PREREQUISITE REVIEW 2.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, find an equation of the line containing P and Q.

1. $P(2, 1),\ Q(2, 4)$

2. $P(2, 2),\ Q(-5, 2)$

In Exercises 3–6, find the limit.

3. $\lim_{\Delta x \to 0} \dfrac{2x\Delta x + (\Delta x)^2}{\Delta x}$

4. $\lim_{\Delta x \to 0} \dfrac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$

5. $\lim_{\Delta x \to 0} \dfrac{1}{x(x + \Delta x)}$

6. $\lim_{\Delta x \to 0} \dfrac{(x + \Delta x)^2 - x^2}{\Delta x}$

In Exercises 7–10, find the domain of the function.

7. $f(x) = \dfrac{1}{x - 1}$

8. $f(x) = \dfrac{1}{5}x^3 - 2x^2 + \dfrac{1}{3}x - 1$

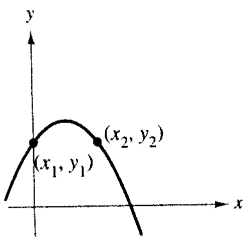
9. $f(x) = \dfrac{6x}{x^3 + x}$

10. $f(x) = \dfrac{x^2 - 2x - 24}{x^2 + x - 12}$

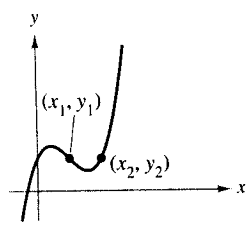
EXERCISES 2.1

In Exercises 1–4, trace the graph and sketch the tangent lines at (x_1, y_1) and (x_2, y_2).

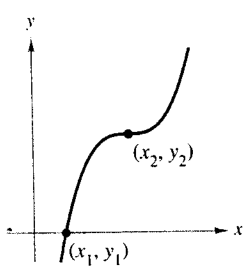
1.



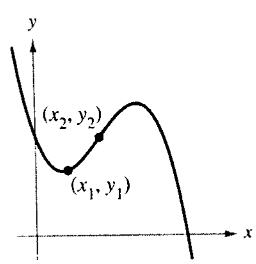
2.



3.

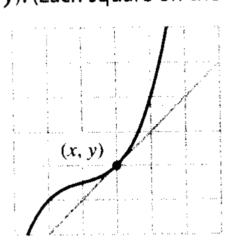


4.



In Exercises 5–10, estimate the slope of the graph at the point (x, y). (Each square on the grid is 1 unit by 1 unit.)

5.



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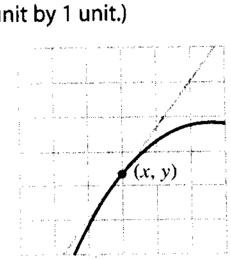
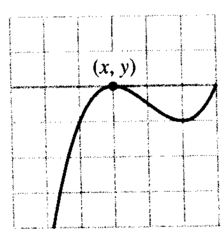


7.



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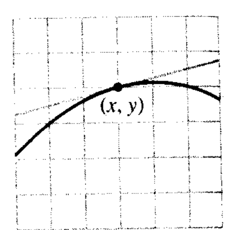
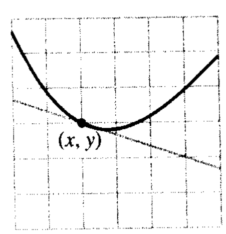
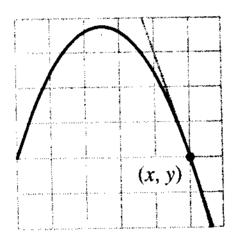


9.

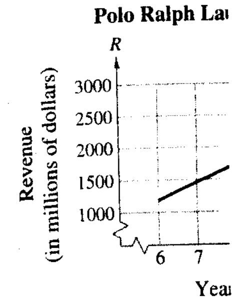


10.

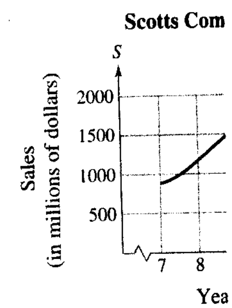


11. *Revenue* The graph (on page 91) represents the revenue R (in millions of dollars per year) for Polo Ralph Lauren from 1996 through 2002, where $t = 6$ corresponds to 1996. Estimate the slopes of the graph for the years 1997, 2000, and 2002. *(Source: Polo Ralph Lauren Corp.)*

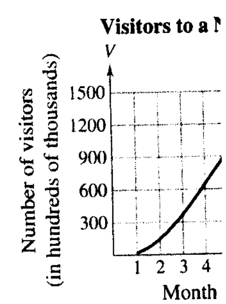
Figure for 11



12. *Sales* The graph rep
dollars per year) for S
2003, where $t = 7$ corre
of the graph for the yea
Scotts Company)

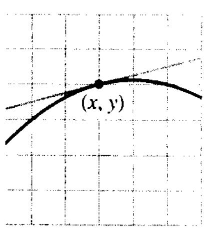


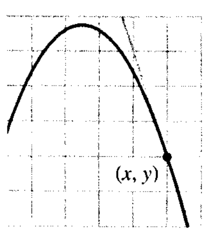
13. *Consumer Trends*
visitors V to a national
a one-year period, wh
Estimate the slopes of



14. *Athletics* Two long
side begin a 10,000-m
$s = f(t)$ and $s = g(t)$,
thousands of meters a

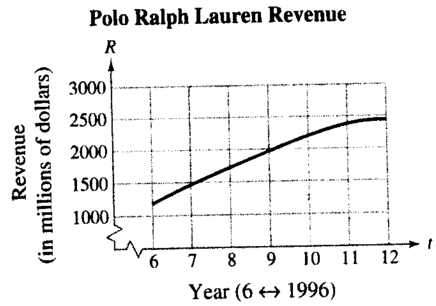
r sections. You will



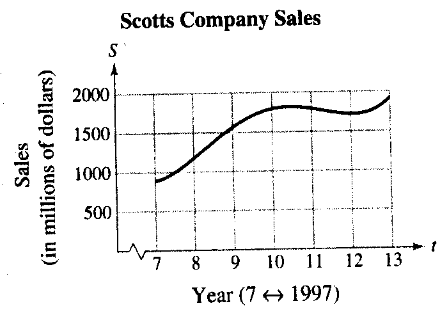


) represents the revenue for Polo Ralph Lauren 6 corresponds to 1996. r the years 1997, 2000, auren Corp.)

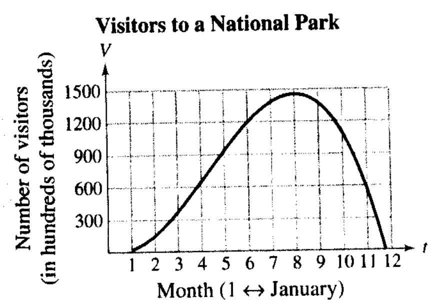
Figure for 11



12. ***Sales*** The graph represents the sales S (in millions of dollars per year) for Scotts Company from 1997 through 2003, where $t = 7$ corresponds to 1997. Estimate the slopes of the graph for the years 1998, 2001, and 2003. *(Source: Scotts Company)*

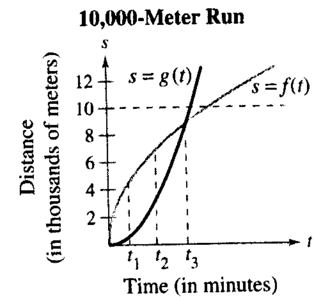


13. ***Consumer Trends*** The graph shows the number of visitors V to a national park in hundreds of thousands during a one-year period, where $t = 1$ corresponds to January. Estimate the slopes of the graph at $t = 1$, 8, and 12.



14. ***Athletics*** Two long distance runners starting out side by side begin a 10,000-meter run. Their distances are given by $s = f(t)$ and $s = g(t)$, respectively, where s is measured in thousands of meters and t is measured in minutes.

Figure for 14



(a) Which runner is running faster at t_1?

(b) What conclusion can you make regarding their rates at t_2?

(c) What conclusion can you make regarding their rates at t_3?

(d) Which runner finishes the race first? Explain.

In Exercises 15–26, use the limit definition to find the derivative of the function.

15. $f(x) = 3$
16. $f(x) = -4$
17. $f(x) = -5x + 3$
18. $f(x) = \frac{1}{2}x + 5$
19. $f(x) = x^2 - 4$
20. $f(x) = 1 - x^2$
21. $h(t) = \sqrt{t - 1}$
22. $f(x) = \sqrt{x + 2}$
23. $f(t) = t^3 - 12t$
24. $f(t) = t^3 + t^2$
25. $f(x) = \dfrac{1}{x + 2}$
26. $g(s) = \dfrac{1}{s - 1}$

In Exercises 27–36, find the slope of the tangent line to the graph of f at the given point.

27. $f(x) = 6 - 2x$; $(2, 2)$
28. $f(x) = 2x + 4$; $(1, 6)$
29. $f(x) = -1$; $(0, -1)$
30. $f(x) = 6$; $(-2, 6)$
31. $f(x) = x^2 - 2$; $(2, 2)$
32. $f(x) = x^2 + 2x + 1$; $(-3, 4)$
33. $f(x) = x^3 - x$; $(2, 6)$
34. $f(x) = x^3 + 2x$; $(1, 3)$
35. $f(x) = \sqrt{1 - 2x}$; $(-4, 3)$
36. $f(x) = \sqrt{2x - 2}$; $(9, 4)$

In Exercises 37–44, find an equation of the tangent line to the graph of f at the given point. Then verify your result by sketching the graph of f and the tangent line.

