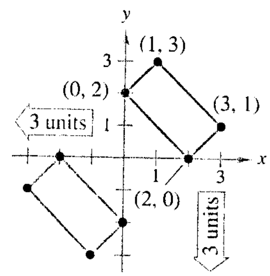
Year	1999	2001	2003
Revenue (millions of $)	256.6		508.6
Profit (millions of $)	34.3		74.8

Computer Graphics In Exercises 41 and 42, the red figure is translated to a new position in the plane to form the blue figure. (a) Find the vertices of the transformed figure. (b) Then use a graphing utility to draw both figures.

41.



42.



43. *Economics* The table shows the numbers of ear infections treated by doctors at HMO clinics of three different sizes: small, medium, and large.

Cases per small clinic	Cases per medium clinic	Cases per large clinic	Number of doctors
0	0	0	0
20	30	35	1
28	42	49	2
35	53	62	3
40	60	70	4

(a) Show the relationship between doctors and treated ear infections using *three* curves, where the number of doctors is on the horizontal axis and the number of ear infections treated is on the vertical axis.

(b) Compare the three relationships.

(Source: Adapted from Taylor, Economics, Fourth Edition)

The symbol indicates an exercise that contains material from textbooks in other disciplines.

1.2

The Graph of ar

In Section 1.1, you relationship between t collection of points in

Frequently, a relat For instance, degrees o scale by the equation

$F = \frac{9}{5}C + 32.$

In this section, you w of such equations. Th solutions of the equati

EXAMPLE 1

Sketch the graph of y =

SOLUTION The simp *plotting method.* With t several solution points when $x = 0$

$y = 7 - 3(0) = 7$

which implies that (0, 7

x	0	1
$y = 7 - 3x$	7	4

From the table, it follo

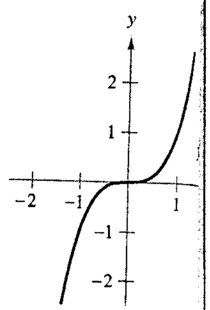
(0, 7), (1, 4), (2, 1)

are solution points of t they appear to lie on a l the line that passes thro

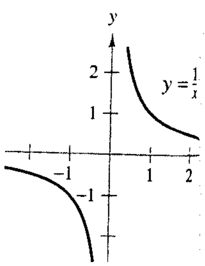
STUDY TIP

Even though we ref $y = 7 - 3x$, it actual is a line that would e

d the behavior of the gra
aphs of six basic algebr
u in the creation and use



(c) Cubic model



(f) Rational model

PREREQUISITE REVIEW 1.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, solve for y.

1. $5y - 12 = x$
2. $-y = 15 - x$
3. $x^3y + 2y = 1$
4. $x^2 + x - y^2 - 6 = 0$
5. $(x - 2)^2 + (y + 1)^2 = 9$
6. $(x + 6)^2 + (y - 5)^2 = 81$

In Exercises 7–10, complete the square to write the expression as a perfect square trinomial.

7. $x^2 - 4x +$
8. $x^2 + 6x +$
9. $x^2 - 5x +$
10. $x^2 + 3x +$

In Exercises 11–14, factor the expression.

11. $x^2 - 3x + 2$
12. $x^2 + 5x + 6$
13. $y^2 - 3y + \frac{9}{4}$
14. $y^2 - 7y + \frac{49}{4}$

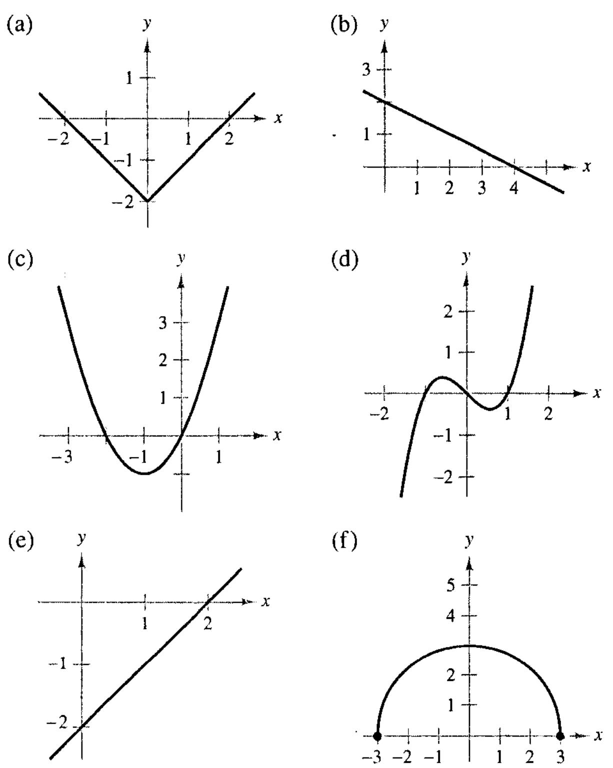
EXERCISES 1.2

In Exercises 1–6, determine whether the points are solution points of the given equation.

1. $2x - y - 3 = 0$
   (a) $(1, 2)$ (b) $(1, -1)$ (c) $(4, 5)$
2. $7x + 4y - 6 = 0$
   (a) $(6, -9)$ (b) $(-5, 10)$ (c) $\left(\frac{1}{2}, \frac{5}{8}\right)$
3. $x^2 + y^2 = 4$
   (a) $\left(1, -\sqrt{3}\right)$ (b) $\left(\frac{1}{2}, -1\right)$ (c) $\left(\frac{3}{2}, \frac{7}{2}\right)$
4. $x^2y + x^2 - 5y = 0$
   (a) $\left(0, \frac{1}{5}\right)$ (b) $(2, 4)$ (c) $(-2, -4)$
5. $x^2 - xy + 4y = 3$
   (a) $(0, 2)$ (b) $\left(-2, -\frac{1}{6}\right)$ (c) $(3, -6)$
6. $3y + 2xy - x^2 = 5$
   (a) $(-7, -5)$ (b) $(-1, 6)$ (c) $\left(1, \frac{6}{5}\right)$

In Exercises 7–12, match the equation with its graph. Use a graphing utility, set for a square setting, to confirm your result. [The graphs are labeled (a)–(f).]

7. $y = x - 2$
8. $y = -\frac{1}{2}x + 2$
9. $y = x^2 + 2x$
10. $y = \sqrt{9 - x^2}$
11. $y = |x| - 2$
12. $y = x^3 - x$



In Exercises 13–22, find the x- and y-intercepts of the graph of the equation.

13. $2x - y - 3 = 0$
14. $4x - 2y - 5 = 0$
15. $y = x^2 + x - 2$
16. $y = x^2 - 4x + 3$
17. $y = x^2\sqrt{9 - x^2}$
18. $y^2 = x^3 - 4x$

19. $y = \dfrac{x^2 - 4}{x - 2}$ **20.** $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

21. $x^2y - x^2 + 4y = 0$ **22.** $2x^2y + 8y - x^2 = 1$

In Exercises 23–38, sketch the graph of the equation and label the intercepts. Use a graphing utility to verify your results.

23. $y = 2x + 3$ **24.** $y = -3x + 2$

25. $y = x^2 - 3$ **26.** $y = x^2 + 6$

27. $y = (x - 1)^2$ **28.** $y = (5 - x)^2$

29. $y = x^3 + 2$ **30.** $y = 1 - x^3$

31. $y = -\sqrt[3]{x + 1}$ **32.** $y = \sqrt{x + 1}$

33. $y = |x + 1|$ **34.** $y = -|x - 2|$

35. $y = 1/(x - 3)$ **36.** $y = 1/(x^2 + 1)$

37. $x = y^2 - 4$ **38.** $x = 4 - y^2$

In Exercises 39–46, write the general form of the equation of the circle.

39. Center: $(0, 0)$; radius: 3 **40.** Center: $(0, 0)$; radius: 5

41. Center: $(2, -1)$; radius: 4

42. Center: $(-4, 3)$; radius: 3

43. Center: $(-1, 2)$; solution point: $(0, 0)$

44. Center: $(3, -2)$; solution point: $(-1, 1)$

45. Endpoints of a diameter: $(3, 3)$, $(-3, 3)$

46. Endpoints of a diameter: $(-4, -1)$, $(4, 1)$

In Exercises 47–54, complete the square to write the equation of the circle in standard form. Then use a graphing utility to graph the circle.

47. $x^2 + y^2 - 2x + 6y + 6 = 0$

48. $x^2 + y^2 - 2x + 6y - 15 = 0$

49. $x^2 + y^2 + 4x + 6y - 3 = 0$

50. $x^2 + y^2 - 4x + 2y + 3 = 0$

51. $2x^2 + 2y^2 - 2x - 2y - 3 = 0$

52. $4x^2 + 4y^2 - 4x + 2y - 1 = 0$

53. $16x^2 + 16y^2 + 16x + 40y - 7 = 0$

54. $3x^2 + 3y^2 - 6y - 1 = 0$

In Exercises 55–62, find the points of intersection (if any) of the graphs of the equations. Use a graphing utility to check your results.

55. $x + y = 2, 2x - y = 1$ **56.** $x + y = 7, 3x - 2y = 11$

57. $x^2 + y^2 = 25, 2x + y = 10$

58. $x^2 + y = 4, 2x - y = 1$

59. $y = x^3, y = 2x$ **60.** $y = \sqrt{x}, y = x$

61. $y = x^4 - 2x^2 + 1, y = 1 - x^2$

62. $y = x^3 - 2x^2 + x - 1, y = -x^2 + 3x - 1$

63. ***Break-Even Analysis*** You are setting up a part-time business with an initial investment of \$15,000. The unit cost of the product is \$11.80, and the selling price is \$19.30.

(a) Find equations for the total cost C and total revenue R for x units.

(b) Find the break-even point by finding the point of intersection of the cost and revenue equations.

(c) How many units would yield a profit of \$1000?

64. ***Break-Even Analysis*** A 2004 Chevrolet Malibu costs \$20,930 with a gasoline engine. A 2004 Toyota Prius costs \$22,052 with a hybrid engine. The Malibu gets 16 miles per gallon of gasoline and the Prius gets 35 miles per gallon of gasoline. Assume that the price of gasoline is \$1.759. *(Source: Adapted from* Consumer Reports, *May 2004)*

(a) Show that the cost C_g of driving the Chevrolet Malibu x miles is

$$C_g = 20{,}930 + 1.759x/16$$

and the cost C_h of driving the Toyota Prius x miles is

$$C_h = 22{,}052 + 1.759x/35.$$

(b) Find the break-even point. That is, find the mileage at which the hybrid-powered Toyota Prius becomes more economical than the gasoline-powered Chevrolet Malibu.

Break-Even Analysis In Exercises 65–68, find the sales necessary to break even for the given cost and revenue equations. (Round your answer up to the nearest whole unit.) Use a graphing utility to graph the equations and then find the break-even point.

65. $C = 0.85x + 35{,}000, R = 1.55x$

66. $C = 6x + 500{,}000, R = 35x$

67. $C = 8650x + 250{,}000, R = 9950x$

68. $C = 5.5\sqrt{x} + 10{,}000, R = 3.29x$

69. ***Supply and Demand*** The demand and supply equations for an electronic organizer are given by

$p = 180 - 4x$ Demand equation

$p = 75 + 3x$ Supply equation

where p is the price in dollars and x represents the number of units, in thousands. Find the equilibrium point for this market.

70. ***Supply and Demand*** The demand and supply equations for a portable CD player are given by

$p = 190 - 15x$ Demand equation

$p = 75 + 8x$ Supply equation

where p is the price in dollars and x represents the number of units, in hundreds of thousands. Find the equilibrium point for this market.

71. ***Consumer Tre*** lions of dollars) States in the yea *(Source: Book In*

Year	199:
Expense	270

Year	199!
Expense	377.

A mathematical n

$y = 2.177t^3 -$

where t represents

(a) Compare the the model. H reasoning.

(b) Use the mode

72. ***Farm Work Fo*** work force in th 1955 to 2000, as p in the table. *(So*

Year	1955
Percent	9.9

Year	1980
Percent	3.1

A mathematical n

$y = \dfrac{-4.97 +}{1 - 0.}$

where y represen with $t = 55$ corre

(a) Compare the model. How g

(b) Use the mod percent of the

(c) Discuss the va

73. ***Weekly Salary*** weekly salary y c estate is given by

$y = \dfrac{292.48 +}{1 + 0.}$

where t represent 1997. *(Source: l*

are setting up a part-ti
ment of $15,000. The u
), and the selling price
l cost C and total revenue
it by finding the point
l revenue equations.
ld a profit of $1000?
004 Chevrolet Malibu co
. A 2004 Toyota Prius co
The Malibu gets 16 miles p
s gets 35 miles per gallon
ice of gasoline is $1.75
ner Reports, May 2004)
riving the Chevrolet Mali
'16
he Toyota Prius x miles is
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5x
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nd x represents the numbe
e equilibrium point for thi
demand and supply equa
re given by
d equation
equation
nd x represents the numbe
nds. Find the equilibrium

71. ***Consumer Trends*** The amounts of money y (in millions of dollars) spent on college textbooks in the United States in the years 1995 to 2002 are shown in the table. *(Source: Book Industry Study Group, Inc.)*

Year	1995	1996	1997	1998
Expense	2708	2920	3110	3365

Year	1999	2000	2001	2002
Expense	3773	3905	4187	4706

A mathematical model for the data is given by

$$y = 2.177t^3 - 41.99t^2 + 497.1t + 985$$

where t represents the year, with $t = 5$ corresponding to 1995.

(a) Compare the actual expenses with those given by the model. How good is the model? Explain your reasoning.

(b) Use the model to predict the expenses in 2010.

72. ***Farm Work Force*** The numbers of workers in farm work force in the United States for selected years from 1955 to 2000, as percents of the total work force, are shown in the table. *(Source: Department of Commerce)*

Year	1955	1960	1965	1970	1975
Percent	9.9	7.8	5.9	4.2	3.6

Year	1980	1985	1990	1995	2000
Percent	3.1	2.8	2.6	2.6	1.7

A mathematical model for the data is given by

$$y = \frac{-4.97 + 0.021t}{1 - 0.025t}$$

where y represents the percent and t represents the year, with $t = 55$ corresponding to 1955.

(a) Compare the actual percents with those given by the model. How good is the model?

(b) Use the model to predict the farm work force as a percent of the total work force in 2010.

(c) Discuss the validity of your prediction in part (b).

73. ***Weekly Salary*** A mathematical model for the average weekly salary y of a person in finance, insurance, or real estate is given by

$$y = \frac{292.48 + 37.72t}{1 + 0.02t}$$

where t represents the year, with $t = 7$ corresponding to 1997. *(Source: U.S. Bureau of Labor Statistics)*

(a) Use the model to complete the table.

Year	1997	1998	1999	2000	2001	2004
Salary						

(b) This model was created using actual data from 1997 through 2002. How accurate do you think the model is in predicting the 2004 average weekly salary? Explain your reasoning.

(c) What does this model predict the average weekly salary to be in 2006? Do you think this prediction is valid?

74. ***Medicine*** A mathematical model for the numbers of kidney transplants performed in the United States in the years 1998 to 2002 is given by

$$y = 60.64t^2 - 544.0t + 12{,}624$$

where y is the number of transplants and t is the time in years, with $t = 8$ corresponding to 1998. *(Source: United Network for Organ Sharing)*

(a) Enter the model into a graphing utility and use it to complete the table.

Year	1998	1999	2000	2001	2002
Transplants					

(b) Use your school's library, the Internet, or some other reference source to find the actual numbers of kidney transplants for the years 1998 to 2002. Compare the actual numbers with those given by the model. How good is the model? Explain your reasoning.

(c) Using this model, what is the prediction for the number of transplants in the year 2008? How valid do you think the prediction is? What factors could affect this model's accuracy?

75. Use a graphing utility to graph the equation $y = cx + 1$ for $c = 1, 2, 3, 4,$ and 5. Then make a conjecture about the x-coefficient and the graph of the equation.

76. Define the break-even point for a business marketing a new product. Give examples of a linear cost equation and a linear revenue equation for which the break-even point is 10,000 units.

In Exercises 77–82, use a graphing utility to graph the equation. Use the graphing utility to approximate the x- and y-intercepts of the graph.

77. $y = 0.24x^2 + 1.32x + 5.36$

78. $y = -0.56x^2 - 5.34x + 6.25$

79. $y = \sqrt{0.3x^2 - 4.3x + 5.7}$

80. $y = \sqrt{-1.21x^2 + 2.34x + 5.6}$

81. $y = \dfrac{0.2x^2 + 1}{0.1x + 2.4}$

82. $y = \dfrac{0.4x - 5.3}{0.4x^2 + 5.3}$

PREREQUISITE REVIEW 1.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, simplify the expression.

1. $\dfrac{5-(-2)}{-3-4}$

2. $\dfrac{-7-(-0)}{4-1}$

3. Evaluate $-\dfrac{1}{m}$ when $m=-3$.

4. Evaluate $-\dfrac{1}{m}$ when $m=\dfrac{6}{7}$.

In Exercises 5–10, solve for y in terms of x.