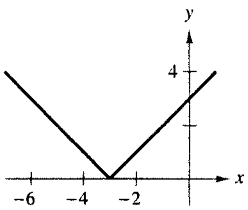
37. $f(x) = \frac{1}{2}x^2$; $(2, 2)$
38. $f(x) = -x^2$; $(-1, -1)$
39. $f(x) = (x - 1)^2$; $(-2, 9)$
40. $f(x) = 2x^2 - 1$; $(0, -1)$
41. $f(x) = \sqrt{x} + 1$; $(4, 3)$
42. $f(x) = \sqrt{x + 2}$; $(7, 3)$
43. $f(x) = \dfrac{1}{x}$; $(1, 1)$
44. $f(x) = \dfrac{1}{x - 1}$; $(2, 1)$

In Exercises 45–48, find an equation of the line that is tangent to the graph of f and parallel to the given line.

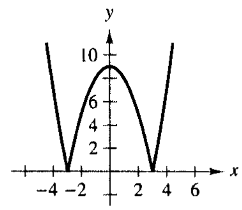
	Function	*Line*
45.	$f(x) = -\frac{1}{4}x^2$	$x + y = 0$
46.	$f(x) = x^2 + 1$	$2x + y = 0$
47.	$f(x) = -\frac{1}{2}x^3$	$6x + y + 4 = 0$
48.	$f(x) = x^2 - x$	$x + 2y - 6 = 0$

In Exercises 49–56, describe the x-values at which the function is differentiable. Explain your reasoning.

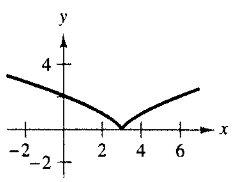
49. $y = |x + 3|$



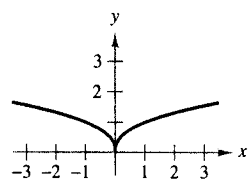
50. $y = |x^2 - 9|$



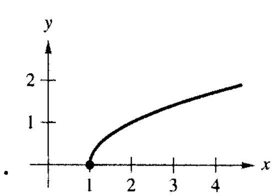
51. $y = (x - 3)^{2/3}$



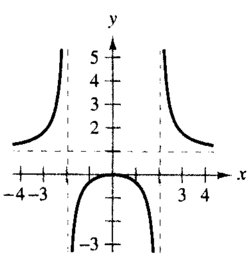
52. $y = x^{2/5}$



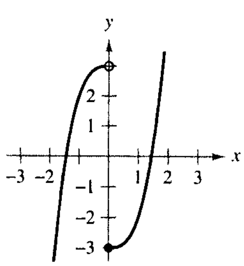
53. $y = \sqrt{x - 1}$



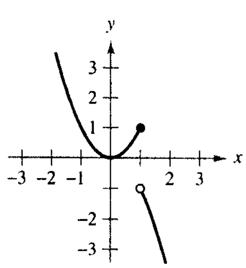
54. $y = \dfrac{x^2}{x^2 - 4}$



55. $y = \begin{cases} x^3 + 3, & x < 0 \\ x^3 - 3, & x \geq 0 \end{cases}$



56. $y = \begin{cases} x^2, & x \leq 1 \\ -x^2, & x > 1 \end{cases}$



Graphical, Numerical, and Analytic Analysis In Exercises 57–60, use a graphing utility to graph f on the interval $[-2, 2]$. Complete the table by graphically estimating the slopes of the graph at the given points. Then evaluate the slopes analytically and compare your results with those obtained graphically.

x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$									
$f'(x)$									

57. $f(x) = \frac{1}{4}x^3$

58. $f(x) = \frac{1}{2}x^2$

59. $f(x) = -\frac{1}{2}x^3$

60. $f(x) = -\frac{3}{2}x^2$

In Exercises 61–64, find the derivative of the given function. Then use a graphing utility to graph f and its derivative in the same viewing window. What does the x-intercept of the derivative indicate about the graph of f?

61. $f(x) = x^2 - 4x$ **62.** $f(x) = 2 + 6x - x^2$

63. $f(x) = x^3 - 3x$ **64.** $f(x) = x^3 - 6x^2$

65. ***Think About It*** Sketch a graph of a function whose derivative is always negative.

66. ***Think About It*** Sketch a graph of a function whose derivative is always positive.

67. ***Writing*** Use a graphing utility to graph the two functions $f(x) = x^2 + 1$ and $g(x) = |x| + 1$ in the same viewing window. Use the *zoom* and *trace* features to analyze the graphs near the point $(0, 1)$. What do you observe? Which function is differentiable at this point? Write a short paragraph describing the geometric significance of differentiability at a point.

True or False? In Exercises 68–71, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

68. The slope of the graph of $y = x^2$ is different at every point on the graph of f.

69. If a function is continuous at a point, then it is differentiable at that point.

70. If a function is differentiable at a point, then it is continuous at that point.

71. A tangent line to a graph can intersect the graph at more than one point.

2.2

The Constant Rul

In Section 2.1, you foun
even for simple functic
differentiation. These r
use of limits.

The Constant Rule

The derivative of a

$\frac{d}{dx}[c] = 0,$

PROOF Let $f(x) = c.$
write

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x -}{}$

So,

$\frac{d}{dx}[c] = 0.$

STUDY TIP

Note in Figure 2.1
slope of a horizonta

EXAMPLE 1

(a) $\frac{d}{dx}[7] = 0$

(c) If $y = 2$, then $\frac{dy}{dx} =$

TRY IT 1

Find the derivative o

(a) $f(x) = -2$

PREREQUISITE REVIEW 2.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, evaluate each expression when $x = 2$.

1. (a) $2x^2$ (b) $(2x)^2$ (c) $2x^{-2}$

2. (a) $\dfrac{1}{(3x)^2}$ (b) $\dfrac{1}{4x^3}$ (c) $\dfrac{(2x)^{-3}}{4x^{-2}}$

In Exercises 3–6, simplify the expression.

3. $4(3)x^3 + 2(2)x$

4. $\frac{1}{2}(3)x^2 - \frac{3}{2}x^{1/2}$

5. $\left(\frac{1}{4}\right)x^{-3/4}$

6. $\frac{1}{3}(3)x^2 - 2\left(\frac{1}{2}\right)x^{-1/2} + \frac{1}{3}x^{-2/3}$

In Exercises 7–10, solve the equation.

7. $3x^2 + 2x = 0$

8. $x^3 - x = 0$

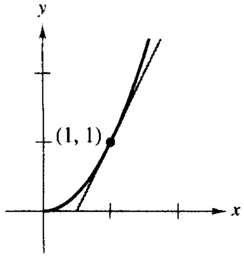
9. $x^2 + 8x - 20 = 0$

10. $x^2 - 10x - 24 = 0$

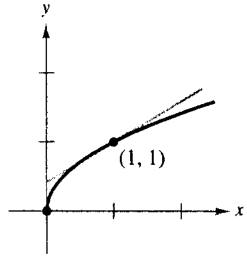
EXERCISES 2.2

In Exercises 1–4, find the slope of the tangent line to $y = x^n$ at the point $(1, 1)$.

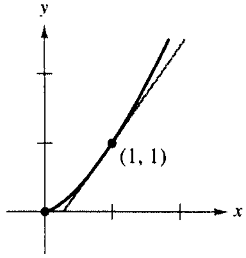
1. (a) $y = x^2$



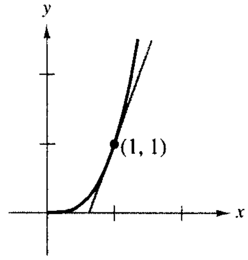
(b) $y = x^{1/2}$



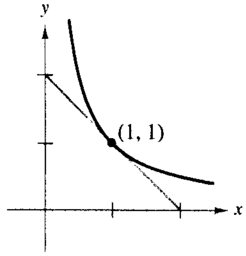
2. (a) $y = x^{3/2}$



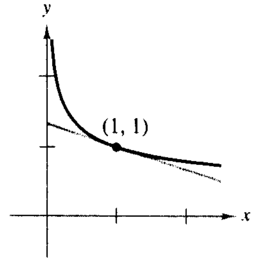
(b) $y = x^3$



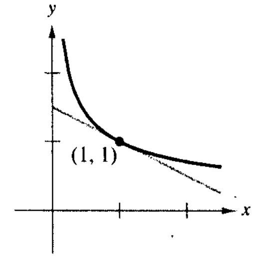
3. (a) $y = x^{-1}$



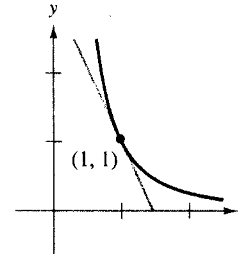
(b) $y = x^{-1/3}$



4. (a) $y = x^{-1/2}$



(b) $y = x^{-2}$



In Exercises 5–20, find the derivative of the function.

5. $y = 3$

6. $f(x) = -2$

7. $f(x) = 4x + 1$

8. $g(x) = 3x - 1$

9. $g(x) = x^2 + 4x - 1$

10. $y = t^2 + 2t - 3$

11. $f(t) = -3t^2 + 2t - 4$

12. $y = x^3 - 9x^2 + 2$

13. $s(t) = t^3 - 2t + 4$

14. $y = 2x^3 - x^2 + 3x - 1$

15. $y = 4t^{4/3}$

16. $h(x) = x^{5/2}$

17. $f(x) = 4\sqrt{x}$

18. $g(x) = 4\sqrt[3]{x} + 2$

19. $y = 4x^{-2} + 2x^2$

20. $s(t) = 4t^{-1} + 1$

In Exercises 21–26, use
derivative.