5. $-4x+y=7$

6. $3x-y=7$

7. $y-2=3(x-4)$

8. $y-(-5)=-1[x-(-2)]$

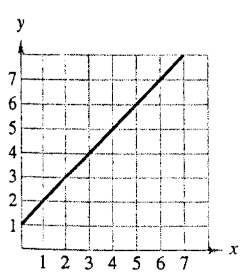
9. $y-(-3)=\dfrac{4-(-3)}{2-1}(x-2)$

10. $y-1=\dfrac{-3-1}{-7-(-1)}[x-(-1)]$

EXERCISES 1.3

In Exercises 1–4, estimate the slope of the line.

1.



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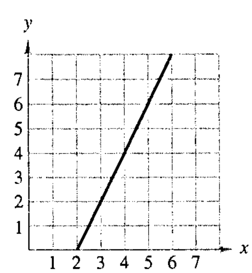

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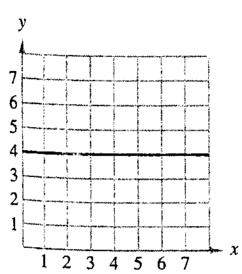

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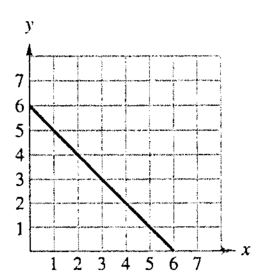

In Exercises 5–16, plot the points and find the slope of the line passing through the pair of points.

5. $(3,-4),\ (5,2)$

6. $(1,2),\ (-2,2)$

7. $(\frac{1}{2},2),\ (6,2)$

8. $(\frac{11}{3},-2),\ (\frac{11}{3},-10)$

9. $(-8,-3),\ (-8,-5)$

10. $(2,-1),\ (-2,-5)$

11. $(-2,1),\ (4,-3)$

12. $(3,-5),\ (-2,-5)$

13. $(\frac{1}{4},-2),\ (-\frac{3}{8},1)$

14. $(-\frac{3}{2},-5),\ (\frac{5}{6},4)$

15. $(\frac{2}{3},\frac{5}{2}),\ (\frac{1}{4},-\frac{5}{6})$

16. $(\frac{7}{8},\frac{3}{4}),\ (\frac{5}{4},-\frac{1}{4})$

In Exercises 17–24, use the point on the line and the slope of the line to find three additional points through which the line passes. (There are many correct answers.)

	Point	*Slope*		*Point*	*Slope*
17.	$(2,1)$	$m=0$	**18.**	$(-3,-1)$	$m=0$
19.	$(6,-4)$	$m=\frac{2}{3}$	**20.**	$(-2,-2)$	$m=\frac{5}{2}$
21.	$(1,7)$	$m=-3$	**22.**	$(10,-6)$	$m=-1$
23.	$(-8,1)$	m is undefined.			
24.	$(-3,4)$	m is undefined.			

In Exercises 25–34, find the slope and y-intercept (if possible) of the equation of the line.

25. $x+5y=20$

26. $2x+y=40$

27. $7x-5y=15$

28. $6x-5y=15$

29. $3x-y=15$

30. $2x-3y=24$

31. $x=4$

32. $x+5=0$

33. $y-4=0$

34. $y+1=0$

In Exercises 35–46, write an equation of the line that passes through the points. Then use the equation to sketch the line.

35. $(4,3),(0,-5)$

36. $(-3,-4),(1,4)$

37. $(0,0),(-1,3)$

38. $(-3,6),(1,2)$

39. $(2,3),(2,-2)$

40. $(6,1),(10,1)$

41. $(3,-1),(-2,-1)$

42. $(2,5),(2,-10)$

43. $(-\frac{1}{3},1),(-\frac{2}{3},\frac{5}{6})$

44. $(\frac{7}{8},\frac{3}{4}),(\frac{5}{4},-\frac{1}{4})$

45. $(-\frac{1}{2},4),(\frac{1}{2},8)$

46. $(4,-1),(\frac{1}{4},-5)$

In Exercises 47–56, write an equation of the line that passes through the given point and has the given slope. Then use a graphing utility to graph the line.

Point	*Slope*	*Point*	*Slope*
47. $(0, 3)$	$m = \frac{3}{4}$	**48.** $(0, 0)$	$m = \frac{2}{3}$
49. $(-1, 2)$	m is undefined.		
50. $(0, 4)$	m is undefined.		
51. $(-2, 7)$	$m = 0$	**52.** $(-2, 4)$	$m = 0$
53. $(0, -2)$	$m = -4$	**54.** $(-1, -4)$	$m = -2$
55. $\left(0, \frac{2}{3}\right)$	$m = \frac{3}{4}$	**56.** $\left(0, -\frac{2}{3}\right)$	$m = \frac{1}{6}$

In Exercises 57 and 58, explain how to use the concept of slope to determine whether the three points are collinear. Then explain how to use the Distance Formula to determine whether the points are collinear.

57. $(-2, 1), (-1, 0), (2, -2)$

58. $(0, 4), (7, -6), (-5, 11)$

59. Write an equation of the vertical line with x-intercept at 3.

60. Write an equation of the horizontal line through $(0, -5)$.

61. Write an equation of the line with y-intercept at -10 and parallel to all horizontal lines.

62. Write an equation of the line with x-intercept at -5 and parallel to all vertical lines.

In Exercises 63–70, write the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line. Then use a graphing utility to graph all three equations in the same viewing window.

Point	*Line*
63. $(-3, 2)$	$x + y = 7$
64. $(2, 1)$	$4x - 2y = 3$
65. $\left(-\frac{2}{3}, \frac{7}{8}\right)$	$3x + 4y = 7$
66. $\left(\frac{7}{8}, \frac{3}{4}\right)$	$5x + 3y = 0$
67. $(-1, 0)$	$y + 3 = 0$
68. $(2, 5)$	$y + 4 = 0$
69. $(1, 1)$	$x - 2 = 0$
70. $(12, -3)$	$x + 4 = 0$

In Exercises 71–78, sketch the graph of the equation. Use a graphing utility to verify your result.

71. $y = -2$ **72.** $y = -4$

73. $2x - y - 3 = 0$ **74.** $x + 2y + 6 = 0$

75. $y = -2x + 1$ **76.** $4x + 5y = 20$

77. $y + 2 = -4(x + 1)$ **78.** $y - 1 = 3(x + 4)$

79. ***Population*** The resident population of South Carolina (in thousands) was 3860 in 1997 and 4107 in 2002. Assume that the relationship between the population y and the year t is linear. Let $t = 7$ represent 1997. *(Source: U.S. Census Bureau)*

(a) Write a linear model for the data. What is the slope and what does it tell you about the population?

(b) Estimate the population in 1999.

(c) Use your model to estimate the population in 2001.

(d) Use your school's library, the Internet, or some other reference source to find the actual populations in 1999 and 2001. How close were your estimates?

(e) Do you think your model could be used to predict the population in 2006? Explain.

80. ***Annual Salary*** Your annual salary was \$26,300 in 2002 and \$29,700 in 2004. Assume your salary can be modeled by a linear equation.

(a) Write a linear equation giving your salary S in terms of the year. Let $t = 2$ represent 2002.

(b) Use the linear model to predict your salary in 2008.

81. ***Temperature Conversion*** Write a linear equation that expresses the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F. Use the fact that water freezes at 0°C (32°F) and boils at 100°C (212°F).

82. ***Chemistry*** Use the result of Exercise 81 to answer the following:

(a) A person has a temperature of 102.5°F. What is this temperature on the Celsius scale?

(b) If the temperature in a room is 74°F, what is this temperature on the Celsius scale?

(Source: Adapted from Zumdahl, Chemistry, Sixth Edition)

83. ***Reimbursed Expenses*** A company reimburses its sales representatives \$150 per day for lodging and meals, plus \$0.34 per mile driven. Write a linear equation giving the daily cost C in terms of x, the number of miles driven.

84. ***Union Negotiation*** You are on a negotiating panel in a union hearing for a large corporation. The union is asking for a base pay of \$9.25 per hour plus an additional piecework rate of \$0.80 per unit produced. The corporation is offering a base pay of \$6.85 per hour plus a piecework rate of \$1.15.

(a) Write a linear equation for the hourly wages W in terms of x, the number of units produced per hour, for each pay schedule.

(b) Use a graphing utility to graph each linear equation and find the point of intersection.

(c) Interpret the meaning of the point of intersection of the graphs. How would you use this information to advise the corporation and the union?

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85. *Chemistry* Ethylene glycol is the main component in automobile antifreeze. To monitor the temperature of an auto cooling system, you intend to use a meter that reads from 0 to 100. You devise a new temperature scale (°A) based on the approximate melting and boiling points of a typical antifreeze solution (−45°C and 115°C). You wish these points to correspond to 0°A and 100°A, respectively.

(a) Derive an expression for converting between °A and °C.

(b) Derive an expression for converting between °F and °A.

(c) At what temperature would your thermometer and a Celsius thermometer give the same numerical reading?

(d) Your thermometer reads 86°A. What is the temperature in °C and in °F?

(e) What is a temperature of 45°C in °A?

(Source: Zumdahl, Chemistry, Sixth Edition)

86. *Linear Depreciation* A company constructs a warehouse for \$825,000. The warehouse has an estimated useful life of 25 years, after which its value is expected to be \$75,000. Write a linear equation giving the value y of the warehouse during its 25 years of useful life. (Let t represent the time in years.)

87. *Linear Depreciation* A small business purchases a piece of equipment for \$1025. After 5 years the equipment will be outdated, having no value.

(a) Write a linear equation giving the value y of the equipment in terms of the time t in years, $0 \le t \le 5$.

(b) Use a graphing utility to graph the equation.

(c) Move the cursor along the graph and estimate (to two-decimal-place accuracy) the value of the equipment when $t = 3$.

(d) Move the cursor along the graph and estimate (to two-decimal-place accuracy) the time when the value of the equipment will be \$600.

88. *College Enrollment* A small college had 2546 students in 2002 and 2702 students in 2004. If the enrollment follows a linear growth pattern, how many students will the college have in 2008?

89. *Consumer Awareness* A real estate office handles an apartment complex with 50 units. When the rent is \$380 per month, all 50 units are occupied. When the rent is \$425, however, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and the demand x is linear. (The term *demand* refers to the number of occupied units.)

(a) Write a linear equation expressing x in terms of p.

(b) *Linear Extrapolation* Predict the number of occupied units when the rent is set at \$455.

(c) *Linear Interpolation* Predict the number of occupied units when the rent is set at \$395.

90. *Profit* You are a contractor and have purchased a piece of equipment for \$26,500. The equipment costs an average of \$5.25 per hour for fuel and maintenance, and the operator is paid \$9.50 per hour.

(a) Write a linear equation giving the total cost C of operating the equipment for t hours.

(b) You charge your customers \$25 per hour of machine use. Write an equation for the revenue R derived from t hours of use.

(c) Use the formula for profit, $P = R - C$, to write an equation for the profit derived from t hours of use.

(d) Find the number of hours you must operate the equipment before you break even.

91. *Personal Income* Personal income (in billions of dollars) in the United States was 6937 in 1997 and 8685 in 2001. Assume that the relationship between the personal income Y and the time t (in years) is linear. Let $t = 0$ correspond to 1990. *(Source: U.S. Bureau of Economic Analysis)*

(a) Write a linear model for the data.

(b) *Linear Interpolation* Estimate the personal income in 1999.

(c) *Linear Extrapolation* Estimate the personal income in 2002.

(d) Use your school's library, the Internet, or some other reference source to find the actual personal income in 1999 and 2002. How close were your estimates?

92. *Sales Commission* As a salesperson, you receive a monthly salary of \$2000, plus a commission of 7% of sales. You are offered a new job at \$2300 per month, plus a commission of 5% of sales.

(a) Write a linear equation for your current monthly wage W in terms of your monthly sales S.

(b) Write a linear equation for the monthly wage W of your job offer in terms of the monthly sales S.

(c) Use a graphing utility to graph both equations in the same viewing window. Find the point of intersection. What does it signify?

(d) You think you can sell \$20,000 worth of a product per month. Should you change jobs? Explain.

In Exercises 93–102, use a graphing utility to graph the cost function. Determine the maximum production level x, given that the cost C cannot exceed \$100,000.

93. $C = 23{,}500 + 3100x$

94. $C = 30{,}000 + 575x$

95. $C = 18{,}375 + 1150x$

96. $C = 24{,}900 + 1785x$

97. $C = 75{,}500 + 89x$

98. $C = 83{,}620 + 67x$

99. $C = 32{,}000 + 650x$

100. $C = 53{,}500 + 495x$

101. $C = 50{,}000 + 0.25x$

102. $C = 75{,}500 + 1.50x$

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PREREQUISITE REVIEW 1.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, simplify the expression.

1. $5(-1)^2 - 6(-1) + 9$

2. $(-2)^3 + 7(-2)^2 - 10$

3. $(x - 2)^2 + 5x - 10$

4. $(3 - x) + (x + 3)^3$

5. $\dfrac{1}{1 - (1 - x)}$

6. $1 + \dfrac{x - 1}{x}$

In Exercises 7–12, solve for y in terms of x.

7. $2x + y - 6 = 11$

8. $5y - 6x^2 - 1 = 0$

9. $(y - 3)^2 = 5 + (x + 1)^2$

10. $y^2 - 4x^2 = 2$

11. $x = \dfrac{2y - 1}{4}$

12. $x = \sqrt[3]{2y - 1}$

EXERCISES 1.4

In Exercises 1–8, decide whether the equation defines y as a function of x.

1. $x^2 + y^2 = 4$

2. $x + y^2 = 4$

3. $\frac{1}{2}x - 6y = -3$

4. $3x - 2y + 5 = 0$

5. $x^2 + y = 4$

6. $x^2 + y^2 - 2x - 4y + 1 = 0$

7. $y^2 = x^2 - 1$

8. $x^2y - x^2 + 4y = 0$

In Exercises 9–16, use a graphing utility to graph the function. Then determine the domain and range of the function.

9. $f(x) = 2x^2 - 5x + 1$

10. $f(x) = 5x^3 + 6x^2 - 1$

11. $f(x) = \dfrac{|x|}{x}$

12. $f(x) = \sqrt{9 - x^2}$

13. $f(x) = \dfrac{x}{\sqrt{x - 4}}$

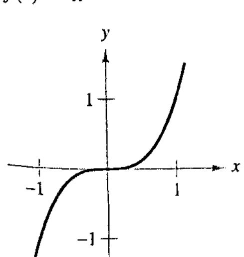
14. $f(x) = \dfrac{2x}{\sqrt{x + 1}}$

15. $f(x) = \dfrac{x - 2}{x + 4}$

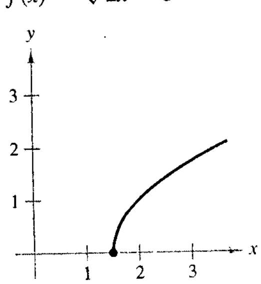
16. $f(x) = \dfrac{x^2}{1 - x}$

In Exercises 17–20, find the domain and range of the function. Use interval notation to write your result.

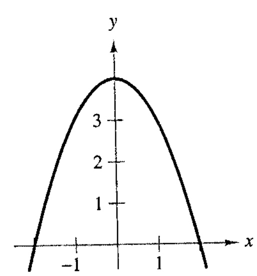
17. $f(x) = x^3$



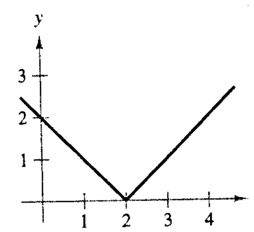
18. $f(x) = \sqrt{2x - 3}$



19. $f(x) = 4 - x^2$



20. $f(x) = |x - 2|$



In Exercises 21–24, evaluate the function at the specified values of the independent variable. Simplify the result.

21. $f(x) = 2x - 3$

(a) $f(0)$ (b) $f(-3)$

(c) $f(x - 1)$ (d) $f(x + \Delta x)$

22. $f(x) = x^2 - 2x + 2$

(a) $f(\frac{1}{2})$ (b) $f(-1)$

(c) $f(c + 2)$ (d) $f(x + \Delta x)$

23. $g(x) = 1/x$

(a) $g(2)$ (b) $g(\frac{1}{4})$

(c) $g(x + 4)$ (d) $g(x + \Delta x) - g(x)$

24. $f(x) = |x| + 4$

(a) $f(2)$ (b) $f(-2)$

(c) $f(x + 2)$ (d) $f(x + \Delta x) - f(x)$

In Exercises 25–30, evaluate the difference quotient and simplify the result.

25. $f(x) = x^2 - 4x + 1$

$\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

26. $h(x) = x^2 - x + 1$

$\dfrac{h(2 + \Delta x) - h(2)}{\Delta x}$

27. $g(x) = \sqrt{x+3}$

$\dfrac{g(x+\Delta x) - g(x)}{\Delta x}$

28. $f(x) = \dfrac{1}{\sqrt{x-1}}$

$\dfrac{f(x) - f(2)}{x - 2}$

29. $f(x) = \dfrac{1}{x-2}$

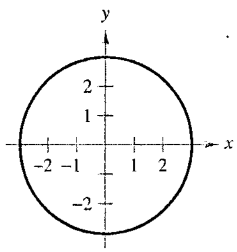
$\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

30. $f(x) = \dfrac{1}{x+4}$

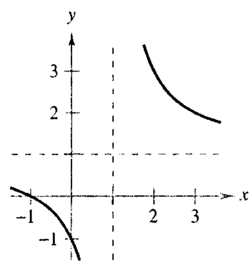
$\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

In Exercises 31–34, use the vertical line test to determine whether y is a function of x.

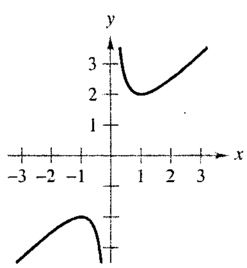
31. $x^2 + y^2 = 9$



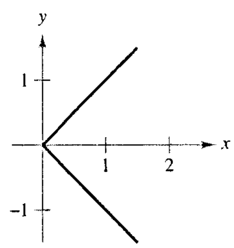
32. $x - xy + y + 1 = 0$



33. $x^2 = xy - 1$



34. $x = |y|$



In Exercises 35–40, find (a) $f(x) + g(x)$, (b) $f(x) \cdot g(x)$, (c) $f(x)/g(x)$, (d) $f(g(x))$, and (e) $g(f(x))$ if defined.

35. $f(x) = 2x - 5$
$g(x) = 5$

36. $f(x) = 2x - 5$
$g(x) = 2 - x$

37. $f(x) = x^2 + 1$
$g(x) = x - 1$

38. $f(x) = x^2 + 5$
$g(x) = \sqrt{1-x}$

39. $f(x) = \dfrac{1}{x}$
$g(x) = \dfrac{1}{x^2}$

40. $f(x) = \dfrac{x}{x+1}$
$g(x) = x^3$

41. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, find the composite functions.

(a) $f(g(1))$ (b) $g(f(1))$
(c) $g(f(0))$ (d) $f(g(-4))$
(e) $f(g(x))$ (f) $g(f(x))$

42. Given $f(x) = 1/x$ and $g(x) = x^2 - 1$, find the compos functions.

(a) $f(g(2))$ (b) $g(f(2))$
(c) $f(g(1/\sqrt{2}))$ (d) $g(f(1/\sqrt{2}))$
(e) $f(g(x))$ (f) $g(f(x))$

In Exercises 43–46, select a function from (a) $f(x) =$ (b) $g(x) = cx^2$, (c) $h(x) = c\sqrt{|x|}$, and (d) $r(x) = c/x$ and det mine the value of the constant c such that the function fits t data in the table.

43.

x	−4	−1	0	1	4
y	−32	−2	0	−2	−32

44.

x	−4	−1	0	1	4
y	−1	$-\frac{1}{4}$	0	$\frac{1}{4}$	1

45.

x	−4	−1	0	1	4
y	−8	−32	Undefined	32	8

46.

x	−4	−1	0	1	4
y	6	3	0	3	6

In Exercises 47–50, show that f and g are inverse functions showing that $f(g(x)) = x$ and $g(f(x)) = x$. Then sketch t graphs of f and g on the same coordinate axes.

47. $f(x) = 5x + 1,\quad g(x) = \dfrac{x-1}{5}$

48. $f(x) = \dfrac{1}{x},\quad g(x) = \dfrac{1}{x}$

49. $f(x) = 9 - x^2,\ x \geq 0,\quad g(x) = \sqrt{9-x},\ x \leq 9$

50. $f(x) = 1 - x^3,\quad g(x) = \sqrt[3]{1-x}$

In Exercises 51–58, find the inverse function of f. Then sketch t graphs of f and f^{-1} on the same coordinate axes.

51. $f(x) = 2x - 3$ **52.** $f(x) = 6 - 3x$

53. $f(x) = x^5$ **54.** $f(x) = x^3 + 1$

55. $f(x) = \sqrt{9 - x^2},\ 0 \leq x \leq 3$

56. $f(x) = \sqrt{x^2 - 4},\ x \geq 2$

57. $f(x) = x^{2/3},\ x \geq 0$ **58.** $f(x) = x^{3/5}$

In Exercises 59–64, use a graphing utility to graph the functio Then use the horizontal line test to determine whether the fu tion is one-to-one. If it is, find its inverse function.

59. $f(x) = 3 - 7x$ **60.** $f(x) = \sqrt{x-2}$

61. $f(x) = x^2$ **62.** $f(x) = x^4$

63. $f(x) = |x - 2|$

65. Use the graph of $f(x)$ each function.

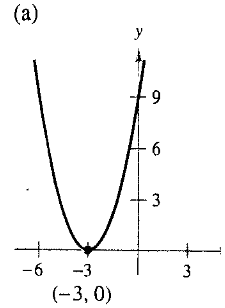
(a) $y = \sqrt{x} + 2$
(b) $y = -\sqrt{x}$
(c) $y = \sqrt{x-2}$
(d) $y = \sqrt{x+3}$
(e) $y = \sqrt{x-4}$
(f) $y = 2\sqrt{x}$

66. Use the graph of $f(x)$ each function.

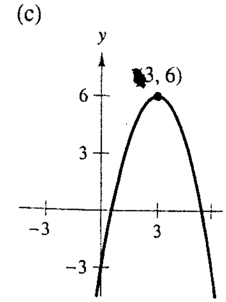
(a) $y = |x| + 3$
(b) $y = -\frac{1}{2}|x|$
(c) $y = |x - 2|$
(d) $y = |x + 1| - 1$
(e) $y = 2|x|$

67. Use the graph of $f(x)$ functions whose grap

(a)



(c)



68. ***Real Estate*** Expre terms of x, the numb acre is valued at $2 $750,000.

$^2 - 1$, find the composit

$g(f(2))$

$g(f(1/\sqrt{2}))$

$g(f(x))$

ction from (a) $f(x) = c$
d (d) $r(x) = c/x$ and deter
h that the function fits th

4	
−32	

1	4
32	8

g are inverse functions b
(x)) = x. Then sketch th
nate axes.

$) = \frac{x-1}{5}$

$) = \frac{1}{x}$

$) = \sqrt{9-x}, \quad x \le 9$

$) = \sqrt[3]{1-x}$

nction of f. Then sketch th
dinate axes.

$f(x) = 6 - 3x$

$f(x) = x^3 + 1$

$f(x) = x^{3/5}$

ility to graph the functio
etermine whether the func
se function.

$f(x) = \sqrt{x-2}$

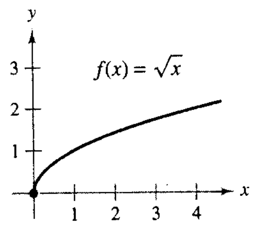
$f(x) = x^4$

63. $f(x) = |x - 2|$

64. $f(x) = 3$

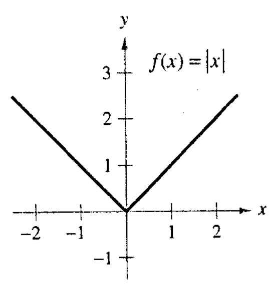
65. Use the graph of $f(x) = \sqrt{x}$ below to sketch the graph of each function.

(a) $y = \sqrt{x} + 2$
(b) $y = -\sqrt{x}$
(c) $y = \sqrt{x-2}$
(d) $y = \sqrt{x+3}$
(e) $y = \sqrt{x} - 4$
(f) $y = 2\sqrt{x}$

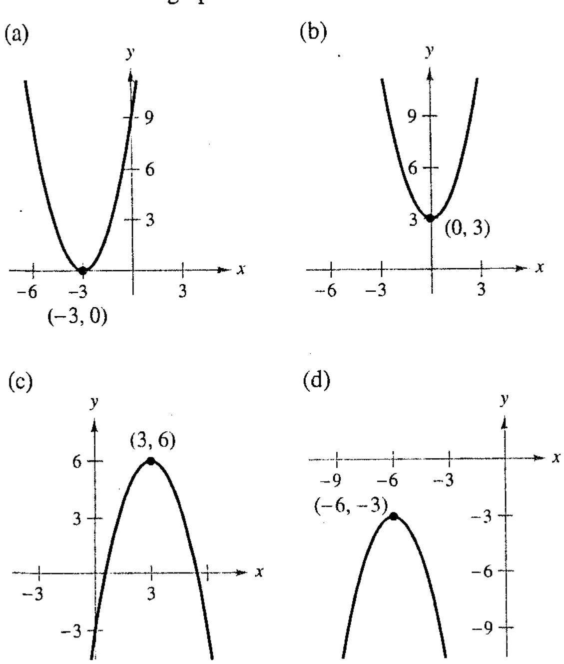


66. Use the graph of $f(x) = |x|$ below to sketch the graph of each function.

(a) $y = |x| + 3$
(b) $y = -\frac{1}{2}|x|$
(c) $y = |x - 2|$
(d) $y = |x + 1| - 1$
(e) $y = 2|x|$



67. Use the graph of $f(x) = x^2$ to find a formula for each of the functions whose graphs are shown.



68. ***Real Estate*** Express the value V of a real estate firm in terms of x, the number of acres of property owned. Each acre is valued at \$2500 and other company assets total \$750,000.

69. ***Owning a Business*** You own two restaurants. From 1998 to 2004, the sales R_1 (in thousands of dollars) for one restaurant can be modeled by

$$R_1 = 480 - 8t - 0.8t^2, \quad t = 0, 1, 2, 3, 4, 5, 6$$

where $t = 0$ represents 1998. During the same seven-year period, the sales R_2 (in thousands of dollars) for the second restaurant can be modeled by

$$R_2 = 254 + 0.78t, \quad t = 0, 1, 2, 3, 4, 5, 6.$$

Write a function that represents the total sales for the two restaurants. Use a graphing utility to graph the total sales function.

70. ***Cost*** The inventor of a new game believes that the variable cost for producing the game is \$0.95 per unit. The fixed cost is \$6000.

(a) Express the total cost C as a function of x, the number of games sold.

(b) Find a formula for the average cost per unit $\overline{C} = C/x$.

(c) The selling price for each game is \$1.69. How many units must be sold before the average cost per unit falls below the selling price?

71. ***Demand*** The demand function for a commodity is

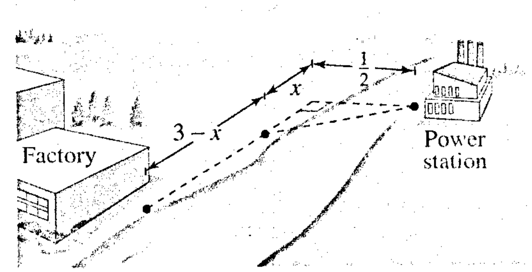
$$p = \frac{14.75}{1 + 0.01x}, \quad x \ge 0$$

where p is the price per unit and x is the number of units sold.

(a) Find x as a function of p.

(b) Find the number of units sold when the price is \$10.

72. ***Cost*** A power station is on one side of a river that is $\frac{1}{2}$ mile wide. A factory is 3 miles downstream on the other side of the river (see figure). It costs \$10/ft to run the power lines on land and \$15/ft to run them under water. Express the cost C of running the lines from the power station to the factory as a function of x.



73. ***Cost*** The weekly cost of producing x units in a manufacturing process is given by the function

$$C(x) = 70x + 375.$$

The number of units produced in t hours is given by $x(t) = 40t$. Find and interpret $C(x(t))$.

74. *Market Equilibrium* The supply function for a product relates the number of units x that producers are willing to supply for a given price per unit p. The supply and demand functions for a market are

$$p = \frac{2}{5}x + 4 \qquad \text{Supply}$$

$$p = -\frac{16}{25}x + 30. \qquad \text{Demand}$$

(a) Use a graphing utility to graph the supply and demand functions in the same viewing window.

(b) Use the *trace* feature of the graphing utility to find the *equilibrium point* for the market.

(c) For what values of x does the demand exceed the supply?

(d) For what values of x does the supply exceed the demand?

75. *Profit* A radio manufacturer charges \$90 per unit for units that cost \$60 to produce. To encourage large orders from distributors, the manufacturer will reduce the price by \$0.01 per unit for each unit in excess of 100 units. (For example, an order of 101 units would have a price of \$89.99 per unit, and an order of 102 units would have a price of \$89.98 per unit.) This price reduction is discontinued when the price per unit drops to \$75.

(a) Express the price per unit p as a function of the order size x.

(b) Express the profit P as a function of the order size x.

76. *Cost, Revenue, and Profit* A company invests \$98,000 for equipment to produce a new product. Each unit of the product costs \$12.30 and is sold for \$17.98. Let x be the number of units produced and sold.

(a) Write the total cost C as a function of x.

(b) Write the revenue R as a function of x.

(c) Write the profit P as a function of x.

77. *Revenue* For groups of 80 or more people, a charter bus company determines the rate r (in dollars per person) according to the formula

$$r = 8 - 0.05(n - 80), \quad n \geq 80$$

where n is the number of people.

(a) Express the revenue R for the bus company as a function of n.

(b) Complete the table.

n	90	100	110	120	130	140	150
R							

(c) Criticize the formula for the rate. Would you use this formula? Explain your reasoning.

78. *Medicine* The temperature of a patient after being given a fever-reducing drug is given by

$$F(t) = 98 + \frac{3}{t + 1}$$

where F is the temperature in degrees Fahrenheit and t is the time in hours since the drug was administered. Use a graphing utility to graph the function. Be sure to choose an appropriate viewing window. For what values of t do you think this function would be valid? Explain.

In Exercises 79–86, use a graphing utility to graph the function. Then use the *zoom* and *trace* features to find the zeros of the function. Is the function one-to-one?

79. $f(x) = 9x - 4x^2$

80. $f(x) = 2\left(3x^2 - \frac{6}{x}\right)$

81. $g(t) = \dfrac{t + 3}{1 - t}$

82. $h(x) = 6x^3 - 12x^2 + 4$

83. $f(x) = \dfrac{4 - x^2}{x}$

84. $g(x) = \left|\frac{1}{2}x^2 - 4\right|$

85. $g(x) = x^2\sqrt{x^2 - 4}$

86. $f(x) = \dfrac{\sqrt{x^2 - 16}}{x^2}$

BUSINESS CAPSULE

Robert L. Boverie/CardSenders

CardSenders *is a home-based greeting card service for businesses. Phyllis and Robert Boverie bought the company in 1990, which has expanded into the United Kingdom, Canada, Asia, and Mexico. Currently there are over 200 licensees and consultants. Start-up costs run from \$995.00 for consultants and \$6,900.00 for licensees.*

87. *Research Project* Use your school's library, the Internet, or some other reference source to find information about the start-up costs of beginning a business, such as the example above. Write a short paper about the company.

1.5

The Limit of a l

In everyday language limit of one's endurai gest that a limit is a l other occasions may

Consider a sprin attached. To determi could attach increasi each weight w, as sh of L, then it is said th limit is much like the

$\lim_{x \to c} f(x) = L$

which is read as "the

EXAMPLE 1

Find the limit: $\lim_{x \to 1}$ (x

SOLUTION Let $f(x)$ that $f(x)$ approaches

$\lim_{x \to 1} (x^2 + 1) =$

The table yields the s $f(x)$ gets closer and c

x approa

x	0.900	0.990
$f(x)$	1.810	1.980

$f(x)$ appro

TRY IT 1

Find the limit: $\lim_{x \to 1}$ (

PREREQUISITE REVIEW 1.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, evaluate the expression and simplify.

1. $f(x) = x^2 - 3x + 3$

(a) $f(-1)$ (b) $f(c)$ (c) $f(x + h)$

2. $f(x) = \begin{cases} 2x - 2, & x < 1 \\ 3x + 1, & x \geq 1 \end{cases}$

(a) $f(-1)$ (b) $f(3)$ (c) $f(t^2 + 1)$

3. $f(x) = x^2 - 2x + 2$ $\quad \dfrac{f(1 + h) - f(1)}{h}$

4. $f(x) = 4x$ $\quad \dfrac{f(2 + h) - f(2)}{h}$

In Exercises 5–8, find the domain and range of the function and sketch its graph.

5. $h(x) = -\dfrac{5}{x}$

6. $g(x) = \sqrt{25 - x^2}$

7. $f(x) = |x - 3|$

8. $f(x) = \dfrac{|x|}{x}$

In Exercises 9 and 10, determine whether y is a function of x.

9. $9x^2 + 4y^2 = 49$

10. $2x^2y + 8x = 7y$

EXERCISES 1.5

In Exercises 1–8, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

1. $\lim_{x \to 2} (5x + 4)$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

2. $\lim_{x \to 2} (x^2 - 3x + 1)$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

3. $\lim_{x \to 2} \dfrac{x - 2}{x^2 - 4}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

4. $\lim_{x \to 2} \dfrac{x^5 - 32}{x - 2}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

5. $\lim_{x \to 0} \dfrac{\sqrt{x + 3} - \sqrt{3}}{x}$

x	−0.1	−0.01	−0.001	0	0.001	0.01	0.
$f(x)$				?			