Function	*Rewrite*
21. $y = \dfrac{1}{4x^3}$	
22. $y = \dfrac{2}{3x^2}$	
23. $y = \dfrac{1}{(4x)^3}$	
24. $y = \dfrac{\pi}{(3x)^2}$	
25. $y = \dfrac{\sqrt{x}}{x}$	
26. $y = \dfrac{4x}{x^{-3}}$	

In Exercises 27–32, find the
at the given point.

Function

27. $f(x) = \dfrac{1}{x}$

28. $f(t) = 4 - \dfrac{4}{3t}$

29. $f(x) = -\frac{1}{2}x(1 + x^2)$

30. $y = 3x\left(x^2 - \dfrac{2}{x}\right)$

31. $y = (2x + 1)^2$

32. $f(x) = 3(5 - x)^2$

In Exercises 33–46, find $f'($

33. $f(x) = x^2 - \dfrac{4}{x} - 3x$

34. $f(x) = x^2 - 3x - 3x$

35. $f(x) = x^2 - 2x - \dfrac{2}{x^4}$

37. $f(x) = x(x^2 + 1)$

39. $f(x) = (x + 4)(2x^2 -$

40. $f(x) = (3x^2 - 5x)(x$

41. $f(x) = \dfrac{2x^3 - 4x^2 +}{x^2}$

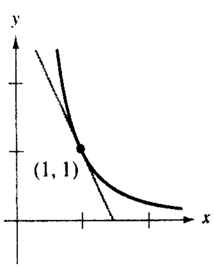
43. $f(x) = \dfrac{4x^3 - 3x^2 +}{x^2}$

44. $f(x) = \dfrac{-6x^3 + 3x^2}{x}$

45. $f(x) = x^{4/5} + x$

lier sections. You will

$y = x^{-2}$



of the function.

$f(x) = -2$

$g(x) = 3x - 1$

In Exercises 21–26, use Example 6 as a model to find the derivative.

Function	*Rewrite*	*Differentiate*	*Simplify*
21. $y = \dfrac{1}{4x^3}$			
22. $y = \dfrac{2}{3x^2}$			
23. $y = \dfrac{1}{(4x)^3}$			
24. $y = \dfrac{\pi}{(3x)^2}$			
25. $y = \dfrac{\sqrt{x}}{x}$			
26. $y = \dfrac{4x}{x^{-3}}$			

In Exercises 27–32, find the value of the derivative of the function at the given point.

Function	*Point*
27. $f(x) = \dfrac{1}{x}$	$(1, 1)$
28. $f(t) = 4 - \dfrac{4}{3t}$	$\left(\dfrac{1}{2}, \dfrac{4}{3}\right)$
29. $f(x) = -\frac{1}{2}x(1 + x^2)$	$(1, -1)$
30. $y = 3x\left(x^2 - \dfrac{2}{x}\right)$	$(2, 18)$
31. $y = (2x + 1)^2$	$(0, 1)$
32. $f(x) = 3(5 - x)^2$	$(5, 0)$

In Exercises 33–46, find $f'(x)$.

33. $f(x) = x^2 - \dfrac{4}{x} - 3x^{-2}$

34. $f(x) = x^2 - 3x - 3x^{-2} + 5x^{-3}$

35. $f(x) = x^2 - 2x - \dfrac{2}{x^4}$

36. $f(x) = x^2 + 4x + \dfrac{1}{x}$

37. $f(x) = x(x^2 + 1)$

38. $f(x) = (x^2 + 2x)(x + 1)$

39. $f(x) = (x + 4)(2x^2 - 1)$

40. $f(x) = (3x^2 - 5x)(x^2 + 2)$

41. $f(x) = \dfrac{2x^3 - 4x^2 + 3}{x^2}$

42. $f(x) = \dfrac{2x^2 - 3x + 1}{x}$

43. $f(x) = \dfrac{4x^3 - 3x^2 + 2x + 5}{x^2}$

44. $f(x) = \dfrac{-6x^3 + 3x^2 - 2x + 1}{x}$

45. $f(x) = x^{4/5} + x$

46. $f(x) = x^{1/3} - 1$

In Exercises 47–50, find an equation of the tangent line to the graph of the function at the given point.

Function	*Point*
47. $y = -2x^4 + 5x^2 - 3$	$(1, 0)$
48. $y = x^3 + x$	$(-1, -2)$
49. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$	$(1, 2)$
50. $f(x) = \dfrac{1}{\sqrt[3]{x^2}} - x$	$(-1, 2)$

In Exercises 51–54, determine the point(s), if any, at which the graph of the function has a horizontal tangent line.

51. $y = -x^4 + 3x^2 - 1$

52. $y = x^3 + 3x^2$

53. $y = \frac{1}{2}x^2 + 5x$

54. $y = x^2 + 2x$

In Exercises 55 and 56,

(a) Sketch the graphs of f, g, and h on the same set of coordinate axes.

(b) Find $f'(1)$, $g'(1)$, and $h'(1)$.

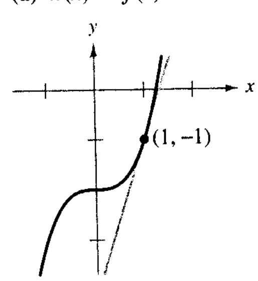
(c) Sketch the graph of the tangent line to each graph when $x = 1$.

55. $f(x) = x^3$
$g(x) = x^3 + 3$
$h(x) = x^3 - 2$

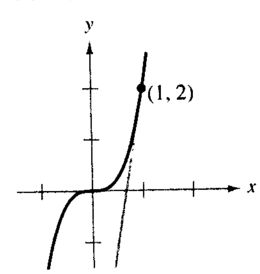
56. $f(x) = \sqrt{x}$
$g(x) = \sqrt{x} + 4$
$h(x) = \sqrt{x} - 2$

57. Use the Constant Rule, the Constant Multiple Rule, and the Sum Rule to find $h'(1)$ given that $f'(1) = 3$.

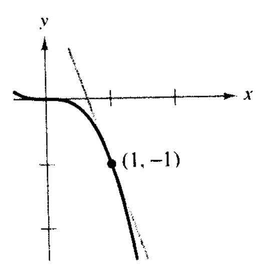
(a) $h(x) = f(x) - 2$



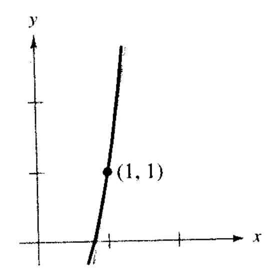
(b) $h(x) = 2f(x)$



(c) $h(x) = -f(x)$



(d) $h(x) = -1 + 2f(x)$

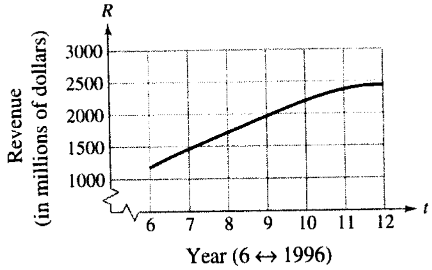


58. ***Revenue*** The revenue R (in millions of dollars per year) for Polo Ralph Lauren from 1996 through 2002 can be modeled by

$$R = -1.17879t^4 + 38.3641t^3 - 469.994t^2 + 2820.22t - 5577.7$$

where $t = 6$ corresponds to 1996. *(Source: Polo Ralph Lauren Corp.)*

Polo Ralph Lauren Revenue



(a) Find the slopes of the graph for the years 1997, 2000, and 2002.

(b) Compare your results with those obtained in Exercise 11 in Section 2.1.

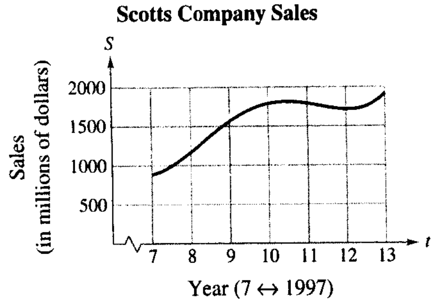
(c) What are the units for the slope of the graph? Interpret the slope of the graph in the context of the problem.

59. ***Sales*** The sales S (in millions of dollars per year) for Scotts Company from 1997 through 2003 can be modeled by

$$S = 8.70947t^4 - 341.0927t^3 + 4885.752t^2 - 30{,}118.17t + 68{,}395.3$$

where $t = 7$ corresponds to 1997. *(Source: Scotts Company)*

Scotts Company Sales



(a) Find the slopes of the graph for the years 1998, 2001, and 2003.

(b) Compare your results with those obtained in Exercise 12 in Section 2.1.

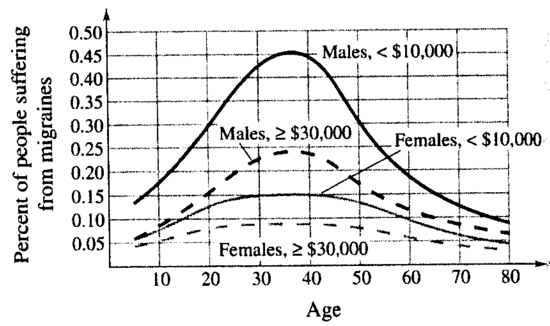
(c) What are the units for the slope of the graph? Interpret the slope of the graph in the context of the problem.