6. $\lim_{x \to 0} \dfrac{\sqrt{x + 2} - \sqrt{2}}{x}$

x	−0.1	−0.01	−0.001	0	0.001	0.01	0.
$f(x)$				?			

7. $\lim_{x \to 0^-} \dfrac{\frac{1}{x + 4} - \frac{1}{4}}{x}$

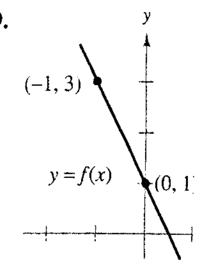
x	−0.5	−
$f(x)$		

8. $\lim_{x \to 0^+} \dfrac{\frac{1}{2 + x} - \frac{1}{2}}{2x}$

x	0.5	0.1
$f(x)$		

In Exercises 9–12, use th

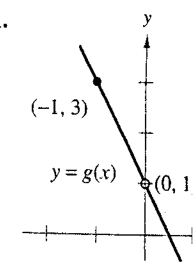
9.



(a) $\lim_{x \to 0} f(x)$

(b) $\lim_{x \to -1} f(x)$

11.



(a) $\lim_{x \to 0} g(x)$

(b) $\lim_{x \to -1} g(x)$

In Exercises 13 and
(b) $f(x)g(x)$, and (c) $f(x)$

13. $\lim_{x \to c} f(x) = 3$

$\lim_{x \to c} g(x) = 9$

In Exercises 15 and 16, f
(c) $[f(x)]^2$ as x approach

15. $\lim_{x \to c} f(x) = 16$

irlier sections. You will

2	2.001	2.01	2.1

01	0	0.001	0.01
	?		

1	0	0.001	0.01
	?		

7. $\lim_{x\to 0^-} \dfrac{\dfrac{1}{x+4} - \dfrac{1}{4}}{x}$

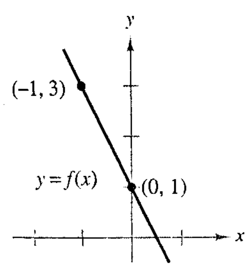
x	-0.5	-0.1	-0.01	-0.001	0
$f(x)$					?

8. $\lim_{x\to 0^+} \dfrac{\dfrac{1}{2+x} - \dfrac{1}{2}}{2x}$

x	0.5	0.1	0.01	0.001	0
$f(x)$					?

In Exercises 9–12, use the graph to find the limit (if it exists).

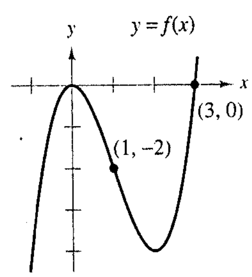
9.



(a) $\lim_{x\to 0} f(x)$

(b) $\lim_{x\to -1} f(x)$

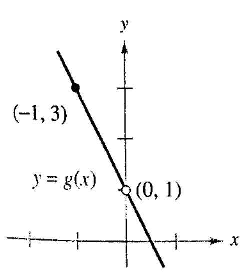
10.



(a) $\lim_{x\to 1} f(x)$

(b) $\lim_{x\to 3} f(x)$

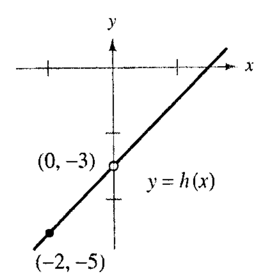
11.



(a) $\lim_{x\to 0} g(x)$

(b) $\lim_{x\to -1} g(x)$

12.



(a) $\lim_{x\to -2} h(x)$

(b) $\lim_{x\to 0} h(x)$

In Exercises 13 and 14, find the limit of (a) $f(x) + g(x)$, (b) $f(x)g(x)$, and (c) $f(x)/g(x)$ as x approaches c.

13. $\lim_{x\to c} f(x) = 3$

$\lim_{x\to c} g(x) = 9$

14. $\lim_{x\to c} f(x) = \frac{3}{2}$

$\lim_{x\to c} g(x) = \frac{1}{2}$

In Exercises 15 and 16, find the limit of (a) $\sqrt{f(x)}$, (b) $[3f(x)]$, and (c) $[f(x)]^2$ as x approaches c.

15. $\lim_{x\to c} f(x) = 16$

16. $\lim_{x\to c} f(x) = 9$

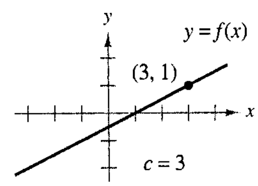
In Exercises 17–22, use the graph to find the limit (if it exists).

(a) $\lim_{x\to c^+} f(x)$

(b) $\lim_{x\to c^-} f(x)$

(c) $\lim_{x\to c} f(x)$

17.



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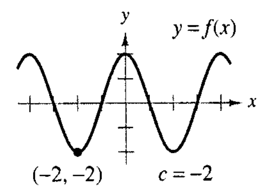

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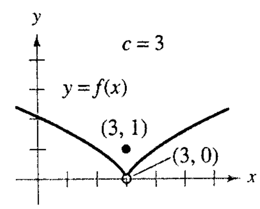

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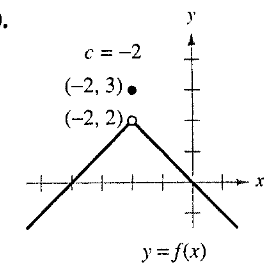

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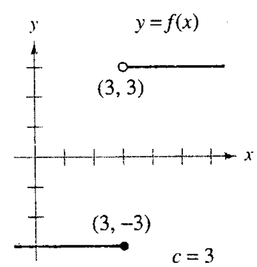

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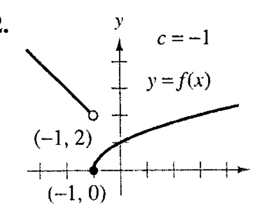


In Exercises 23–40, find the limit.

23. $\lim_{x\to 2} x^4$

24. $\lim_{x\to -2} x^3$

25. $\lim_{x\to -3} (3x + 2)$

26. $\lim_{x\to 0} (2x - 3)$

27. $\lim_{x\to 1} (1 - x^2)$

28. $\lim_{x\to 2} (-x^2 + x - 2)$

29. $\lim_{x\to 3} \sqrt{x+1}$

30. $\lim_{x\to 4} \sqrt[3]{x+4}$

31. $\lim_{x\to -3} \dfrac{2}{x+2}$

32. $\lim_{x\to -2} \dfrac{3x+1}{2-x}$

33. $\lim_{x\to -2} \dfrac{x^2-1}{2x}$

34. $\lim_{x\to -1} \dfrac{4x-5}{3-x}$

35. $\lim_{x\to 7} \dfrac{5x}{x+2}$

36. $\lim_{x\to 3} \dfrac{\sqrt{x+1}}{x-4}$

37. $\lim_{x\to 3} \dfrac{\sqrt{x+1}-1}{x}$

38. $\lim_{x\to 5} \dfrac{\sqrt{x+4}-2}{x}$

39. $\lim_{x\to 1} \dfrac{\dfrac{1}{x+4} - \dfrac{1}{4}}{x}$

40. $\lim_{x\to 2} \dfrac{\dfrac{1}{x+2} - \dfrac{1}{2}}{x}$

In Exercises 41–58, find the limit (if it exists).

41. $\lim\limits_{x\to -1} \dfrac{x^2-1}{x+1}$

42. $\lim\limits_{x\to -1} \dfrac{2x^2-x-3}{x+1}$

43. $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4x+4}$

44. $\lim\limits_{x\to 2} \dfrac{2-x}{x^2-4}$

45. $\lim\limits_{t\to 5} \dfrac{t-5}{t^2-25}$

46. $\lim\limits_{t\to 1} \dfrac{t^2+t-2}{t^2-1}$

47. $\lim\limits_{x\to -2} \dfrac{x^3+8}{x+2}$

48. $\lim\limits_{x\to 1} \dfrac{x^3-1}{x-1}$

49. $\lim\limits_{x\to -2} \dfrac{|x+2|}{x+2}$

50. $\lim\limits_{x\to 2} \dfrac{|x-2|}{x-2}$

51. $\lim\limits_{x\to 3} f(x)$, where $f(x) = \begin{cases} \frac{1}{3}x - 2, & x \le 3 \\ -2x+5, & x > 3 \end{cases}$

52. $\lim\limits_{s\to 1} f(s)$, where $f(s) = \begin{cases} s, & s \le 1 \\ 1-s, & s > 1 \end{cases}$

53. $\lim\limits_{\Delta x\to 0} \dfrac{2(x+\Delta x)-2x}{\Delta x}$

54. $\lim\limits_{\Delta x\to 0} \dfrac{4(x+\Delta x)-5-(4x-5)}{\Delta x}$

55. $\lim\limits_{\Delta x\to 0} \dfrac{\sqrt{x+2+\Delta x}-\sqrt{x+2}}{\Delta x}$

56. $\lim\limits_{\Delta x\to 0} \dfrac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}$

57. $\lim\limits_{\Delta t\to 0} \dfrac{(t+\Delta t)^2-5(t+\Delta t)-(t^2-5t)}{\Delta t}$

58. $\lim\limits_{\Delta t\to 0} \dfrac{(t+\Delta t)^2-4(t+\Delta t)+2-(t^2-4t+2)}{\Delta t}$

Graphical, Numerical, and Analytic Analysis In Exercises 59–62, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.

59. $\lim\limits_{x\to 1^-} \dfrac{2}{x^2-1}$

60. $\lim\limits_{x\to 1^+} \dfrac{5}{1-x}$

61. $\lim\limits_{x\to -2^-} \dfrac{1}{x+2}$

62. $\lim\limits_{x\to 0^-} \dfrac{x+1}{x}$

In Exercises 63–66, use a graphing utility to estimate the limit (if it exists).

63. $\lim\limits_{x\to 2} \dfrac{x^2-5x+6}{x^2-4x+4}$

64. $\lim\limits_{x\to 1} \dfrac{x^2+6x-7}{x^3-x^2+2x-2}$

65. $\lim\limits_{x\to -4} \dfrac{x^3+4x^2+x+4}{2x^2+7x-4}$

66. $\lim\limits_{x\to -2} \dfrac{4x^3+7x^2+x+6}{3x^2-x-14}$

67. The limit of

$$f(x) = (1+x)^{1/x}$$

is a natural base for many business applications, as you will see in Section 4.2.

$$\lim_{x\to 0} (1+x)^{1/x} = e \approx 2.718$$

(a) Show the reasonableness of this limit by completing the table.

x	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01
$f(x)$							

(b) Use a graphing utility to graph f and to confirm the answer in part (a).

(c) Find the domain and range of the function.

68. Find $\lim\limits_{x\to 0} f(x)$, given

$$4 - x^2 \le f(x) \le 4 + x^2, \text{ for all } x.$$

69. ***Environment*** The cost (in dollars) of removing $p\%$ of the pollutants from the water in a small lake is given by

$$C = \frac{25{,}000p}{100-p}, \quad 0 \le p < 100$$

where C is the cost and p is the percent of pollutants.

(a) Find the cost of removing 50% of the pollutants.

(b) What percent of the pollutants can be removed for \$100,000?

(c) Evaluate $\lim\limits_{p\to 100^-} C$. Explain your results.

70. ***Compound Interest*** You deposit \$1000 in an account that is compounded quarterly at an annual rate of r (in decimal form). The balance A after 10 years is

$$A = 1000\left(1 + \frac{r}{4}\right)^{40}.$$

Does the limit of A exist as the interest rate approaches 6%? If so, what is the limit?

71. ***Compound Interest*** Consider a certificate of deposit that pays 10% (annual percentage rate) on an initial deposit of \$500. The balance A after 10 years is

$$A = 500(1 + 0.1x)^{10/x}$$

where x is the length of the compounding period (in years).

(a) Use a graphing utility to graph A, where $0 \le x \le 1$.

(b) Use the *zoom* and *trace* features to estimate the balance for quarterly compounding and daily compounding.

(c) Use the *zoom* and *trace* features to estimate

$$\lim_{x\to 0^+} A.$$

What do you think this limit represents? Explain your reasoning.

1.4

Continuity

In mathematics, the
everyday use. To say
interruption in the g
no holes, jumps, or g
tion eluded mathem
1800s that a precise
Before looking a
in Figure 1.60. This
not continuous.

1. At $x = c_1$, $f(c_1)$ is
2. At $x = c_2$, $\lim\limits_{x\to c_2} f($
3. At $x = c_3$, $f(c_3) \neq$

At all other points in
implies that the functi

Definition of Con

Let c be a number
domain contains t
point c if the follo

1. $f(c)$ is defined.
2. $\lim\limits_{x\to c} f(x)$ exists.
3. $\lim\limits_{x\to c} f(x) = f(c)$.

If f is continuous at
on an open interv

Roughly, you can
on the interval can be
from the paper, as show

est

rest is paid to an accoun
rest is paid on previous
npounded. Suppose, f
it pays 6% interest, com
est rate, the quarterly ra
st five quarters are show

= $10,150.00
= $10,302.25
= $10,456.78
= $10,613.63

st

d above.

nt and let t represent th
to represent the balanc

nction has a discontinui

nt in Example 6 if the

PREREQUISITE REVIEW 1.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, simplify the expression.

1. $\dfrac{x^2 + 6x + 8}{x^2 - 6x - 16}$

2. $\dfrac{x^2 - 5x - 6}{x^2 - 9x + 18}$

3. $\dfrac{2x^2 - 2x - 12}{4x^2 - 24x + 36}$

4. $\dfrac{x^3 - 16x}{x^3 + 2x^2 - 8x}$

In Exercises 5–8, solve for x.

5. $x^2 + 7x = 0$

6. $x^2 + 4x - 5 = 0$

7. $3x^2 + 8x + 4 = 0$

8. $x^3 + 5x^2 - 24x = 0$

In Exercises 9 and 10, find the limit.

9. $\lim\limits_{x \to 3} (2x^2 - 3x + 4)$

10. $\lim\limits_{x \to -2} (3x^3 - 8x + 7)$

EXERCISES 1.6

In Exercises 1–10, determine whether the function is continuous on the entire real line. Explain your reasoning.

1. $f(x) = 5x^3 - x^2 + 2$

2. $f(x) = (x^2 - 1)^3$

3. $f(x) = \dfrac{1}{x^2 - 4}$

4. $f(x) = \dfrac{1}{9 - x^2}$

5. $f(x) = \dfrac{1}{4 + x^2}$

6. $f(x) = \dfrac{3x}{x^2 + 1}$

7. $f(x) = \dfrac{2x - 1}{x^2 - 8x + 15}$

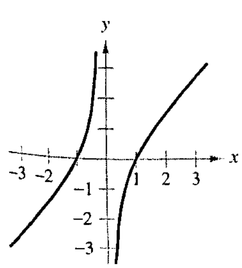
8. $f(x) = \dfrac{x + 4}{x^2 - 6x + 5}$

9. $g(x) = \dfrac{x^2 - 4x + 4}{x^2 - 4}$

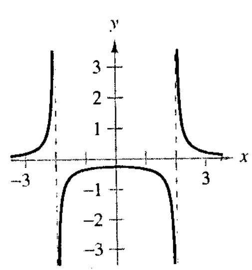
10. $g(x) = \dfrac{x^2 - 9x + 20}{x^2 - 16}$

In Exercises 11–34, describe the interval(s) on which the function is continuous.

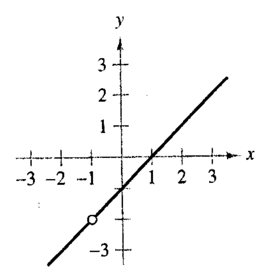
11. $f(x) = \dfrac{x^2 - 1}{x}$



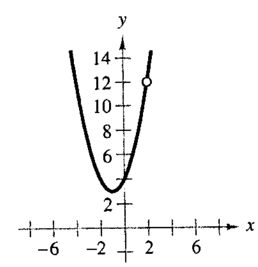
12. $f(x) = \dfrac{1}{x^2 - 4}$



13. $f(x) = \dfrac{x^2 - 1}{x + 1}$



14. $f(x) = \dfrac{x^3 - 8}{x - 2}$



15. $f(x) = x^2 - 2x + 1$

16. $f(x) = 3 - 2x - x^2$

17. $f(x) = \dfrac{x}{x^2 - 1}$

18. $f(x) = \dfrac{x - 3}{x^2 - 9}$

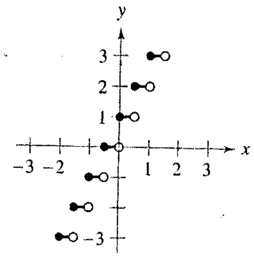
19. $f(x) = \dfrac{x}{x^2 + 1}$

20. $f(x) = \dfrac{1}{x^2 + 1}$

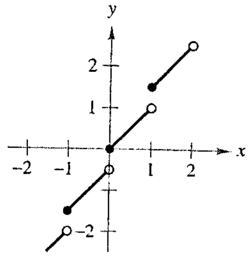
21. $f(x) = \dfrac{x - 5}{x^2 - 9x + 20}$

22. $f(x) = \dfrac{x - 1}{x^2 + x - 2}$

23. $f(x) = [\![2x]\!] + 1$



24. $f(x) = \dfrac{[\![x]\!]}{2} + x$



25. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$

26. $f(x) = \begin{cases} 3 + x, & x \leq 2 \\ x^2 + 1, & x > 2 \end{cases}$

27. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x, & x > 2 \end{cases}$

28. $f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 3x + 1, & x > 0 \end{cases}$

29. $f(x) = \dfrac{|x + 1|}{x + 1}$

30. $f(x) = \dfrac{|4 - x|}{4 - x}$

31. $f(x) = [\![x - 1]\!]$

32. $f(x) = x - [\![x]\!]$

33. $h(x) = f(g(x)), \quad f(x) = \dfrac{1}{\sqrt{x}}, \quad g(x) = x - 1, x > 1$

34. $h(x) = f(g(x)), \quad f(x) = \dfrac{1}{x - 1}, \quad g(x) = x^2 + 5$

In Exercises 35–38, discuss the continuity of the function on the closed interval. If there are any discontinuities, determine whether they are removable.