60. ***Cost*** The variable cost for manufacturing an electrical component is \$7.75 per unit, and the fixed cost is \$500. Write the cost C as a function of x, the number of units produced. Show that the derivative of this cost function is a constant and is equal to the variable cost.

61. ***Profit*** A college club raises funds by selling candy bars for \$1.00 each. The club pays \$0.60 for each candy bar and has annual fixed costs of \$250. Write the profit P as a function of x, the number of candy bars sold. Show that the derivative of the profit function is a constant and is equal to the profit on each candy bar sold.

62. ***Psychology: Migraine Prevalence*** The graph illustrates the prevalence of migraine headaches in males and females in selected income groups. *(Source: Adapted from Sue/Sue/Sue, Understanding Abnormal Behavior, Seventh Edition)*

Prevalence of Migraine Headaches



(a) Write a short paragraph describing your general observations about the prevalence of migraines in females and males with respect to age group and income bracket.

(b) Describe the graph of the derivative of each curve, and explain the significance of each derivative. Include an explanation of the units of the derivatives, and indicate the time intervals in which the derivatives would be positive and negative.

In Exercises 63 and 64, use a graphing utility to graph f and f' over the given interval. Determine any points at which the graph of f has horizontal tangents.

Function	*Interval*
63. $f(x) = 4.1x^3 - 12x^2 + 2.5x$	$[0, 3]$
64. $f(x) = x^3 - 1.4x^2 - 0.96x + 1.44$	$[-2, 2]$

True or False? In Exercises 65 and 66, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

65. If $f'(x) = g'(x)$, then $f(x) = g(x)$.

66. If $f(x) = g(x) + c$, then $f'(x) = g'(x)$.

2.3

Average Rate of

In Sections 2.1 and 2.2

1. **Slope** The derivati
a point $(x, f(x))$.

2. **Rate of Change** T
of $f(x)$ with respec

In this section, you wi
change. A few are velc
rates, production rates
involve change with re
variable with respect t

When determining
you must be careful t
change. The distinctio
distinction between the
the slope of the tangen

Definition of Aver

If $y = f(x)$, then th
the interval $[a, b]$ i

Average rate c

Note that $f(a)$ is th
interval, $f(b)$ is the
interval, and $b - a$

STUDY TI

In real-life problem
change. The units f
measured in miles
miles per hour.

PREREQUISITE REVIEW 2.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, evaluate the expression.

1. $\dfrac{-63 - (-105)}{21 - 7}$

2. $\dfrac{-37 - 54}{16 - 3}$

In Exercises 3–10, find the derivative of the function.

3. $y = 4x^2 - 2x + 7$

4. $y = -3t^3 + 2t^2 - 8$

5. $s = -16t^2 + 24t + 30$

6. $y = -16x^2 + 54x + 70$

7. $A = \frac{1}{10}(-2r^3 + 3r^2 + 5r)$

8. $y = \frac{1}{9}(6x^3 - 18x^2 + 63x - 15)$

9. $y = 12x - \dfrac{x^2}{5000}$

10. $y = 138 + 74x - \dfrac{x^3}{10{,}000}$

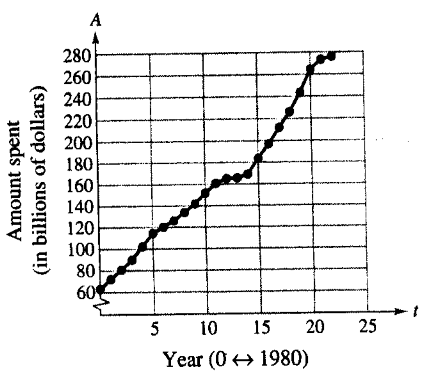
EXERCISES 2.3

1. ***Research and Development*** The graph shows the amounts A (in billions of dollars per year) spent on R&D in the United States from 1980 through 2002. Approximate the average rate of change of A during each period. *(Source: U.S. National Science Foundation)*

   (a) 1980–1985 (b) 1985–1990 (c) 1990–1995

   (d) 1995–2000 (e) 1980–2002 (f) 1990–2002

Research and Development



2. ***Trade Deficit*** The graph shows the values I (in billions of dollars per year) of goods imported to the United States and the value E (in billions of dollars per year) of goods exported from the United States from 1980 through 2002. Approximate each indicated average rate of change. *(Source: U.S. International Trade Administration)*

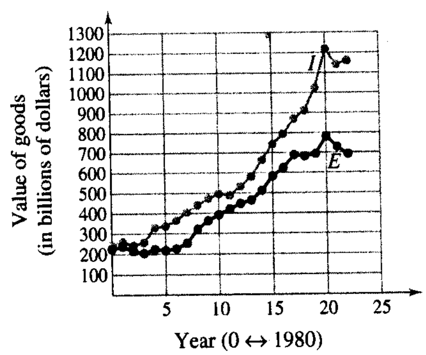
   (a) Imports: 1980–1990 (b) Exports: 1980–1990

   (c) Imports: 1990–2000 (d) Exports: 1990–2000

   (e) Imports: 1980–2002 (f) Exports: 1980–2002

Figure for 2

Trade Deficit



In Exercises 3–8, sketch the graph of the function and find its average rate of change on the interval. Compare this rate with the instantaneous rates of change at the endpoints of the interval.

3. $f(t) = 2t + 7; [1, 2]$

4. $h(x) = 1 - x; [0, 1]$

5. $h(x) = x^2 - 4x + 2; [-2, 2]$

6. $f(x) = x^2 - 6x - 1; [-1, 3]$

7. $f(x) = \dfrac{1}{x}; [1, 4]$

8. $f(x) = \dfrac{1}{\sqrt{x}}; [1, 4]$

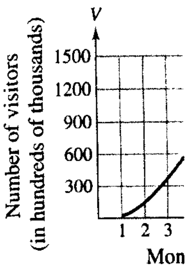
In Exercises 9 and 10, use a graphing utility to graph the function and find its average rate of change on the interval. Compare this rate with the instantaneous rates of change at the endpoints of the interval.

9. $g(x) = x^4 - x^2 + 2; [1, 3]$

10. $g(x) = x^3 - 1; [-1, 1]$

11. ***Consumer Trend*** visitors V to a nati during a one-year pe

Visitors to

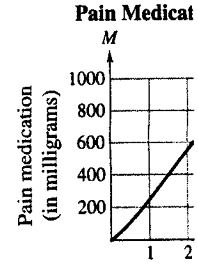


   (a) Estimate the ra [9, 12] and expl

   (b) Over what inte approximately e Explain your rea

12. ***Medicine*** The gr milligrams of a pai hours after a 1000- given.

Pain Medicat



   (a) Estimate the on rate of change is

   (b) Over what inte approximately e Explain your rea

13. ***Medicine*** The eff a pain-killing drug t given by

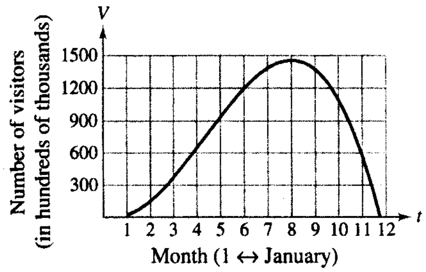
   $E = \frac{1}{27}(9t + 3t$

   Find the average ra interval and compa of change at the enc

   (a) $[0, 1]$ (b) [

11. ***Consumer Trends*** The graph shows the number of visitors V to a national park in hundreds of thousands during a one-year period, where $t = 1$ represents January.

Visitors to a National Park

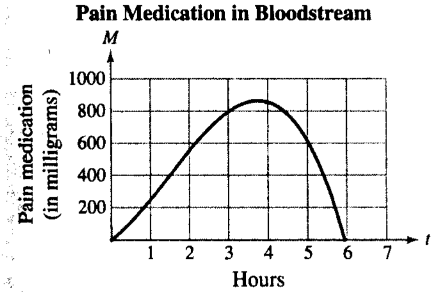


(a) Estimate the rate of change of V over the interval $[9, 12]$ and explain your results.

(b) Over what interval is the average rate of change approximately equal to the rate of change at $t = 8$? Explain your reasoning.

12. ***Medicine*** The graph shows the estimated number of milligrams of a pain medication M in the bloodstream t hours after a 1000-milligram dose of the drug has been given.

Pain Medication in Bloodstream



(a) Estimate the one-hour interval over which the average rate of change is the greatest.

(b) Over what interval is the average rate of change approximately equal to the rate of change at $t = 4$? Explain your reasoning.

13. ***Medicine*** The effectiveness E (on a scale from 0 to 1) of a pain-killing drug t hours after entering the bloodstream is given by

$$E = \frac{1}{27}(9t + 3t^2 - t^3), \quad 0 \le t \le 4.5.$$

Find the average rate of change of E on each indicated interval and compare this rate with the instantaneous rates of change at the endpoints of the interval.

(a) $[0, 1]$ (b) $[1, 2]$ (c) $[2, 3]$ (d) $[3, 4]$

14. ***Chemistry: Wind Chill*** At 0° Celsius, the heat loss H (in kilocalories per square meter per hour) from a person's body can be modeled by

$$H = 33(10\sqrt{v} - v + 10.45)$$

where v is the wind speed (in meters per second).

(a) Find $\frac{dH}{dv}$ and interpret its meaning in this situation.

(b) Find the rates of change of H when $v = 2$ and when $v = 5$.

15. ***Velocity*** The height s (in feet) at time t (in seconds) of a silver dollar dropped from the top of the Washington Monument is given by

$$s = -16t^2 + 555.$$

(a) Find the average velocity on the interval $[2, 3]$.

(b) Find the instantaneous velocities when $t = 2$ and when $t = 3$.

(c) How long will it take the dollar to hit the ground?

(d) Find the velocity of the dollar when it hits the ground.

16. ***Physics: Velocity*** A racecar travels northward on a straight, level track at a constant speed, traveling 0.750 kilometer in 20.0 seconds. The return trip over the same track is made in 25.0 seconds.

(a) What is the average velocity of the car in meters per second for the first leg of the run?

(b) What is the average velocity for the total trip?

(*Source: Shipman/Wilson/Todd,* An Introduction to Physical Science, *Tenth Edition*)

Marginal Cost In Exercises 17–20, find the marginal cost for producing x units. (The cost is measured in dollars.)

17. $C = 4500 + 1.47x$

18. $C = 104{,}000 + 7200x$

19. $C = 55{,}000 + 470x - 0.25x^2, \quad 0 \le x \le 940$

20. $C = 100(9 + 3\sqrt{x})$

Marginal Revenue In Exercises 21–24, find the marginal revenue for producing x units. (The revenue is measured in dollars.)

21. $R = 50x - 0.5x^2$

22. $R = 30x - x^2$

23. $R = -6x^3 + 8x^2 + 200x$

24. $R = 50(20x - x^{3/2})$

Marginal Profit In Exercises 25–28, find the marginal profit for producing x units. (The profit is measured in dollars.)

25. $P = -2x^2 + 72x - 145$

26. $P = -0.25x^2 + 2000x - 1{,}250{,}000$

27. $P = -0.00025x^2 + 12.2x - 25{,}000$

28. $P = -0.5x^3 + 30x^2 - 164.25x - 1000$

29. *Marginal Cost* The cost (in dollars) of producing x units of a product is given by

$$C = 3.6\sqrt{x} + 500.$$

(a) Find the additional cost when the production increases from 9 to 10 units.

(b) Find the marginal cost when $x = 9$.

(c) Compare the results of parts (a) and (b).

30. *Marginal Revenue* The revenue (in dollars) from renting x apartments can be modeled by

$$R = 2x(900 + 32x - x^2).$$

(a) Find the additional revenue when the number of rentals is increased from 14 to 15.

(b) Find the marginal revenue when $x = 14$.

(c) Compare the results of parts (a) and (b).

31. *Marginal Profit* The profit (in dollars) from selling x units of calculus textbooks is given by

$$p = -0.05x^2 + 20x - 1000.$$

(a) Find the additional profit when the sales increase from 150 to 151 units.

(b) Find the marginal profit when $x = 150$.

(c) Compare the results of parts (a) and (b).

32. *Population Growth* The population of a developing rural area has been growing according to the model

$$P = 22t^2 + 52t + 10{,}000$$

where t is time in years, with $t = 0$ corresponding to 1990.

(a) Evaluate P for $t = 0, 10, 15, 20,$ and 25. Explain these values.

(b) Determine the population growth rate, dP/dt.

(c) Evaluate dP/dt for the same values as in part (a). Explain your results.

33. *Health* The temperature of a person during an illness is given by $T = -0.0375t^2 + 0.3t + 100.4$, where t is time in hours since the person started to show signs of a fever.

(a) Use a graphing utility to graph the function. Be sure to choose an appropriate window.

(b) Do the slopes of the tangent lines appear to be positive or negative? What does this tell you?

(c) Evaluate the function for $t = 0, 4, 8,$ and 12.

(d) Find dT/dt and explain its meaning in this situation.

(e) Evaluate dT/dt for $t = 0, 4, 8,$ and 12.

34. *Marginal Profit* The profit (in dollars) from selling x units of a product is given by

$$P = 36{,}000 + 2048\sqrt{x} - \frac{1}{8x^2}, \quad 150 \le x \le 275.$$

Find the marginal profit for each of the following sales.

(a) $x = 150$ (b) $x = 175$ (c) $x = 200$

(d) $x = 225$ (e) $x = 250$ (f) $x = 275$

35. *Profit* The monthly demand function and cost function for x newspapers at a newsstand are given by $p = 5 - 0.001x$ and $C = 35 + 1.5x$.

(a) Find the monthly revenue R as a function of x.

(b) Find the monthly profit P as a function of x.

(c) Complete the table.

x	600	1200	1800	2400	3000
dR/dx					
dP/dx					
P					

36. *Economics* Use the table to answer the questions below.

Quantity produced and sold (Q)	Price (p)	Total revenue (TR)	Marginal revenue (MR)
0	160	0	—
2	140	280	130
4	120	480	90
6	100	600	50
8	80	640	10
10	60	600	−30

(a) Use the *regression* feature of a graphing utility to find a quadratic model that relates the total revenue (TR) to the quantity produced and sold (Q).

(b) Using derivatives, find a model for marginal revenue from the model you found in part (a).

(c) Calculate the marginal revenue for all values of Q using your model in part (b), and compare these values with the actual values given. How good is your model?

(*Source: Adapted from Taylor,* Economics, *Fourth Edition*)

37. *Marginal Profit* When a glass of lemonade at a lemonade stand was \$0.75, 400 glasses were sold. When the price was lowered to \$0.50, 500 glasses were sold. Assume that the demand function is linear and that the variable and fixed costs are \$0.05 and \$20, respectively.

(a) Find the profit P as a function of x, the number of glasses of lemonade sold.

(b) Use a graphing utility to graph P, and comment about the slopes of P when $x = 200$ and when $x = 400$.

(c) Find the marginal profits when 200 glasses of lemonade are sold and when 400 glasses of lemonade are sold.

38. *Marginal Profit* baseball game was \$ When the price was sold. Assume that th variable and fixed c and \$85,000, respec

(a) Find the profit tickets sold.

(b) Use a graphing the slopes of P v

(c) Find the margin and when 36,00

39. *Marginal Profit* decreased to 30,000 would this change t

40. *Marginal Cost* modeled by $C = v($ cost and k represent cost is independent

41. *Profit* The dema $p = 50/\sqrt{x}$ for 1 given by $C = 0.5x$

Find the marginal (c) $x = 2500$, and (

If you were in char what price would y

42. *Inventory Manag* a manufacturer is where Q is the order Find the change in 350 to 351, and com change when $Q =$

43. *Center of Popula* ulation of the United ward. Use the figure at which the center during the given pe

(a) From 1790 to 1

(c) From 1790 to 2

1 of the following sales.

(c) $x = 200$

(f) $x = 275$

function and cost function
ewsstand are given by
+ 1.5x.

as a function of x.

a function of x.

	2400	3000

nswer the questions below.

Marginal revenue (MR)
—
130
90
50
10
−30

f a graphing utility to find
s the total revenue (TR) to
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odel for marginal revenue
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ie for all values of Q using
compare these values with
good is your model?
conomics, Fourth Edition)

s of lemonade at a lemon-
were sold. When the price
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spectively.

ion of x, the number of

ph P, and comment about
0 and when $x = 400$.

n 200 glasses of lemonade
s of lemonade are sold.

38. ***Marginal Profit*** When the admission price for a baseball game was \$6 per ticket, 36,000 tickets were sold. When the price was raised to \$7, only 33,000 tickets were sold. Assume that the demand function is linear and that the variable and fixed costs for the ballpark owners are \$0.20 and \$85,000, respectively.

(a) Find the profit P as a function of x, the number of tickets sold.

(b) Use a graphing utility to graph P, and comment about the slopes of P when $x = 18{,}000$ and when $x = 36{,}000$.

(c) Find the marginal profits when 18,000 tickets are sold and when 36,000 tickets are sold.

39. ***Marginal Profit*** In Exercise 38, suppose ticket sales decreased to 30,000 when the price increased to \$7. How would this change the answers?

40. ***Marginal Cost*** The cost C of producing x units is modeled by $C = v(x) + k$, where v represents the variable cost and k represents the fixed cost. Show that the marginal cost is independent of the fixed cost.

41. ***Profit*** The demand function for a product is given by $p = 50/\sqrt{x}$ for $1 \le x \le 8000$, and the cost function is given by $C = 0.5x + 500$ for $0 \le x \le 8000$.

Find the marginal profits for (a) $x = 900$, (b) $x = 1600$, (c) $x = 2500$, and (d) $x = 3600$.

If you were in charge of setting the price for this product, what price would you set? Explain your reasoning.

42. ***Inventory Management*** The annual inventory cost for a manufacturer is given by $C = 1{,}008{,}000/Q + 6.3Q$, where Q is the order size when the inventory is replenished. Find the change in annual cost when Q is increased from 350 to 351, and compare this with the instantaneous rate of change when $Q = 350$.

43. ***Center of Population*** Since 1790, the center of population of the United States has been gradually moving westward. Use the figure to estimate the rate (in miles per year) at which the center of population was moving *westward* during the given period. *(Source: U.S. Census Bureau)*

(a) From 1790 to 1900 (b) From 1900 to 2000

(c) From 1790 to 2000

44. ***Consumer Awareness*** A car is driven 15,000 miles a year and gets x miles per gallon. The average fuel cost is \$1.30 per gallon. Find the annual cost C of fuel as a function of x and use this function to complete the table.

x	10	15	20	25	30	35	40
C							
dC/dx							

Who would benefit more from a 1 mile per gallon increase in fuel efficiency—the driver who gets 15 miles per gallon or the driver who gets 35 miles per gallon? Explain.