	Function	*Interval*
35.	$f(x) = x^2 - 4x - 5$	$[-1, 5]$
36.	$f(x) = \dfrac{5}{x^2 + 1}$	$[-2, 2]$
37.	$f(x) = \dfrac{1}{x - 2}$	$[1, 4]$
38.	$f(x) = \dfrac{x}{x^2 - 4x + 3}$	$[0, 4]$

In Exercises 39–44, sketch the graph of the function and describe the interval(s) on which the function is continuous.

39. $f(x) = \dfrac{x^2 - 16}{x - 4}$

40. $f(x) = \dfrac{2x^2 + x}{x}$

41. $f(x) = \dfrac{x^3 + x}{x}$

42. $f(x) = \dfrac{x - 3}{4x^2 - 12x}$

43. $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$

44. $f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 2x + 4, & x > 0 \end{cases}$

In Exercises 45 and 46, find the constant a (Exercise 45) and constants a and b (Exercise 46) such that the function is continu on the entire real line.

45. $f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$

46. $f(x) = \begin{cases} 2, & x \leq -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$

In Exercises 47–52, use a graphing utility to graph the functi Use the graph to determine any x-values at which the functior not continuous.

47. $h(x) = \dfrac{1}{x^2 - x - 2}$

48. $k(x) = \dfrac{x - 4}{x^2 - 5x + 4}$

49. $f(x) = \begin{cases} 2x - 4, & x \leq 3 \\ x^2 - 2x, & x > 3 \end{cases}$

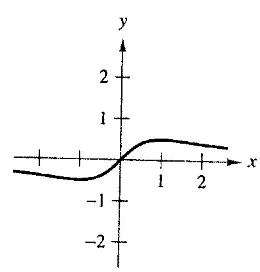
50. $f(x) = \begin{cases} 3x - 1, & x \leq 1 \\ x + 1, & x > 1 \end{cases}$

51. $f(x) = x - 2[\![x]\!]$

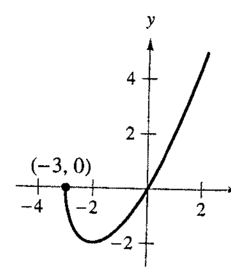
52. $f(x) = [\![2x - 1]\!]$

In Exercises 53–56, describe the interval(s) on which the funct is continuous.

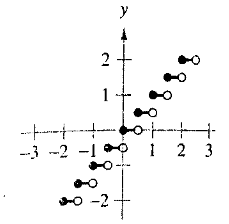
53. $f(x) = \dfrac{x}{x^2 + 1}$



54. $f(x) = x\sqrt{x + 3}$



55. $f(x) = \frac{1}{2}[\![2x]\!]$



Writing In Exercises 57 the function on the interv tion appear to be contin fact continuous on $[-4$ the importance of exami graphically.

57. $f(x) = \dfrac{x^2 + x}{x}$

58. $f(x) = \dfrac{x^3 - 8}{x - 2}$

59. ***Compound Interes*** account that pays 6% A in the account afte

$A = 7500(1.015)^{[}$

(a) Sketch the graph your reasoning.

(b) What is the balar

60. ***Environmental Co.*** of removing x perce smokestack of a fact

$C = \dfrac{2x}{100 - x}.$

(a) What is the im reasoning.

(b) Use a graphing u function continu reasoning.

(c) Find the cost of r the smokestack.

61. ***Consumer Awaren*** for sending an overni is $9.80 for the first pound or fraction the to create a model for a package weighing graph the function, ar

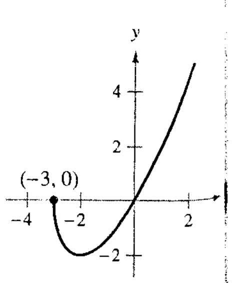
constant a (Exercise 45) and
ch that the function is continu

< 3

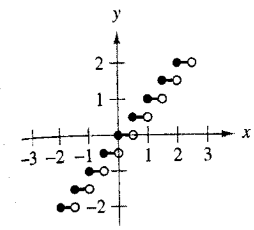
ng utility to graph the functi
x-values at which the functio

nterval(s) on which the funct

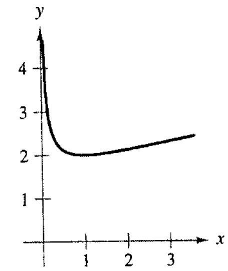
54. $f(x) = x\sqrt{x+3}$



55. $f(x) = \frac{1}{2}[\![2x]\!]$



56. $f(x) = \dfrac{x+1}{\sqrt{x}}$



Writing In Exercises 57 and 58, use a graphing utility to graph the function on the interval $[-4, 4]$. Does the graph of the function appear to be continuous on this interval? Is the function in fact continuous on $[-4, 4]$? Write a short paragraph about the importance of examining a function analytically as well as graphically.

57. $f(x) = \dfrac{x^2 + x}{x}$

58. $f(x) = \dfrac{x^3 - 8}{x - 2}$

59. ***Compound Interest*** A deposit of \$7500 is made in an account that pays 6% compounded quarterly. The amount A in the account after t years is

$$A = 7500(1.015)^{[\![4t]\!]}, \quad t \geq 0.$$

(a) Sketch the graph of A. Is the graph continuous? Explain your reasoning.

(b) What is the balance after 7 years?

60. ***Environmental Cost*** The cost C (in millions of dollars) of removing x percent of the pollutants emitted from the smokestack of a factory can be modeled by

$$C = \frac{2x}{100 - x}.$$

(a) What is the implied domain of C? Explain your reasoning.

(b) Use a graphing utility to graph the cost function. Is the function continuous on its domain? Explain your reasoning.

(c) Find the cost of removing 75% of the pollutants from the smokestack.

61. ***Consumer Awareness*** A shipping company's charge for sending an overnight package from New York to Atlanta is \$9.80 for the first pound and \$2.50 for each additional pound or fraction thereof. Use the greatest integer function to create a model for the charge C for overnight delivery of a package weighing x pounds. Use a graphing utility to graph the function, and discuss its continuity.

62. ***Consumer Awareness*** A cab company charges \$3 for the first mile and \$0.25 for each additional mile or fraction thereof. Use the greatest integer function to create a model for the cost C of a cab ride n miles long. Use a graphing utility to graph the function, and discuss its continuity.

63. ***Consumer Awareness*** A dial-direct long distance call between two cities costs \$1.04 for the first 2 minutes and \$0.36 for each additional minute or fraction thereof.

(a) Use the greatest integer function to write the cost C of a call in terms of the time t (in minutes). Sketch the graph of the cost function and discuss its continuity.

(b) Find the cost of a nine-minute call.

64. ***Salary Contract*** A union contract guarantees a 9% yearly increase for 5 years. For a current salary of \$28,500, the salary for the next 5 years is given by

$$S = 28{,}500(1.09)^{[\![t]\!]}$$

where $t = 0$ represents the present year.

(a) Use the greatest integer function of a graphing utility to graph the salary function, and discuss its continuity.

(b) Find the salary during the fifth year (when $t = 5$).

65. ***Inventory Management*** The number of units in inventory in a small company is

$$N = 25\left(2\left[\!\!\left[\frac{t+2}{2}\right]\!\!\right] - t\right), \quad 0 \leq t \leq 12$$

where the real number t is the time in months.

(a) Use the greatest integer function of a graphing utility to graph this function, and discuss its continuity.

(b) How often must the company replenish its inventory?

66. ***Owning a Franchise*** You have purchased a franchise. You have determined a linear model for your revenue as a function of time. Is the model a continuous function? Would your actual revenue be a continuous function of time? Explain your reasoning.

67. ***Biology*** The gestation period of rabbits is only 26 to 30 days. Therefore, the population of a form (rabbits' home) can increase dramatically in a short period of time. The table gives the population of a form, where t is the time in months and N is the rabbit population.

t	0	1	2	3	4	5	6
N	2	8	10	14	10	15	12

Graph the population as a function of time. Find any points of discontinuity in the function. Explain your reasoning.

PREREQUISITE REVIEW 3.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, find the limit.

1. $\lim_{x\to 2} (x + 1)$
2. $\lim_{x\to -1} (3x + 4)$
3. $\lim_{x\to -3} \dfrac{2x^2 + x - 15}{x + 3}$
4. $\lim_{x\to 2} \dfrac{3x^2 - 8x + 4}{x - 2}$
5. $\lim_{x\to 2^+} \dfrac{x^2 - 5x + 6}{x^2 - 4}$
6. $\lim_{x\to 1^-} \dfrac{x^2 - 6x + 5}{x^2 - 1}$
7. $\lim_{x\to 0^+} \sqrt{x}$
8. $\lim_{x\to 1^+} \left(x + \sqrt{x - 1}\right)$

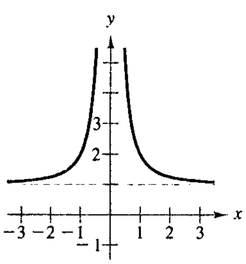
In Exercises 9–12, find the average cost and the marginal cost.

9. $C = 150 + 3x$
10. $C = 1900 + 1.7x + 0.002x^2$
11. $C = 0.005x^2 + 0.5x + 1375$
12. $C = 760 + 0.05x$

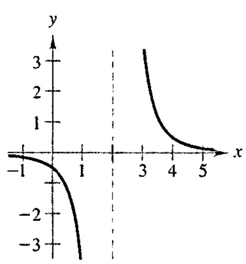
EXERCISES 3.6

In Exercises 1–8, find the vertical and horizontal asymptotes. Write the asymptotes as equations of lines.

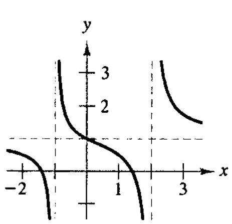
1. $f(x) = \dfrac{x^2 + 1}{x^2}$



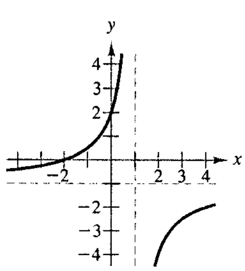
2. $f(x) = \dfrac{4}{(x - 2)^3}$



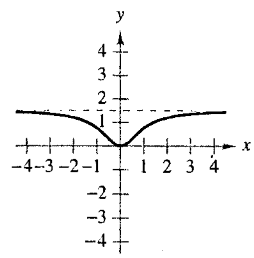
3. $f(x) = \dfrac{x^2 - 2}{x^2 - x - 2}$



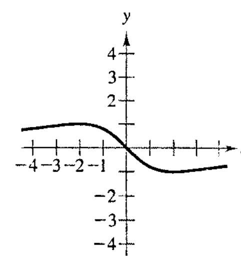
4. $f(x) = \dfrac{2 + x}{1 - x}$



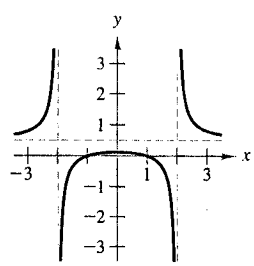
5. $f(x) = \dfrac{3x^2}{2(x^2 + 1)}$



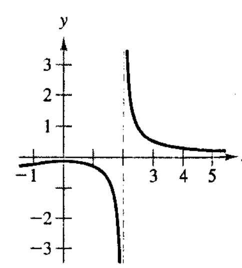
6. $f(x) = \dfrac{-4x}{x^2 + 4}$



7. $f(x) = \dfrac{x^2 - 1}{2x^2 - 8}$

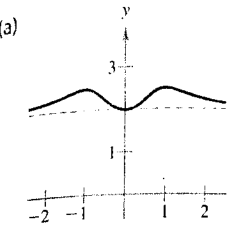


8. $f(x) = \dfrac{x^2 + 1}{x^3 - 8}$

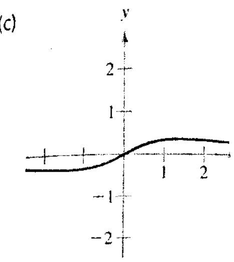


In Exercises 9–14, matc
horizontal asymptotes as

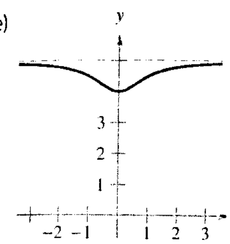
(a)



(c)



(e)



9. $f(x) = \dfrac{3x^2}{x^2 + 2}$

11. $f(x) = \dfrac{x}{x^2 + 2}$

13. $f(x) = 5 - \dfrac{1}{x^2 + 1}$

In Exercises 15–22, find the

15. $\lim_{x\to -2^-} \dfrac{1}{(x + 2)^2}$

17. $\lim_{x\to 3^+} \dfrac{x - 4}{x - 3}$

19. $\lim_{x\to 4^-} \dfrac{x^2}{x^2 - 16}$

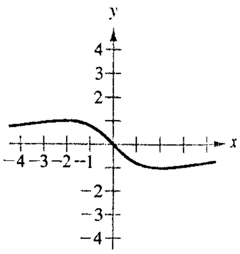
21. $\lim_{x\to 0^-} \left(1 + \dfrac{1}{x}\right)$

In Exercises 23–32, find the l

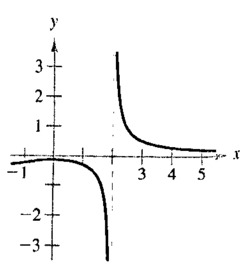
23. $\lim_{x\to\infty} \dfrac{2x - 1}{3x + 2}$

n earlier sections. You will

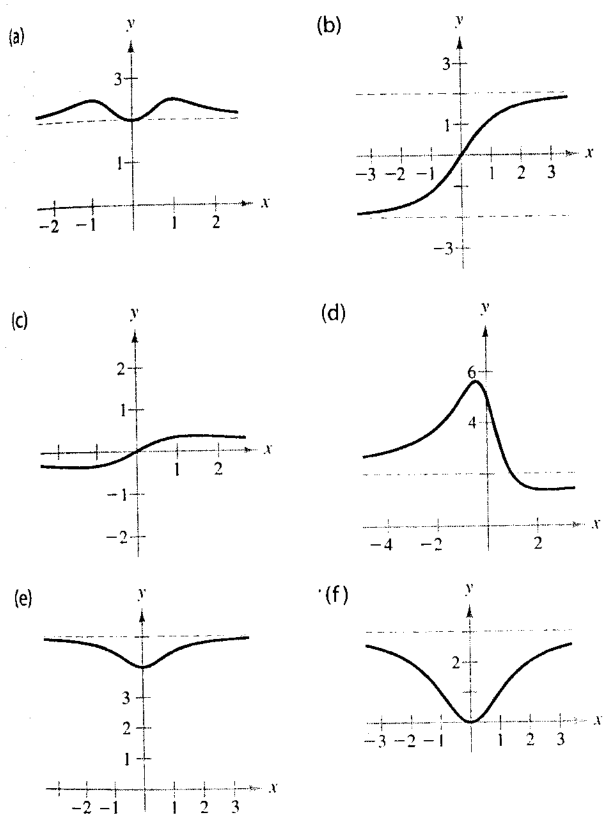
6. $f(x) = \dfrac{-4x}{x^2 + 4}$



8. $f(x) = \dfrac{x^2 + 1}{x^3 - 8}$



In Exercises 9–14, match the function with its graph. Use horizontal asymptotes as an aid. [The graphs are labeled (a)–(f).]



9. $f(x) = \dfrac{3x^2}{x^2 + 2}$

10. $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$

11. $f(x) = \dfrac{x}{x^2 + 2}$

12. $f(x) = 2 + \dfrac{x^2}{x^4 + 1}$

13. $f(x) = 5 - \dfrac{1}{x^2 + 1}$

14. $f(x) = \dfrac{2x^2 - 3x + 5}{x^2 + 1}$

In Exercises 15–22, find the limit.

15. $\lim\limits_{x \to -2^-} \dfrac{1}{(x + 2)^2}$

16. $\lim\limits_{x \to -2^-} \dfrac{1}{x + 2}$

17. $\lim\limits_{x \to 3^+} \dfrac{x - 4}{x - 3}$

18. $\lim\limits_{x \to 1^+} \dfrac{2 + x}{1 - x}$

19. $\lim\limits_{x \to 4^-} \dfrac{x^2}{x^2 - 16}$

20. $\lim\limits_{x \to 4} \dfrac{x^2}{x^2 + 16}$

21. $\lim\limits_{x \to 0^-} \left(1 + \dfrac{1}{x}\right)$

22. $\lim\limits_{x \to 0^-} \left(x^2 - \dfrac{1}{x}\right)$

In Exercises 23–32, find the limit.

23. $\lim\limits_{x \to \infty} \dfrac{2x - 1}{3x + 2}$

24. $\lim\limits_{x \to \infty} \dfrac{5x^3 + 1}{10x^3 - 3x^2 + 7}$

25. $\lim\limits_{x \to \infty} \dfrac{3x}{4x^2 - 1}$

26. $\lim\limits_{x \to \infty} \dfrac{2x^{10} - 1}{10x^{11} - 3}$

27. $\lim\limits_{x \to -\infty} \dfrac{5x^2}{x + 3}$

28. $\lim\limits_{x \to \infty} \dfrac{x^3 - 2x^2 + 3x + 1}{x^2 - 3x + 2}$

29. $\lim\limits_{x \to \infty} (2x - x^{-2})$

30. $\lim\limits_{x \to \infty} (2 - x^{-3})$

31. $\lim\limits_{x \to -\infty} \left(\dfrac{2x}{x - 1} + \dfrac{3x}{x + 1}\right)$

32. $\lim\limits_{x \to \infty} \left(\dfrac{2x^2}{x - 1} + \dfrac{3x}{x + 1}\right)$

In Exercises 33 and 34, complete the table. Then use the result to estimate the limit of $f(x)$ as x approaches infinity.

33. $f(x) = \dfrac{x + 1}{x\sqrt{x}}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

34. $f(x) = x^2 - x\sqrt{x(x - 1)}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

In Exercises 35 and 36, use a spreadsheet software program to complete the table and use the result to estimate the limit of $f(x)$ as x approaches infinity.

35. $f(x) = \dfrac{x^2 - 1}{0.02x^2}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

36. $f(x) = \dfrac{3x^2}{0.1x^2 + 1}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

In Exercises 37 and 38, use a graphing utility to complete the table and use the result to estimate the limit of $f(x)$ as approaches infinity and as x approaches negative infinity.

37. $f(x) = \dfrac{2x}{\sqrt{x^2 + 4}}$

x	-10^6	-10^4	-10^2	10^0	10^2	10^4	10^6
$f(x)$							

38. $f(x) = x - \sqrt{x(x-1)}$

x	-10^6	-10^4	-10^2	10^0	10^2	10^4	10^6
$f(x)$							

In Exercises 39–56, sketch the graph of the equation. Use intercepts, extrema, and asymptotes as sketching aids.

39. $y = \dfrac{2+x}{1-x}$

40. $y = \dfrac{x-3}{x-2}$

41. $f(x) = \dfrac{x^2}{x^2+9}$

42. $f(x) = \dfrac{x}{x^2+4}$

43. $g(x) = \dfrac{x^2}{x^2-16}$

44. $g(x) = \dfrac{x}{x^2-4}$

45. $xy^2 = 4$

46. $x^2y = 4$

47. $y = \dfrac{2x}{1-x}$

48. $y = \dfrac{2x}{1-x^2}$

49. $y = 3(1 - x^{-2})$

50. $y = 1 + x^{-1}$

51. $f(x) = \dfrac{1}{x^2-x-2}$

52. $f(x) = \dfrac{x-2}{x^2-4x+3}$

53. $g(x) = \dfrac{x^2-x-2}{x-2}$

54. $g(x) = \dfrac{x^2-9}{x+3}$

55. $y = \dfrac{2x^2-6}{(x-1)^2}$

56. $y = \dfrac{x}{(x+1)^2}$

57. ***Cost*** The cost C (in dollars) of producing x units of a product is $C = 1.35x + 4570$.

(a) Find the average cost function $\overline{C}$.

(b) Find $\overline{C}$ when $x = 100$ and when $x = 1000$.

(c) What is the limit of $\overline{C}$ as x approaches infinity?

58. ***Average Cost*** A business has a cost (in dollars) of $C = 0.5x + 500$ for producing x units.

(a) Find the average cost function $\overline{C}$.

(b) Find $\overline{C}$ when $x = 250$ and when $x = 1250$.

(c) What is the limit of $\overline{C}$ as x approaches infinity?

59. ***Cost*** The cost C (in millions of dollars) for the federal government to seize $p\%$ of a type of illegal drug as it enters the country is modeled by

$$C = 528p/(100 - p), \qquad 0 \le p < 100.$$

(a) Find the cost of seizing 25%, 50%, and 75%.

(b) Find the limit of C as $p \to 100^-$.

(c) Use a graphing utility to verify the result of part (b).

60. ***Cost*** The cost C (in dollars) of removing $p\%$ of the air pollutants in the stack emission of a utility company that burns coal is modeled by

$$C = 80{,}000p/(100 - p), \qquad 0 \le p < 100.$$

(a) Find the cost of removing 15%, 50%, and 90%.

(b) Find the limit of C as $p \to 100^-$.

(c) Use a graphing utility to verify the result of part (b).

61. ***Learning Curve*** Psychologists have developed mathematical models to predict performance P (the percent of correct responses) as a function of n, the number of times a task is performed. One such model is

$$P = \frac{b + \theta a(n-1)}{1 + \theta(n-1)}$$

where a, b, and θ are constants that depend on the actual learning situation. Find the limit of P as n approaches infinity.

62. ***Learning Curve*** Consider the learning curve given by

$$P = \frac{0.5 + 0.9(n-1)}{1 + 0.9(n-1)}, \qquad 0 < n.$$

(a) Complete the table for the model.

n	1	2	3	4	5	6	7	8	9	10
P										

(b) Find the limit as n approaches infinity.

(c) Use a graphing utility to graph this learning curve, and interpret the graph in the context of the problem.

63. ***Biology: Wildlife Management*** The state game commission introduces 30 elk into a new state park. The population N of the herd is modeled by

$$N = [10(3 + 4t)]/(1 + 0.1t)$$

where t is the time in years.

(a) Find the size of the herd after 5, 10, and 25 years.

(b) According to this model, what is the limiting size of the herd as time progresses?

64. ***Average Profit*** The cost and revenue functions for a product are $C = 34.5x + 15{,}000$ and $R = 69.9x$.

(a) Find the average profit function

$$\overline{P} = (R - C)/x.$$

(b) Find the average profit when x is 1000, 10,000, and 100,000.

(c) What is the limit of the average profit function as x approaches infinity? Explain your reasoning.

65. ***Average Profit*** The cost and revenue functions for a product are $C = 25.5x + 1000$ and $R = 75.5x$.

(a) Find the average profit function $\overline{P} = \dfrac{R - C}{x}$.

(b) Find the average profit when x is 100, 500, and 1000.

(c) What is the limit of the average profit function as x approaches infinity? Explain your reasoning.

3.7

Summary of Cu

It would be difficult t
Descartes's introduct
rapid advances in cal
So far, you have
graph of a function.

- x-intercepts and y-i
- Domain and range
- Continuity
- Differentiability
- Relative extrema
- Concavity
- Points of inflection
- Vertical asymptote
- Horizontal asympt

When you are s
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the viewing windows

$f(x) = x^3 - 25x$

The lower viewing w
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Guidelines for A

1. Determine the mode a real-
2. Determine the
3. Locate the x-
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PREREQUISITE REVIEW 8.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, find the area of the triangle.

1. Base: 10 cm; height: 7 cm

2. Base: 4 in.; height: 6 in.

In Exercises 3–6, let a and b represent the lengths of the legs, and let c represent the length of the hypotenuse, of a right triangle. Solve for the missing side length.

3. $a = 5, b = 12$

4. $a = 3, c = 5$

5. $a = 8, c = 17$

6. $b = 8, c = 10$

In Exercises 7–10, let a, b, and c represent the side lengths of a triangle. Use the information below to determine whether the figure is a right triangle, an isosceles triangle, or an equilateral triangle.

7. $a = 4, b = 4, c = 4$

8. $a = 1, b = 1, c = 2$

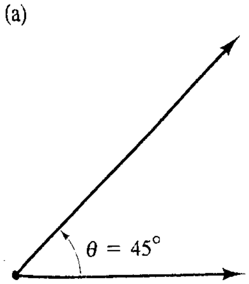
9. $a = 12, b = 16, c = 20$

10. $a = 1, b = 1, c = \sqrt{2}$

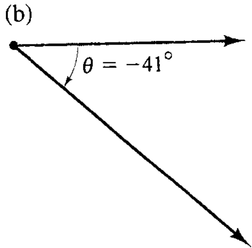
EXERCISES 8.1

In Exercises 1–4, determine two coterminal angles (one positive and one negative) for the given angle. Give the answers in degrees.

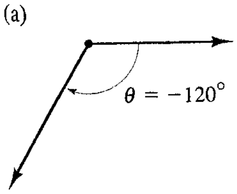
1. (a)



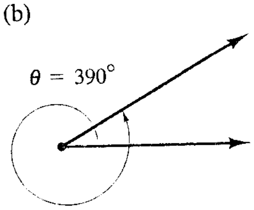
(b)



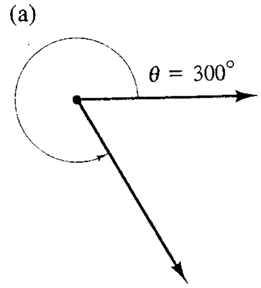
2. (a)



(b)



3. (a)



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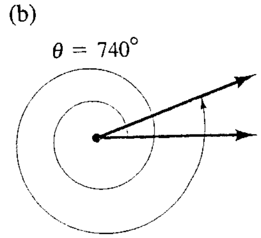
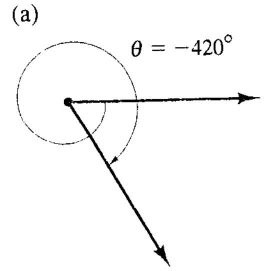


4. (a)



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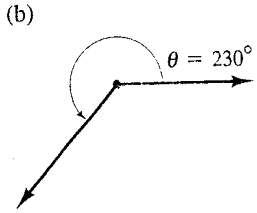
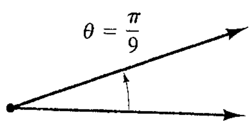
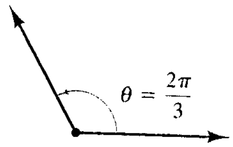


In Exercises 5–8, determine two coterminal angles (one positive and one negative) for each angle. Give the answers in radians.

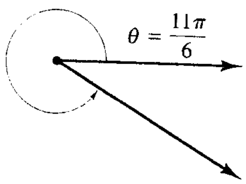
5. (a)



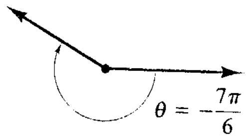
(b)



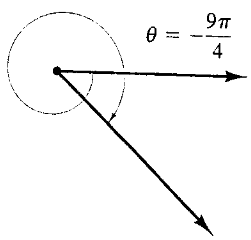
6. (a)



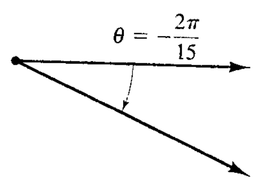
(b)



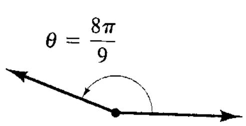
7. (a)



(b)



8. (a)



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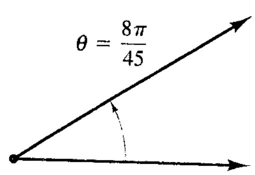


In Exercises 9–20, express the angle in radian measure as a multiple of π. Use a calculator to verify your result.

9. 30° **10.** 150°

11. 225° **12.** 210°

13. 315° **14.** 120°

15. −30° **16.** −240°

17. −270° **18.** −330°

19. 390° **20.** 405°

In Exercises 21–30, express the angle in degree measure. Use a calculator to verify your result.

21. $\frac{3\pi}{2}$ **22.** $\frac{7\pi}{6}$

23. $\frac{11\pi}{6}$ **24.** $\frac{7\pi}{4}$

25. $-\frac{5\pi}{3}$ **26.** $-\frac{3\pi}{4}$

27. $\frac{9\pi}{4}$ **28.** $\frac{5\pi}{2}$

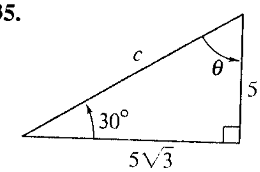
29. $\frac{19\pi}{6}$ **30.** $\frac{8\pi}{3}$

In Exercises 31–34, find the indicated measure of the angle. Express radian measure as a multiple of π.

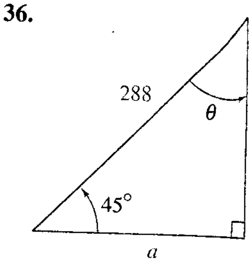
	Degree Measure	*Radian Measure*
31.	−270°	
32.		$\frac{\pi}{9}$
33.	144°	
34.		$-\frac{7\pi}{12}$

In Exercises 35–42, solve the triangle for the indicated side and/or angle.

35.



36.



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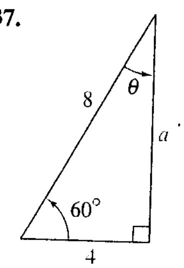

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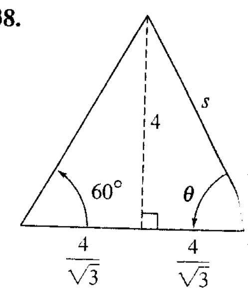

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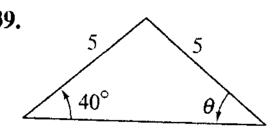

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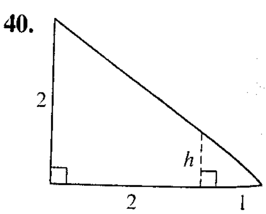

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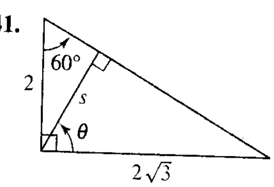

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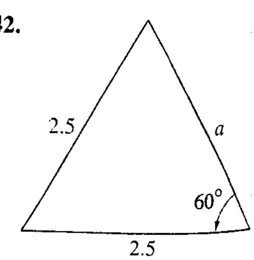


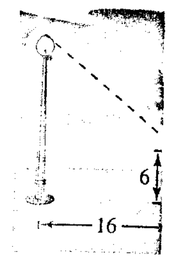
In Exercises 43–46,
sides of length s.

43. $s = 6$ in.

45. $s = 5$ ft

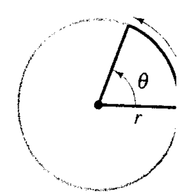
47. ***Height*** A pe
streetlight casts
the height of th

Figure for 47



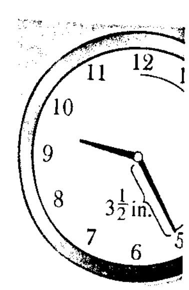
48. ***Length*** A gu
tower at a point
feet from the bas

49. Let r represent t
(measured in rad
by the angle (se
complete the tab



r	8 ft	15
s	12 ft	
θ		1.6

50. ***Arc Length*** Th
long (see figure).
minute hand mov



In Exercises 43–46, find the area of the equilateral triangle with sides of length s.

43. $s = 6$ in. **44.** $s = 8$ m

45. $s = 5$ ft **46.** $s = 12$ cm

47. ***Height*** A person 6 feet tall standing 16 feet from a streetlight casts a shadow 8 feet long (see figure). What is the height of the streetlight?

Figure for 47

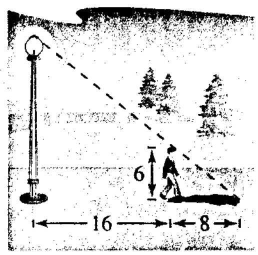
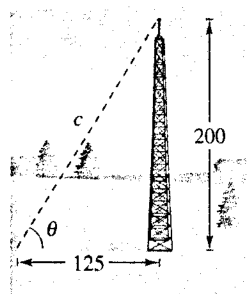
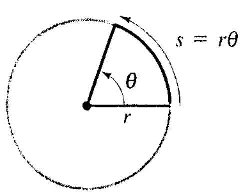


Figure for 48



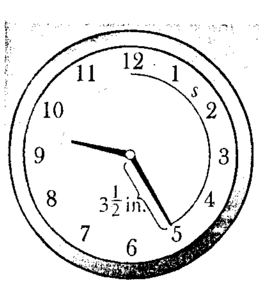
48. ***Length*** A guy wire is stretched from a broadcasting tower at a point 200 feet above the ground to an anchor 125 feet from the base (see figure). How long is the wire?