45. ***Writing*** The number N of gallons of regular unleaded gasoline sold by a gasoline station at a price of p dollars per gallon is given by $N = f(p)$.

(a) Describe the meaning of $f'(1.479)$.

(b) Is $f'(1.479)$ usually positive or negative? Explain.

46. Consider the function given by $f(x) = \dfrac{4}{x}$, $0 < x \le 5$.

(a) Use a graphing utility to graph f and f' in the same viewing window.

(b) Complete the table.

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4	5
$f(x)$								
$f'(x)$								

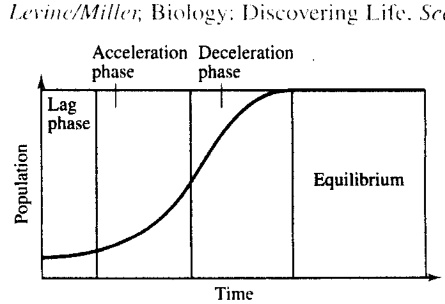
(c) Find the average rate of change of f over the intervals determined by consecutive x-values in the table.

In Exercises 47 and 48, use a graphing utility to graph f and f'. Then determine the points (if any) at which f has a horizontal tangent.

47. $f(x) = \frac{1}{4}x^3$, $-2 \le x \le 2$

48. $f(x) = x^4 - 12x^3 + 52x^2 - 96x + 64$, $1 \le x \le 5$

49. ***Biology*** Many populations in nature exhibit logistic growth, which consists of four phases, as shown in the figure. Describe the rate of growth of the population in each phase, and give possible reasons as to why the rates might be changing from phase to phase. *(Source: Adapted from Levine/Miller, Biology: Discovering Life, Second Edition)*



PREREQUISITE REVIEW 2.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–10, simplify the expression.

1. $(x^2 + 1)(2) + (2x + 7)(2x)$
2. $(2x - x^3)(8x) + (4x^2)(2 - 3x^2)$
3. $x(4)(x^2 + 2)^3(2x) + (x^2 + 4)(1)$
4. $x^2(2)(2x + 1)(2) + (2x + 1)^4(2x)$
5. $\dfrac{(2x + 7)(5) - (5x + 6)(2)}{(2x + 7)^2}$
6. $\dfrac{(x^2 - 4)(2x + 1) - (x^2 + x)(2x)}{(x^2 - 4)^2}$
7. $\dfrac{(x^2 + 1)(2) - (2x + 1)(2x)}{(x^2 + 1)^2}$
8. $\dfrac{(1 - x^4)(4) - (4x - 1)(-4x^3)}{(1 - x^4)^2}$
9. $(x^{-1} + x)(2) + (2x - 3)(-x^{-2} + 1)$
10. $\dfrac{(1 - x^{-1})(1) - (x - 4)(x^{-2})}{(1 - x^{-1})^2}$

In Exercises 11–14, find $f'(2)$.

11. $f(x) = 3x^2 - x + 4$
12. $f(x) = -x^3 + x^2 + 8x$
13. $f(x) = \dfrac{1}{x}$
14. $f(x) = x^2 - \dfrac{1}{x^2}$

EXERCISES 2.4

In Exercises 1–14, find the value of the derivative of the function at the given point.

	Function	*Point*
1.	$f(x) = x^2(3x^3 - 1)$	$(1, 2)$
2.	$f(x) = (x^2 + 1)(2x + 5)$	$(-1, 6)$
3.	$f(x) = \frac{1}{3}(2x^3 - 4)$	$\left(0, -\frac{4}{3}\right)$
4.	$f(x) = \frac{1}{7}(5 - 6x^2)$	$\left(1, -\frac{1}{7}\right)$
5.	$g(x) = (x^2 - 4x + 3)(x - 2)$	$(4, 6)$
6.	$g(x) = (x^2 - 2x + 1)(x^3 - 1)$	$(1, 0)$
7.	$h(x) = \dfrac{x}{x - 5}$	$(6, 6)$
8.	$h(x) = \dfrac{x^2}{x + 3}$	$\left(-1, \dfrac{1}{2}\right)$
9.	$f(t) = \dfrac{2t^2 - 3}{3t + 1}$	$\left(3, \dfrac{3}{2}\right)$
10.	$f(x) = \dfrac{3x}{x^2 + 4}$	$\left(-1, -\dfrac{3}{5}\right)$
11.	$g(x) = \dfrac{2x + 1}{x - 5}$	$(6, 13)$
12.	$f(x) = \dfrac{x + 1}{x - 1}$	$(2, 3)$
13.	$f(t) = \dfrac{t^2 - 1}{t + 4}$	$(1, 0)$
14.	$g(x) = \dfrac{4x - 5}{x^2 - 1}$	$(0, 5)$

In Exercises 15–22, find the derivative of the function. Use Example 7 as a model.

	Function	*Rewrite*	*Differentiate*	*Simplify*
15.	$y = \dfrac{x^2 + 2x}{x}$			
16.	$y = \dfrac{4x^{3/2}}{x}$			
17.	$y = \dfrac{7}{3x^3}$			
18.	$y = \dfrac{4}{5x^2}$			
19.	$y = \dfrac{4x^2 - 3x}{8\sqrt{x}}$			
20.	$y = \dfrac{3x^2 - 4x}{6x}$			
21.	$y = \dfrac{x^2 - 4x + 3}{x - 1}$			
22.	$y = \dfrac{x^2 - 4}{x + 2}$			

In Exercises 23–38, find th

23. $f(x) = (x^3 - 3x)(2x$
24. $h(t) = (t^5 - 1)(4t^2 -$
25. $g(t) = (2t^3 - 1)^2$
26. $h(p) = (p^3 - 2)^2$
27. $f(x) = \sqrt[3]{x}\,(\sqrt{x} + 3$
28. $f(x) = \sqrt[3]{x}(x + 1)$
30. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$
32. $f(x) = (x^5 - 3x)\Big(\dfrac{1}{x}$
33. $f(x) = x\Big(1 - \dfrac{2}{x + 1}$
34. $h(t) = \dfrac{t + 2}{t^2 + 5t + 6}$
35. $g(s) = \dfrac{s^2 - 2s + 5}{\sqrt{s}}$
36. $f(x) = \dfrac{x + 1}{\sqrt{x}}$
37. $g(x) = \left(\dfrac{x - 3}{x + 4}\right)(x^2$
38. $f(x) = (3x^3 + 4x)(x$

In Exercises 39–44, find
graph of the function at
utility to graph the funct
viewing window.

Function

39. $f(x) = (x - 1)^2(x -$
40. $h(x) = (x^2 - 1)^2$
41. $f(x) = \dfrac{x - 2}{x + 1}$
42. $f(x) = \dfrac{2x + 1}{x - 1}$
43. $f(x) = \left(\dfrac{x + 5}{x - 1}\right)(2x$
44. $g(x) = (x + 2)\Big(\dfrac{x -}{x +}$

In Exercises 45–48, find th
f has a horizontal tangen

45. $f(x) = \dfrac{x^2}{x - 1}$
47. $f(x) = \dfrac{x^4}{x^3 + 1}$

r sections. You will

Point

(1, 0)

(0, 5)

ve of the function. Use

Differentiate *Simplify*

In Exercises 23–38, find the derivative of the function.

23. $f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$

24. $h(t) = (t^5 - 1)(4t^2 - 7t - 3)$

25. $g(t) = (2t^3 - 1)^2$

26. $h(p) = (p^3 - 2)^2$

27. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

28. $f(x) = \sqrt[3]{x}(x + 1)$

29. $f(x) = \dfrac{3x - 2}{2x - 3}$

30. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$

31. $f(x) = \dfrac{3 - 2x - x^2}{x^2 - 1}$

32. $f(x) = (x^5 - 3x)\left(\dfrac{1}{x^2}\right)$

33. $f(x) = x\left(1 - \dfrac{2}{x + 1}\right)$

34. $h(t) = \dfrac{t + 2}{t^2 + 5t + 6}$

35. $g(s) = \dfrac{s^2 - 2s + 5}{\sqrt{s}}$

36. $f(x) = \dfrac{x + 1}{\sqrt{x}}$

37. $g(x) = \left(\dfrac{x - 3}{x + 4}\right)(x^2 + 2x + 1)$

38. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

In Exercises 39–44, find an equation of the tangent line to the graph of the function at the given point. Then use a graphing utility to graph the function and the tangent line in the same viewing window.

Function	*Point*
39. $f(x) = (x - 1)^2(x - 2)$	$(0, -2)$
40. $h(x) = (x^2 - 1)^2$	$(-2, 9)$
41. $f(x) = \dfrac{x - 2}{x + 1}$	$\left(1, -\frac{1}{2}\right)$
42. $f(x) = \dfrac{2x + 1}{x - 1}$	$(2, 5)$
43. $f(x) = \left(\dfrac{x + 5}{x - 1}\right)(2x + 1)$	$(0, -5)$
44. $g(x) = (x + 2)\left(\dfrac{x - 5}{x + 1}\right)$	$(0, -10)$

In Exercises 45–48, find the point(s), if any, at which the graph of f has a horizontal tangent.

45. $f(x) = \dfrac{x^2}{x - 1}$

46. $f(x) = \dfrac{x^2}{x^2 + 1}$

47. $f(x) = \dfrac{x^4}{x^3 + 1}$

48. $f(x) = \dfrac{x^4 + 3}{x^2 + 1}$

In Exercises 49–52, use a graphing utility to graph f and f' on the interval $[-2, 2]$.