49. Let r represent the radius of a circle, θ the central angle (measured in radians), and s the length of the arc interceped by the angle (see figure). Use the relationship $\theta = s/r$ to complete the table.



r	8 ft	15 in.	85 cm		
s	12 ft			96 in.	8642 mi
θ		1.6	$\frac{3\pi}{4}$	4	$\frac{2\pi}{3}$

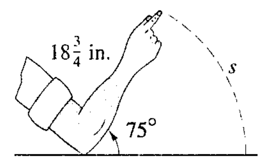
50. ***Arc Length*** The minute hand on a clock is $3\frac{1}{2}$ inches long (see figure). Through what distance does the tip of the minute hand move in 25 minutes?



51. ***Distance*** A man bends his elbow through 75°. The distance from his elbow to the tip of his index finger is $18\frac{3}{4}$ inches (see figure).

(a) Find the radian measure of this angle.

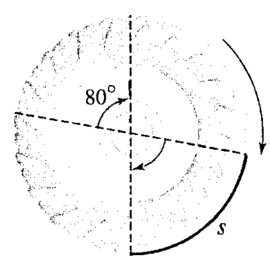
(b) Find the distance the tip of the index finger moves.



52. ***Distance*** A tractor tire that is 5 feet in diameter d is partially filled with a liquid ballast for additional traction. To check the air pressure, the tractor operator rotates the tire until the valve stem is at the top so that the liquid will not enter the gauge. On a given occasion, the operator notes that the tire must be rotated 80° to have the stem in the proper position (see figure).

(a) Find the radian measure of this rotation.

(b) How far must the tractor be moved to get the valve stem in the proper position?



53. ***Speed of Revolution*** A compact disc can have an angular speed up to 3142 radians per minute.

(a) At this angular speed, how many revolutions per minute would the CD make?

(b) How long would it take the CD to make 10,000 revolutions?

54. ***Speed of Revolution*** The radius of the magnetic disk in a 3.5-inch diskette is 1.68 inches. Find the linear speed of a point on the circumference of the disk if it rotates at a speed of 360 revolutions per minute.

True or False? In Exercises 55–58, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

55. An angle whose measure is 75° is obtuse.

56. $\theta = -35°$ is coterminal to 325°.

57. A right triangle can have one angle whose measure is 89°.

58. An angle whose measure is π radians is a straight angle.

PREREQUISITE REVIEW 8.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, convert the angle to radian measure.

1. 135°
2. 315°
3. −210°
4. −300°
5. −120°
6. −225°
7. 540°
8. 390°

In Exercises 9–16, solve for *x*.

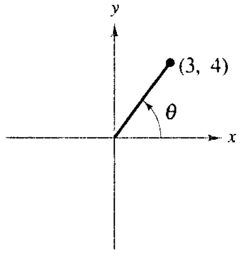
9. $x^2 - x = 0$
10. $2x^2 + x = 0$
11. $2x^2 - x = 1$
12. $x^2 - 2x = 3$
13. $x^2 - 2x = -1$
14. $2x^2 + x = 1$
15. $x^2 - 5x = -6$
16. $x^2 + x = 2$

In Exercises 17–20, solve for *t*.

17. $\frac{2\pi}{24}(t - 4) = \frac{\pi}{2}$
18. $\frac{2\pi}{12}(t - 2) = \frac{\pi}{4}$
19. $\frac{2\pi}{365}(t - 10) = \frac{\pi}{4}$
20. $\frac{2\pi}{12}(t - 4) = \frac{\pi}{2}$

EXERCISES 8.2

In Exercises 1–6, determine all six trigonometric functions for the angle θ.

1.


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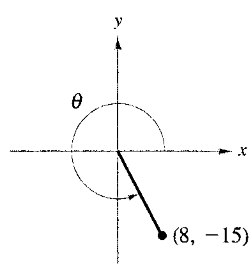

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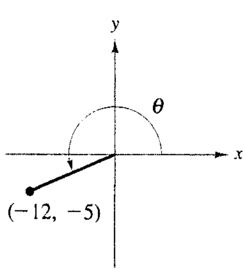

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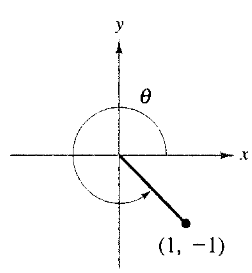

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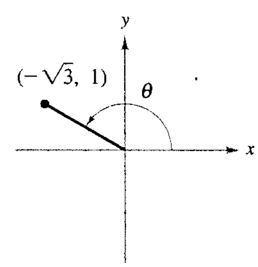

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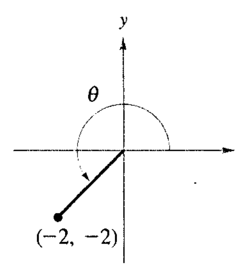
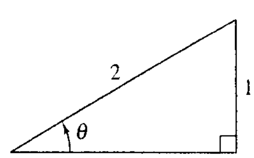
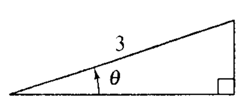


In Exercises 7–12, find the indicated trigonometric function from the given function.

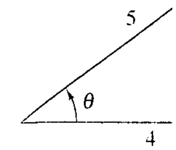
7. Given $\sin\theta = \frac{1}{2}$, find $\csc\theta$.



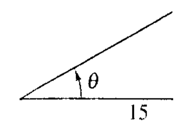
8. Given $\sin\theta = \frac{1}{3}$, find $\tan\theta$.



9. Given $\cos\theta = \frac{4}{5}$
find $\cot\theta$.



11. Given $\cot\theta = \frac{15}{8}$
find $\sec\theta$.



In Exercises 13–18, sk
trigonometric functic
trigonometric functio

13. $\sin\theta = \frac{1}{3}$
15. $\sec\theta = \frac{3}{2}$
17. $\tan\theta = 3.5$

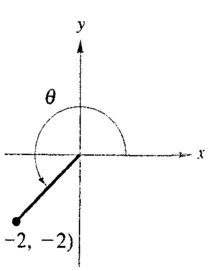
In Exercises 19–24, de

19. $\sin\theta < 0$, $\cos\theta$
20. $\sin\theta > 0$, $\cos\theta$
21. $\sin\theta > 0$, $\sec\theta$
22. $\cot\theta < 0$, $\cos\theta$
23. $\csc\theta > 0$, $\tan\theta$
24. $\cos\theta > 0$, $\tan\theta$

In Exercises 25–32, ev
the angles *without* usi

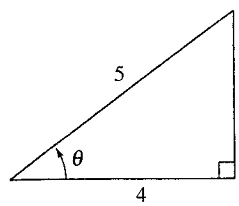
25. (a) 60°
26. (a) $\frac{\pi}{4}$
27. (a) $-\frac{\pi}{6}$
28. (a) $-\frac{\pi}{2}$
29. (a) 225°
30. (a) 300°
31. (a) 750°

sections. You will

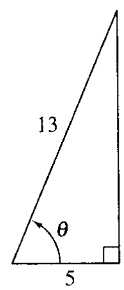


nometric function from

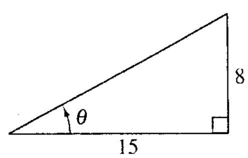
9. Given $\cos\theta = \frac{4}{5}$, find $\cot\theta$.



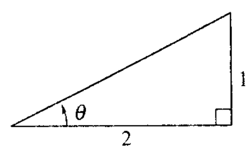
10. Given $\sec\theta = \frac{13}{5}$, find $\cot\theta$.



11. Given $\cot\theta = \frac{15}{8}$, find $\sec\theta$.



12. Given $\tan\theta = \frac{1}{2}$, find $\sin\theta$.



In Exercises 13–18, sketch a right triangle corresponding to the trigonometric function of the angle θ and find the other five trigonometric functions of θ.

13. $\sin\theta = \frac{1}{3}$ **14.** $\cot\theta = 5$

15. $\sec\theta = \frac{3}{2}$ **16.** $\cos\theta = \frac{5}{7}$

17. $\tan\theta = 3.5$ **18.** $\csc\theta = 4.25$

In Exercises 19–24, determine the quadrant in which θ lies.

19. $\sin\theta < 0, \cos\theta > 0$

20. $\sin\theta > 0, \cos\theta < 0$

21. $\sin\theta > 0, \sec\theta > 0$

22. $\cot\theta < 0, \cos\theta > 0$

23. $\csc\theta > 0, \tan\theta < 0$

24. $\cos\theta > 0, \tan\theta < 0$

In Exercises 25–32, evaluate the sines, cosines, and tangents of the angles *without* using a calculator.

25. (a) $60°$ (b) $-\frac{2\pi}{3}$

26. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$

27. (a) $-\frac{\pi}{6}$ (b) $150°$

28. (a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{2}$

29. (a) $225°$ (b) $-225°$

30. (a) $300°$ (b) $330°$

31. (a) $750°$ (b) $510°$

32. (a) $\frac{10\pi}{3}$ (b) $\frac{17\pi}{3}$

In Exercises 33–40, use a calculator to evaluate the trigonometric functions to four decimal places.

33. (a) $\sin 12°$ (b) $\csc 12°$

34. (a) $\sec 225°$ (b) $\sec 135°$

35. (a) $\tan\frac{\pi}{9}$ (b) $\tan\frac{10\pi}{9}$

36. (a) $\cot 1.35$ (b) $\tan 1.35$

37. (a) $\cos(-110°)$ (b) $\cos 250°$

38. (a) $\tan 240°$ (b) $\cot 210°$

39. (a) $\csc 2.62$ (b) $\csc 150°$

40. (a) $\sin(-0.65)$ (b) $\sin 5.63$

In Exercises 41–46, find two values of θ corresponding to each function. List the measure of θ in radians $(0 \le \theta \le 2\pi)$. Do not use a calculator.

41. (a) $\sin\theta = \frac{1}{2}$ (b) $\sin\theta = -\frac{1}{2}$

42. (a) $\cos\theta = \frac{\sqrt{2}}{2}$ (b) $\cos\theta = -\frac{\sqrt{2}}{2}$

43. (a) $\csc\theta = \frac{2\sqrt{3}}{3}$ (b) $\cot\theta = -1$

44. (a) $\sec\theta = 2$ (b) $\sec\theta = -2$

45. (a) $\tan\theta = -1$ (b) $\cot\theta = -\sqrt{3}$

46. (a) $\sin\theta = \frac{\sqrt{3}}{2}$ (b) $\sin\theta = -\frac{\sqrt{3}}{2}$

In Exercises 47–56, solve the equation for θ $(0 \le \theta \le 2\pi)$. For some of the equations you should use the trigonometric identities listed in this section. Use the *trace* feature of a graphing utility to verify your results.

47. $2\sin^2\theta = 1$ **48.** $\tan^2\theta = 3$

49. $\tan^2\theta - \tan\theta = 0$ **50.** $2\cos^2\theta - \cos\theta = 1$

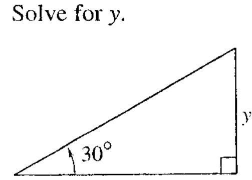
51. $\sin 2\theta - \cos\theta = 0$ **52.** $\cos 2\theta + 3\cos\theta + 2 = 0$

53. $\sin\theta = \cos\theta$ **54.** $\sec\theta\csc\theta = 2\csc\theta$

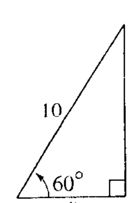
55. $\cos^2\theta + \sin\theta = 1$ **56.** $\cos\frac{\theta}{2} - \cos\theta = 1$

In Exercises 57–62, solve for x, y, or r as indicated.

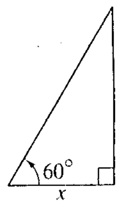
57. Solve for y.



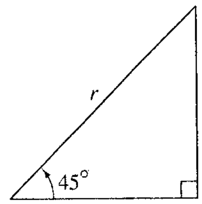
58. Solve for x.



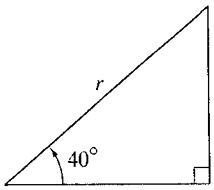
59. Solve for x.



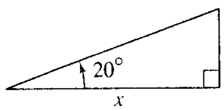
60. Solve for r.



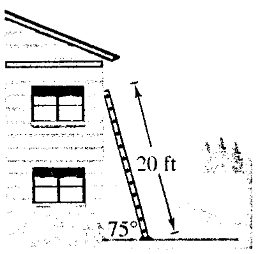
61. Solve for r.



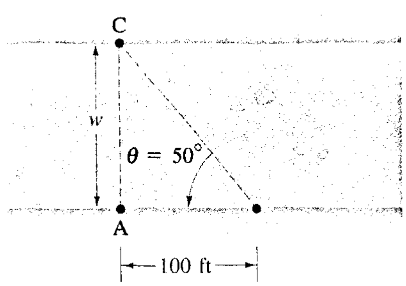
62. Solve for x.



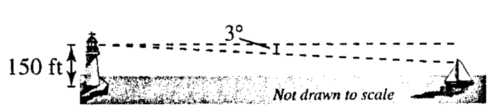
63. ***Length*** A 20-foot ladder leaning against the side of a house makes a 75° angle with the ground (see figure). How far up the side of the house does the ladder reach?



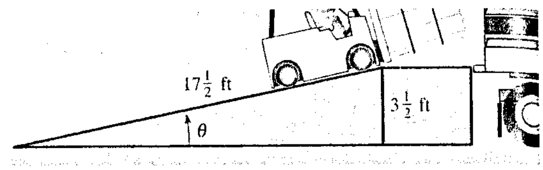
64. ***Width*** A biologist wants to know the width w of a river in order to set instruments to study the pollutants in the water. From point A the biologist walks downstream 100 feet and sights to point C. From this sighting it is determined that $\theta = 50°$ (see figure). How wide is the river?



65. ***Distance*** From a 150-foot observation tower on the coast, a Coast Guard officer sights a boat in difficulty. The angle of depression of the boat is 3° (see figure). How far is the boat from the shoreline?



66. ***Angle Measure*** A ramp $17\frac{1}{2}$ feet in length rises to a loading platform that is $3\frac{1}{2}$ feet off the ground (see figure). Find the angle that the ramp makes with the ground.



67. ***Medicine*** The temperature T in degrees Fahrenheit of a patient t hours after arriving at the emergency room of a hospital at 10:00 P.M. is given by

$$T(t) = 98.6 + 4\cos\frac{\pi t}{36}, \quad 0 \le t \le 18.$$

Find the patient's temperature at each time.

(a) 10:00 P.M.

(b) 4:00 A.M.

(c) 10:00 A.M.

At what time do you expect the patient's temperature to return to normal? Explain your reasoning.

68. ***Sales*** A company that produces a window and door insulating kit forecasts monthly sales over the next 2 years to be

$$S = 23.1 + 0.442t + 4.3\sin\frac{\pi t}{6}$$

where S is measured in thousands of units and t is the time in months, with $t = 1$ corresponding to January 2006. Use a graphing utility to estimate sales for each month.

(a) February 2006 (b) February 2007

(c) September 2006 (d) September 2007

In Exercises 69 and 70, use a graphing utility to complete the table. Then graph the function.

x	0	2	4	6	8	10
$f(x)$						

69. $f(x) = \frac{2}{5}x + 2\sin\frac{\pi x}{5}$

70. $f(x) = \frac{1}{2}(5 - x) + 3\cos\frac{\pi x}{5}$

8.3

Graphs of Trigo

When you are sketch
use x (rather than θ) a
sketch the graph of a

$f(x) = \sin x$

by constructing a tab
them with a smooth c
shown in the table be

x	0	$\frac{\pi}{6}$
$\sin x$	0.00	0.50

In Figure 8.21, note t
is −1. The **amplitud**
be half of the differe
amplitude of $f(x) =$
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that as x increases be
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x-axis) between succ

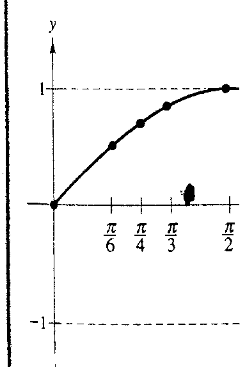


FIGURE 8.21

PREREQUISITE REVIEW 8.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, find the limit.

1. $\lim_{x \to 2} (x^2 + 4x + 2)$

2. $\lim_{x \to 3} (x^3 - 2x^2 + 1)$

In Exercises 3–10, evaluate the trigonometric function without using a calculator.

3. $\cos \frac{\pi}{2}$ **4.** $\sin \pi$

5. $\tan \frac{5\pi}{4}$ **6.** $\cot \frac{2\pi}{3}$

7. $\sin \frac{11\pi}{6}$ **8.** $\cos \frac{5\pi}{6}$

9. $\cos \frac{5\pi}{3}$ **10.** $\sin \frac{4\pi}{3}$

In Exercises 11–18, use a calculator to evaluate the trigonometric function to four decimal places.

11. $\cos 15°$ **12.** $\sin 220°$

13. $\sin 275°$ **14.** $\cos 310°$

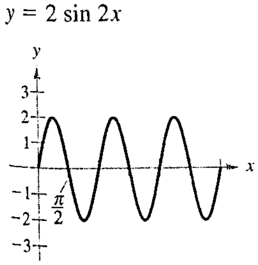
15. $\sin 103°$ **16.** $\cos 72°$

17. $\tan 327°$ **18.** $\tan 140°$

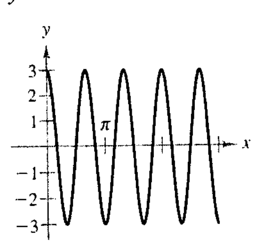
EXERCISES 8.3

In Exercises 1–14, determine the period and amplitude of the function.

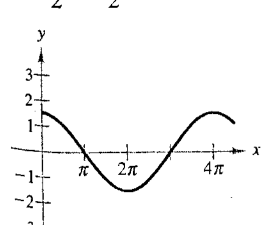
1. $y = 2 \sin 2x$



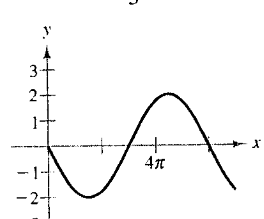
2. $y = 3 \cos 3x$



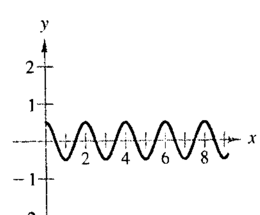
3. $y = \frac{3}{2} \cos \frac{x}{2}$



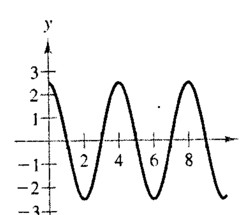
4. $y = -2 \sin \frac{x}{3}$



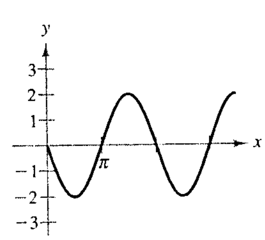
5. $y = \frac{1}{2} \cos \pi x$



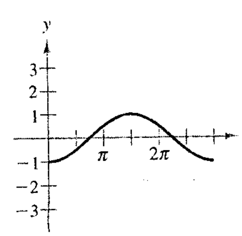
6. $y = \frac{5}{2} \cos \frac{\pi x}{2}$



7. $y = -2 \sin x$



8. $y = -\cos \frac{2x}{3}$



9. $y = -2 \sin 10x$

10. $y = \frac{1}{3} \sin 8x$

11. $y = \dfrac{1}{2} \sin \dfrac{2x}{3}$

12. $y = 5 \cos \dfrac{x}{4}$

13. $y = 3 \sin 4\pi x$

14. $y = \dfrac{2}{3} \cos \dfrac{\pi x}{10}$

In Exercises 15–20, find the period of the function.

15. $y = 5 \tan 2x$

16. $y = 7 \tan 2\pi x$

17. $y = 3 \sec 5x$

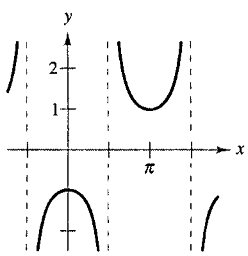
18. $y = \csc 4x$

19. $y = \cot \dfrac{\pi x}{6}$

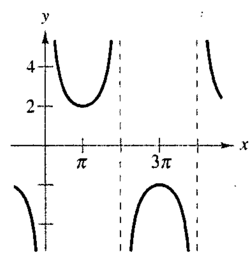
20. $y = 5 \tan \dfrac{2\pi x}{3}$

In Exercises 21–26, match the trigonometric function with the correct graph and give the period of the function. [The graphs are labeled (a)–(f).]

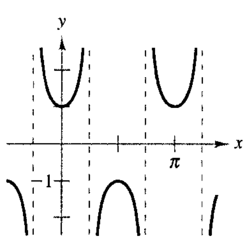
(a)



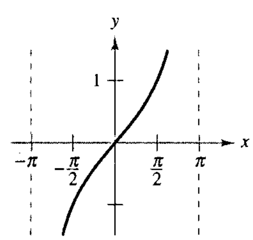
(b)



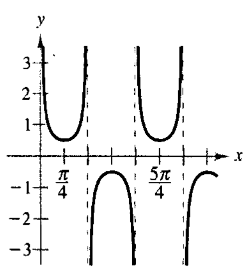
(c)



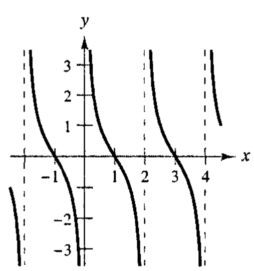
(d)



(e)



(f)



21. $y = \sec 2x$

22. $y = \frac{1}{2} \csc 2x$

23. $y = \cot \dfrac{\pi x}{2}$

24. $y = -\sec x$

25. $y = 2 \csc \dfrac{x}{2}$

26. $y = \tan \dfrac{x}{2}$

In Exercises 27–36, sketch the graph of the function by hand. Use a graphing utility to verify your sketch.

27. $y = \sin \dfrac{x}{2}$

28. $y = 4 \sin \dfrac{x}{3}$

29. $y = 2 \cos \dfrac{2x}{3}$

30. $y = \dfrac{3}{2} \cos \dfrac{2x}{3}$

31. $y = -2 \sin 6x$

32. $y = -3 \cos 4x$

33. $y = \cos 2\pi x$

34. $y = \dfrac{3}{2} \sin \dfrac{\pi x}{4}$

35. $y = 2 \tan x$

36. $y = 2 \cot x$

In Exercises 37–46, sketch the graph of the function.

37. $y = -\sin \dfrac{2\pi x}{3}$

38. $y = 10 \cos \dfrac{\pi x}{6}$

39. $y = \cot 2x$

40. $y = 3 \tan \pi x$

41. $y = \csc \dfrac{2x}{3}$

42. $y = \csc \dfrac{x}{3}$

43. $y = 2 \sec 2x$

44. $y = \sec \pi x$

45. $y = \csc 2\pi x$

46. $y = -\tan x$

In Exercises 47–56, complete the table (using a graphing utility set in *radian* mode) to estimate $\lim_{x \to 0} f(x)$.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

47. $f(x) = \dfrac{1 - \cos 2x}{2x}$

48. $f(x) = \dfrac{\sin 2x}{\sin 3x}$

49. $f(x) = \dfrac{\sin x}{5x}$

50. $f(x) = \dfrac{1 - \cos 2x}{x}$

51. $f(x) = \dfrac{3(1 - \cos x)}{x}$

52. $f(x) = \dfrac{2 \sin(x/4)}{x}$

53. $f(x) = \dfrac{\tan 2x}{x}$

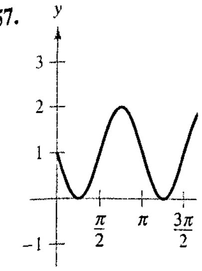
54. $f(x) = \dfrac{\tan 4x}{3x}$

55. $f(x) = \dfrac{1 - \cos^2 4x}{x}$

56. $f(x) = \dfrac{1 - \cos^2 x}{2x}$

In Exercises 57 and 58,
that the graph of $y =$

57.



59. ***Health*** For a p
second) of air flo
piratory cycle is g

$v = 0.9 \sin \dfrac{\pi t}{3}$

where t is the ti
$v > 0$, and exhala

(a) Find the time

(b) Find the numb

(c) Use a graphin

60. ***Health*** After e
a respiratory cycl
approximated by

$y = 1.75 \sin \frac{\pi}{2}$

Use this model to

61. ***Music*** When tu
fork for the A abo
can be approxima

$y = 0.001 \sin$

where t is the time

(a) What is the pe

(b) What is the fr

(c) Use a graphin

62. ***Health*** The fun

$P = 100 - 20$

approximates the
mercury) at time t

(a) Find the perio

(b) Find the numb

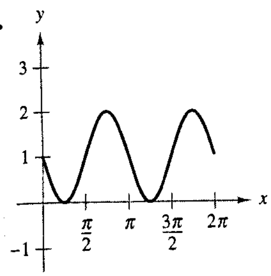
(c) Use a graphin

63. ***Biology: Predat***
predator at time t (

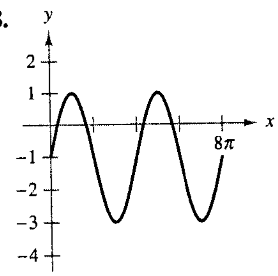
$P = 8000 + 2$

In Exercises 57 and 58, determine the values of $a, b, c,$ and d such that the graph of $y = a\sin(bx + c) + d$ is shown.

57.



58.



59. ***Health*** For a person at rest, the velocity v (in liters per second) of air flow into and out of the lungs during a respiratory cycle is given by

$$v = 0.9 \sin \frac{\pi t}{3}$$

where t is the time in seconds. Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.

(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Use a graphing utility to graph the velocity function.

60. ***Health*** After exercising for a few minutes, a person has a respiratory cycle for which the velocity of air flow is approximated by

$$y = 1.75 \sin \frac{\pi t}{2}.$$

Use this model to repeat Exercise 59.

61. ***Music*** When tuning a piano, a technician strikes a tuning fork for the A above middle C and sets up wave motion that can be approximated by

$$y = 0.001 \sin 880\pi t$$

where t is the time in seconds.

(a) What is the period p of this function?

(b) What is the frequency f of this note $(f = 1/p)$?

(c) Use a graphing utility to graph this function.

62. ***Health*** The function

$$P = 100 - 20 \cos \frac{5\pi t}{3}$$

approximates the blood pressure P (in millimeters of mercury) at time t in seconds for a person at rest.

(a) Find the period of the function.

(b) Find the number of heartbeats per minute.

(c) Use a graphing utility to graph the pressure function.

63. ***Biology: Predator-Prey Cycle*** The population P of a predator at time t (in months) is modeled by

$$P = 8000 + 2500 \sin \frac{2\pi t}{24}$$

and the population p of its prey is modeled by

$$p = 12{,}000 + 4000 \cos \frac{2\pi t}{24}.$$

(a) Use a graphing utility to graph both models in the same viewing window.

(b) Explain the oscillations in the size of each population.

64. ***Biology: Predator-Prey Cycle*** The population P of a predator at time t (in months) is modeled by

$$P = 5700 + 1200 \sin \frac{2\pi t}{24}$$

and the population p of its prey is modeled by

$$p = 9800 + 2750 \cos \frac{2\pi t}{24}.$$

(a) Use a graphing utility to graph both models in the same viewing window.

(b) Explain the oscillations in the size of each population.

Sales In Exercises 65 and 66, sketch the graph of the sales function over 1 year where S is sales in thousands of units and t is the time in months, with $t = 1$ corresponding to January.

65. $S = 22.3 - 3.4 \cos \dfrac{\pi t}{6}$ **66.** $S = 74.50 + 43.75 \sin \dfrac{\pi t}{6}$

67. ***Biorhythms*** For the person born on July 20, 1984, use the biorhythm cycles given in Example 6 to calculate this person's three energy levels on December 31, 2008. Assume this is the 8930th day of the person's life.

68. ***Biorhythms*** Use your birthday and the concept of biorhythms as given in Example 6 to calculate your three energy levels on December 31, 2008. Use the internet or some other reference source to calculate the number of days between your birthday and December 31, 2008.

In Exercises 69 and 70, use a graphing utility to graph the functions in the same viewing window where $0 \le x \le 2$.

69. (a) $y = \dfrac{4}{\pi}\sin \pi x$

(b) $y = \dfrac{4}{\pi}\left(\sin \pi x + \dfrac{1}{3}\sin 3\pi x\right)$

70. (a) $y = \dfrac{1}{2} - \dfrac{4}{\pi^2}\cos \pi x$

(b) $y = \dfrac{1}{2} - \dfrac{4}{\pi^2}\left(\cos \pi x + \dfrac{1}{9}\cos 3\pi x\right)$

In Exercises 71–74, use a graphing utility to graph the function f and find $\lim_{x \to 0} f(x)$.

71. $f(x) = \dfrac{\sin x}{x}$ **72.** $f(x) = \dfrac{\sin 5x}{2x}$

73. $f(x) = \dfrac{\sin 5x}{\sin 2x}$ **74.** $f(x) = \dfrac{\tan 2x}{3x}$

75. ***Sales*** The sales S (in thousands of units) of snowmobiles are modeled by

$$S = 58.3 + 32.5 \cos \frac{\pi t}{6}$$

where t is the time in months, with $t = 1$ corresponding to January and $t = 12$ corresponding to December.

(a) Use a graphing utility to graph S.

(b) Determine the months when sales exceed 75,000 units.

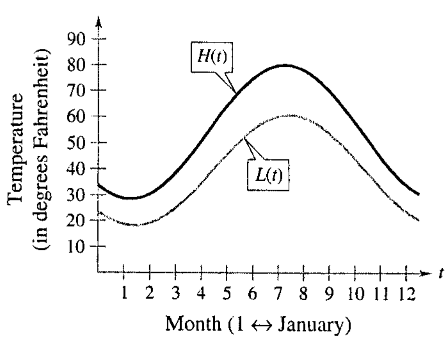
76. ***Meteorology*** The normal monthly high temperatures for Erie, Pennsylvania are approximated by

$$H(t) = 54.33 - 20.38 \cos \frac{\pi t}{6} - 15.69 \sin \frac{\pi t}{6}$$

and the normal monthly low temperatures are approximated by

$$L(t) = 39.36 - 15.70 \cos \frac{\pi t}{6} - 14.16 \sin \frac{\pi t}{6}$$

where t is the time in months, with $t = 1$ corresponding to January. *(Source: NOAA)* Use the figure to answer the questions below.

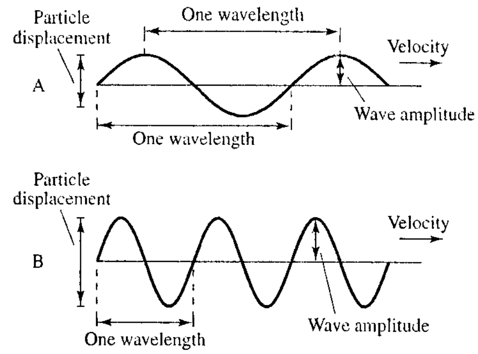


(a) During what part of the year is the difference between the normal high and low temperatures greatest? When is it smallest?

(b) The sun is the farthest north in the sky around June 21, but the graph shows the highest temperatures at a later date. Approximate the lag time of the temperatures relative to the position of the sun.

77. ***Finance: Cyclical Stocks*** The term "cyclical stock" describes the stock of a company whose profits are greatly influenced by changes in the economic business cycle. The market prices of cyclical stocks mirror the general state of the economy and reflect the various phases of the business cycle. Give a description and sketch of the graph of a given corporation's stock prices during recurrent periods of prosperity and recession. *(Source: Adapted from Garman/Forgue,* Personal Finance, *Fifth Edition)*

78. ***Physics*** Use the graphs below to answer each question.



(a) Which graph (A or B) has a longer wavelength, or period?

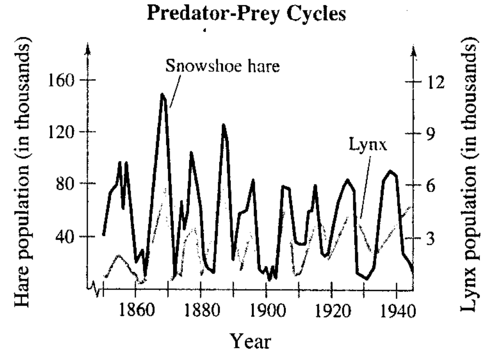
(b) Which graph (A or B) has a greater amplitude?

(c) The frequency of a graph is the number of oscillations or cycles that occur during a given period of time. Which graph (A or B) has a greater frequency?

(d) Based on the definition of frequency, how are frequency and period related?

(Source: Adapted from Shipman/Wilson/Todd, An Introduction to Physical Science, *Tenth Edition)*

79. ***Biology: Predator-Prey Cycles*** The graph below demonstrates snowshoe hare and lynx population fluctuations. The cycles of each population follow a periodic pattern. Describe several factors that could be contributing to the cyclical patterns. *(Source: Adapted from Levine/Miller,* Biology: Discovering Life, *Second Edition)*



True or False? In Exercises 80–83, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

80. The amplitude of $f(x) = -3 \cos 2x$ is -3.

81. The period of $f(x) = 5 \cot\left(-\frac{4x}{3}\right)$ is $\frac{3\pi}{2}$.

82. $\lim_{x \to 0} \frac{\sin 5x}{3x} = \frac{5}{3}$

83. One solution of $\tan \frac{x}{2} = 1$ is $\frac{5\pi}{4}$.

3.4

Derivatives of

In Example 4 and Ti trigonometric limits:

$\lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} =$

These two limits are

$\frac{d}{dx}[\sin x] = \lim_{\Delta x \to}$

$= \lim_{\Delta x \to}$

$= \lim_{\Delta x \to}$

= cos

= (co

= cos

This differentiation slope of the sine curv of x, the Chain Rule

$\frac{d}{dx}[\sin u] = \cos$

The Chain Rule ver functions are listed b

Derivatives of Tr

$\frac{d}{dx}[\sin u] = \cos$

$\frac{d}{dx}[\tan u] = \sec^2$

$\frac{d}{dx}[\sec u] = \sec$

STUDY TI

To help you remer function that begi