49. $f(x) = x(x + 1)$

50. $f(x) = x^2(x + 1)$

51. $f(x) = x(x + 1)(x - 1)$

52. $f(x) = x^2(x + 1)(x - 1)$

Demand In Exercises 53 and 54, use the demand function to find the rate of change in the demand x for the given price p.

53. $x = 275\left(1 - \dfrac{3p}{5p + 1}\right), \; p = \$4$

54. $x = 300 - p - \dfrac{2p}{p + 1}, \; p = \$3$

55. ***Environment*** The model

$$f(t) = \frac{t^2 - t + 1}{t^2 + 1}$$

measures the percent of the normal level of oxygen in a pond, where t is the time (in weeks) after organic waste is dumped into the pond. Find the rates of change of f with respect to t when (a) $t = 0.5$, (b) $t = 2$, and (c) $t = 8$.

56. ***Physical Science*** The temperature T of food placed in a refrigerator is modeled by

$$T = 10\left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$$

where t is the time (in hours). What is the initial temperature of the food? Find the rates of change of T with respect to t when (a) $t = 1$, (b) $t = 3$, (c) $t = 5$, and (d) $t = 10$.

57. ***Population Growth*** A population of bacteria is introduced into a culture. The number of bacteria P can be modeled by

$$P = 500\left(1 + \frac{4t}{50 + t^2}\right)$$

where t is the time (in hours). Find the rate of change of the population when $t = 2$.

58. ***Quality Control*** The percent P of defective parts produced by a new employee t days after the employee starts work can be modeled by

$$P = \frac{t + 1750}{50(t + 2)}.$$

Find the rates of change of P when (a) $t = 1$ and (b) $t = 10$.

59. ***Profit*** You decide to form a partnership with another business. Your business determines that the demand x for your product is inversely proportional to the square of the price for $x \geq 5$.

(a) The price is \$1000 and the demand is 16 units. Find the demand function.

(b) Your partner determines that the product costs \$250 per unit and the fixed cost is \$10,000. Find the cost function.

(c) Find the profit function and use a graphing utility to graph it. From the graph, what price would you negotiate with your partner for this product? Explain your reasoning.

60. ***Profit*** You are managing a store and have been adjusting the price of an item. You have found that you make a profit of \$50 when 10 units are sold, \$60 when 12 units are sold, and \$65 when 14 units are sold.

(a) Fit these data to the model $P = ax^2 + bx + c$.

(b) Use a graphing utility to graph P.

(c) Find the point on the graph at which the marginal profit is zero. Interpret this point in the context of the problem.

61. ***Demand Function*** Given $f(x) = x^2 + 1$, which function would most likely represent a demand function?

(a) $p = f(x)$

(b) $p = xf(x)$

(c) $p = 1/f(x)$

Explain your reasoning. Use a graphing utility to graph each function, and use each graph as part of your explanation.

62. ***Cost*** The cost of producing x units of a product is given by

$$C = x^3 - 15x^2 + 87x - 73, \qquad 4 \le x \le 9.$$

(a) Use a graphing utility to graph the marginal cost function and the average cost function, C/x, in the same viewing window.

(b) Find the point of intersection of the graphs of dC/dx and C/x. Does this point have any significance?

63. ***Inventory Replenishment*** The ordering and transportation cost C (in thousands of dollars) of the components used in manufacturing a product is given by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \qquad 1 \le x$$

where x is the order size (in hundreds). Find the rate of change of C with respect to x for each order size.

(a) $x = 10$

(b) $x = 15$

(c) $x = 20$

What do these rates of change imply about increasing the size of an order?

64. ***Sales Analysis*** The monthly sales of memberships M at a newly built fitness center are modeled by

$$M(t) = \frac{300t}{t^2 + 1} + 8$$

where t is the number of months since the center opened.

(a) Find $M'(t)$.

(b) Find $M(3)$ and $M'(3)$ and interpret the results.

(c) Find $M(24)$ and $M'(24)$ and interpret the results.

65. ***Consumer Awareness*** The prices of 1 pound of 100% ground beef in the United States from 1995 to 2002 can be modeled by

$$P = \frac{1.47 - 0.311t + 0.0173t^2}{1 - 0.206t + 0.0112t^2}$$

where t is the year, with $t = 5$ corresponding to 1995. Find dP/dt and evaluate it for $t = 5, 7, 9,$ and 11. Interpret the meaning of these values. *(Source: U.S. Bureau of Labor Statistics)*

BUSINESS CAPSULE

The Blackwood Centre for Adolescent Development, a state-sponsored secondary school for young people at risk in Victoria, Australia, joined forces in 2000 with the Centre for Executive Development (CED). With the CED providing fundraising and logistical support, the Blackwood Centre has been able to transform its program into a model of success, offering their students training and educational opportunities for entry into the business world. The CED has gained team-building skills, improved workplace assessment methods, and a stronger connection to the community.

66. ***Research Project*** Use your school's library, the Internet, or some other reference source to find information about partnerships between companies and federal, state, or local government that have benefited their communities. (One such partnership is described above.) Write a short paper about the partnership.

2.5

The Chain Rule

In this section, you v
calculus—the **Chain R**
and adds versatility to
compare the functions
Chain Rule, whereas tl

Without the Chair

$y = x^2 + 1$

$y = x + 1$

$y = 3x + 2$

$y = \frac{x + 5}{x^2 + 2}$

The Chain Rule

If $y = f(u)$ is a dil
tiable function of .

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

or, equivalently,

$\frac{d}{dx}[f(g(x))] =$

Basically, the Cha
u changes du/dx time

$\frac{dy}{du} \cdot \frac{du}{dx}$

times as fast as x, as
notation for derivative
as the Chain Rule. Fo

$dy/dx = (dy/du)$

you can imagine that

PREREQUISITE REVIEW 8.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the derivative of the function.

1. $f(x) = 3x^3 - 2x^2 + 4x - 7$

2. $g(x) = (x^3 + 4)^4$

3. $f(x) = (x - 1)(x^2 + 2x + 3)$

4. $g(x) = \dfrac{2x}{x^2 + 5}$

In Exercises 5 and 6, find the relative extrema of the function.

5. $f(x) = x^2 + 4x + 1$

6. $f(x) = \frac{1}{3}x^3 - 4x + 2$

In Exercises 7–10, solve the trigonometric equation for x where $0 \le x \le 2\pi$.

7. $\sin x = \dfrac{\sqrt{3}}{2}$

8. $\cos x = -\dfrac{1}{2}$

9. $\cos \dfrac{x}{2} = 0$

10. $\sin \dfrac{x}{2} = -\dfrac{\sqrt{2}}{2}$

EXERCISES 8.4

In Exercises 1–26, find the derivative of the function.

1. $y = \frac{1}{2} - 3 \sin x$

2. $y = 5 + \sin x$

3. $y = x^2 - \cos x$

4. $g(t) = \pi \cos t - \dfrac{1}{t^2}$

5. $f(x) = 4\sqrt{x} + 3 \cos x$

6. $f(x) = \sin x + \cos x$

7. $f(t) = t^2 \cos t$

8. $f(x) = (x + 1) \cos x$

9. $g(t) = \dfrac{\cos t}{t}$

10. $f(x) = \dfrac{\sin x}{x}$

11. $y = \tan x + x^2$

12. $y = x + \cot x$

13. $y = e^{x^2} \sec x$

14. $y = e^{-x} \sin x$

15. $y = \cos 3x + \sin^2 x$

16. $y = \csc^2 x - \cos 2x$

17. $y = \sin \pi x$

18. $y = \frac{1}{2} \csc 2x$

19. $y = x \sin \dfrac{1}{x}$

20. $y = x^2 \sin \dfrac{1}{x}$

21. $y = 3 \tan 4x$

22. $y = \tan e^x$

23. $y = 2 \tan^2 4x$

24. $y = -\sin^4 2x$

25. $y = e^{2x} \sin 2x$

26. $y = e^{-x} \cos \dfrac{x}{2}$

In Exercises 27–38, find the derivative of the function and simplify your answer by using the trigonometric identities listed in Section 8.2.

27. $y = \cos^2 x$

28. $y = \frac{1}{4} \sin^2 2x$

29. $y = \cos^2 x - \sin^2 x$

30. $y = \dfrac{x}{2} + \dfrac{\sin 2x}{4}$

31. $y = \ln|\sin x|$

32. $y = -\ln|\cos x|$

33. $y = \ln|\csc x^2 - \cot x^2|$

34. $y = \ln|\sec x + \tan x|$

35. $y = \tan x - x$

36. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

37. $y = \ln(\sin^2 x)$

38. $y = \frac{1}{2}(x \tan x - \sec x)$

In Exercises 39–46, find an equation of the tangent line to the graph of the function at the given point.

	Function	*Point*
39.	$y = \tan x$	$\left(-\frac{\pi}{4}, -1\right)$
40.	$y = \sec x$	$\left(\frac{\pi}{3}, 2\right)$
41.	$y = \sin 4x$	$(\pi, 0)$
42.	$y = \csc^2 x$	$\left(\frac{\pi}{2}, 1\right)$
43.	$y = \dfrac{\cos x}{\sin x}$	$\left(\frac{3\pi}{4}, -1\right)$
44.	$y = \sin x \cos x$	$\left(\frac{3\pi}{2}, 0\right)$
45.	$y = \ln|\cot x|$	$\left(\frac{\pi}{4}, 0\right)$
46.	$y = \sqrt{\sin x}$	$\left(\frac{\pi}{6}, \frac{\sqrt{2}}{2}\right)$

In Exercises 47 and 48, use implicit differentiation to find dy/dx and evaluate the derivative at the given point.

Function	*Point*
47. $\sin x + \cos 2y = 1$	$\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$
48. $\tan(x + y) = x$	$(0, 0)$

In Exercises 49–52, show that the function satisfies the differential equation.

49. $y = 2 \sin x + 3 \cos x$

$y'' + y = 0$

50. $y = \dfrac{10 - \cos x}{x}$

$xy' + y = \sin x$

51. $y = \cos 2x + \sin 2x$

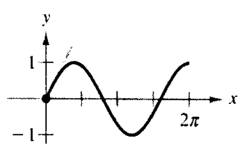
$y'' + 4y = 0$

52. $y = e^x(\cos \sqrt{2}x + \sin \sqrt{2}x)$

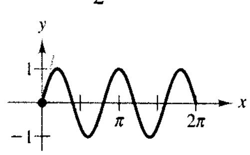
$y'' - 2y' + 3y = 0$

In Exercises 53–58, find the slope of the tangent line to the given sine function at the origin. Compare this value with the number of complete cycles in the interval $[0, 2\pi]$.

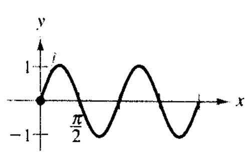
53. $y = \sin \dfrac{5x}{4}$



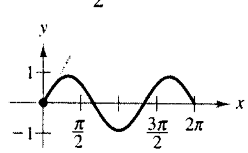
54. $y = \sin \dfrac{5x}{2}$



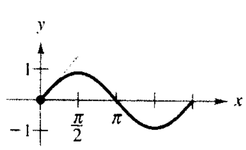
55. $y = \sin 2x$



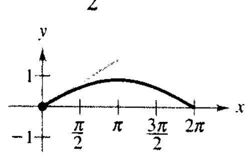
56. $y = \sin \dfrac{3x}{2}$



57. $y = \sin x$



58. $y = \sin \dfrac{x}{2}$



In Exercises 59–64, determine the relative extrema of the function on the interval $(0, 2\pi)$. Use a graphing utility to confirm your result.

59. $y = 2 \sin x + \sin 2x$

60. $y = 2 \sin x + \cos 2x$

61. $y = x - 2 \sin x$

62. $y = e^{-x} \sin x$

63. $y = e^{-x} \cos x$

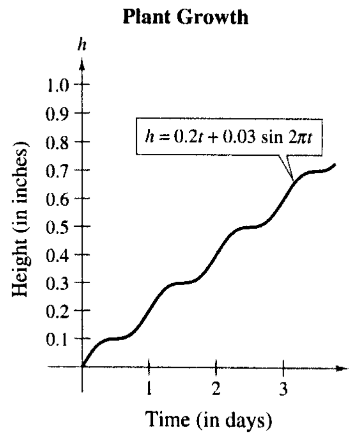
64. $y = \sec \dfrac{x}{2}$

65. ***Biology*** Plants do not grow at constant rates during a normal 24-hour period because their growth is affected by sunlight. Suppose that the growth of a certain plant species in a controlled environment is given by the model

$$h = 0.2t + 0.03 \sin 2\pi t$$

where h is the height of the plant in inches and t is the time in days, with $t = 0$ corresponding to midnight of day 1 (see figure). During what time of day is the rate of growth of this plant

(a) a maximum? (b) a minimum?



66. ***Meteorology*** The normal average daily temperature in degrees Fahrenheit for a city is given by

$$T = 55 - 21 \cos \frac{2\pi(t - 32)}{365}$$

where t is the time in days, with $t = 1$ corresponding to January 1. Find the expected date of

(a) the warmest day. (b) the coldest day.

67. ***Physics*** An amusement park ride is constructed such that its height h in feet above ground in terms of the horizontal distance x in feet from the starting point can be modeled by

$$h = 50 + 45 \sin \frac{\pi x}{150}, \quad 0 \le x \le 300.$$

(a) Use a graphing utility to graph the model. Be sure to choose an appropriate viewing window.

(b) Determine dh/dx and evaluate for $x = 50$, 150, 200, and 250. Interpret these values of dh/dx.

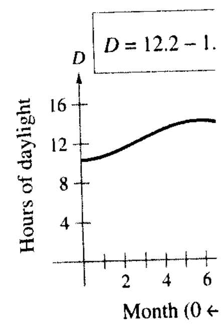
(c) Find the maximum height and the minimum height of the ride.

(d) Find the distance from the starting point at which the ride's rate of change is the greatest.

68. ***Meteorology*** The nu
Orleans can be modele

$D = 12.2 - 1.9$ co

where t represents the
January. Find the mont
imum number of dayl
number of daylight ho



69. For $f(x) = \sec^2 x$ and
70. For $f(x) = \sin^2 x$ and $g($
71. ***Physics*** A 15-centi
the equation

$\theta = 0.2 \cos 8t$

where θ is the angul
radians and t is the tin

(a) Determine the ma
(b) Find the rate of c

72. ***Tides*** Throughout t
at the end of a dock v
particular day can be

$D = 3.5 + 1.5$ co

where $t = 0$ represen

(a) Determine dD/dt
(b) Evaluate dD/dt f
results.
(c) Find the time(s)
and the time(s) w

grow at constant rates during a
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growth of a certain plant species
t is given by the model

t

plant in inches and t is the time
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3
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park ride is constructed such
oove ground in terms of the
from the starting point can be

) ≤ x ≤ 300.

graph the model. Be sure to
ewing window.

valuate for x = 50, 150, 200,
values of dh/dx.

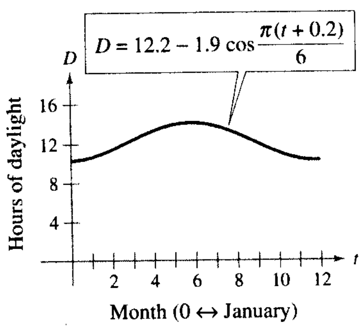
nt and the minimum height of

he starting point at which the
ne greatest.

68. ***Meteorology*** The number of hours of daylight D in New Orleans can be modeled by

$$D = 12.2 - 1.9\cos\frac{\pi(t + 0.2)}{6}, \quad 0 \le t \le 12$$

where t represents the month, with $t = 0$ corresponding to January. Find the month t when New Orleans has the maximum number of daylight hours. What is this maximum number of daylight hours?



69. For $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$, show that $f'(x) = g'(x)$.

70. For $f(x) = \sin^2 x$ and $g(x) = \cos^2 x$, show that $f'(x) = -g'(x)$.

71. ***Physics*** A 15-centimeter pendulum moves according to the equation

$$\theta = 0.2\cos 8t$$

where θ is the angular displacement from the vertical in radians and t is the time in seconds.

(a) Determine the maximum angular displacement.

(b) Find the rate of change of θ when $t = 3$ seconds.

72. ***Tides*** Throughout the day, the depth of water D in meters at the end of a dock varies with the tides. The depth for one particular day can be modeled by

$$D = 3.5 + 1.5\cos\frac{\pi t}{6}, \quad 0 \le t \le 24$$

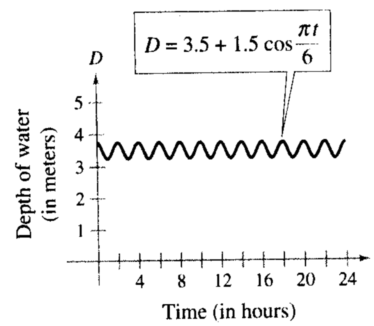
where $t = 0$ represents midnight.

(a) Determine dD/dt.

(b) Evaluate dD/dt for $t = 4$ and $t = 20$ and interpret your results.

(c) Find the time(s) when the water depth is the greatest and the time(s) when the water depth is the least.

(d) What is the greatest depth? What is the least depth? Did you have to use calculus to determine these depths? Explain your reasoning.



In Exercises 73–78, use a graphing utility (a) to graph f and f' on the same coordinate axes over the specified interval, (b) to find the critical numbers of f, and (c) to find the interval(s) on which f' is positive and the interval(s) on which it is negative. Note the behavior of f in relation to the sign of f'.

	Function	*Interval*
73.	$f(t) = t^2 \sin t$	$(0, 2\pi)$
74.	$f(x) = \frac{x}{2} + \cos\frac{x}{2}$	$(0, 4\pi)$
75.	$f(x) = \sin x - \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x$	$(0, \pi)$
76.	$f(x) = x\sin x$	$(0, \pi)$
77.	$f(x) = \sqrt{2x}\sin x$	$(0, 2\pi)$
78.	$f(x) = 4e^{-0.5x}\sin \pi x$	$(0, 4)$

In Exercises 79–84, use a graphing utility to find the relative extrema of the trigonometric function. Let $0 < x < 2\pi$.

79. $f(x) = \dfrac{x}{\sin x}$

80. $f(x) = \dfrac{x^2 - 2}{\sin x} - 5x$

81. $f(x) = \ln x \cos x$

82. $f(x) = \ln x \sin x$

83. $f(x) = \sin(0.1x^2)$

84. $f(x) = \sin\sqrt{x}$

True or False? In Exercises 85–88, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

85. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}$.

86. If $f(x) = \sin^2(2x)$, then $f'(x) = 2(\sin 2x)(\cos 2x)$.

87. If $y = x\sin^3 x$, then $y' = 3x\sin^2 x$.

88. The maximum value of $y = 3\sin x + 2\cos x$ is 5.