PREREQUISITE REVIEW 2.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, rewrite the expression with rational exponents.

1. $\sqrt[5]{(1-5x)^2}$

2. $\sqrt[4]{(2x-1)^3}$

3. $\dfrac{1}{\sqrt{4x^2+1}}$

4. $\dfrac{1}{\sqrt[3]{x-6}}$

5. $\dfrac{\sqrt{x}}{\sqrt[3]{1-2x}}$

6. $\dfrac{\sqrt{(3-7x)^3}}{2x}$

In Exercises 7–10, factor the expression.

7. $3x^3 - 6x^2 + 5x - 10$

8. $5x\sqrt{x} - x - 5\sqrt{x} + 1$

9. $4(x^2+1)^2 - x(x^2+1)^3$

10. $-x^5 + 3x^3 + x^2 - 3$

EXERCISES 2.5

In Exercises 1–8, identify the inside function, $u = g(x)$, and the outside function, $y = f(u)$.

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1. $y = (6x-5)^4$		
2. $y = (x^2-2x+3)^3$		
3. $y = (4-x^2)^{-1}$		
4. $y = (x^2+1)^{4/3}$		
5. $y = \sqrt{5x-2}$		
6. $y = \sqrt{9-x^2}$		
7. $y = (3x+1)^{-1}$		
8. $y = (x+1)^{-1/2}$		

In Exercises 9–16, match the function with the rule that you would use to find the derivative *most efficiently*.

(a) Simple Power Rule (b) Constant Rule

(c) General Power Rule (d) Quotient Rule

9. $f(x) = \dfrac{2}{1-x^3}$

10. $f(x) = \dfrac{2x}{1-x^3}$

11. $f(x) = \sqrt[3]{8^2}$

12. $f(x) = \sqrt[3]{x^2}$

13. $f(x) = \dfrac{x^2+2}{x}$

14. $f(x) = \dfrac{x^4-2x+1}{\sqrt{x}}$

15. $f(x) = \dfrac{2}{x-2}$

16. $f(x) = \dfrac{5}{x^2+1}$

In Exercises 17–34, use the General Power Rule to find the derivative of the function.

17. $y = (2x-7)^3$

18. $y = (3x^2+1)^4$

19. $g(x) = (4-2x)^3$

20. $h(t) = (1-t^2)^4$

21. $h(x) = (6x-x^3)^2$

22. $f(x) = (4x-x^2)^3$

23. $f(x) = (x^2-9)^{2/3}$

24. $f(t) = (9t+2)^{2/3}$

25. $f(t) = \sqrt{t+1}$

26. $g(x) = \sqrt{2x+3}$

27. $s(t) = \sqrt{2t^2+5t+2}$

28. $y = \sqrt[3]{3x^3+4x}$

29. $y = \sqrt[3]{9x^2+4}$

30. $y = 2\sqrt{4-x^2}$

31. $f(x) = -3\sqrt[4]{2-9x}$

32. $f(x) = (25+x^2)^{-1/2}$

33. $h(x) = (4-x^3)^{-4/3}$

34. $f(x) = (4-3x)^{-5/2}$

In Exercises 35–40, find an equation of the tangent line to the graph of f at the point $(2, f(2))$. Use a graphing utility to check your result by graphing the original function and the tangent line in the same viewing window.

35. $f(x) = 2(x^2-1)^3$

36. $f(x) = 3(9x-4)^4$

37. $f(x) = \sqrt{4x^2-7}$

38. $f(x) = x\sqrt{x^2+5}$

39. $f(x) = \sqrt{x^2-2x+1}$

40. $f(x) = (4-3x^2)^{-2/3}$

In Exercises 41–44, use a symbolic differentiation utility to find the derivative of the function. Graph the function and its derivative in the same viewing window. Describe the behavior of the function when the derivative is zero.

41. $f(x) = \dfrac{\sqrt{x}+1}{x^2+1}$

42. $f(x) = \sqrt{\dfrac{2x}{x+1}}$

43. $f(x) = \sqrt{\dfrac{x+1}{x}}$

44. $f(x) = \sqrt{x}(2-x^2)$

In Exercises 45–64, find the derivative of the function.

45. $y = \dfrac{1}{x-2}$

46. $s(t) = \dfrac{1}{t^2+3t-1}$

47. $y = -\dfrac{4}{(t+2)^2}$

48. $f(x) = \dfrac{3}{(x^3-4)^2}$

49. $f(x) = \dfrac{1}{(x^2-3x)^2}$

50. $y = \dfrac{1}{\sqrt{x+2}}$

51. $g(t) = \dfrac{1}{t^2-2}$

53. $f(x) = x(3x-9)^3$

55. $y = x\sqrt{2x+3}$

57. $y = t^2\sqrt{t-2}$

59. $f(x) = \sqrt{\dfrac{3-2x}{4x}}$

61. $f(x) = \sqrt{x^2+1} -$ ...

62. $y = \sqrt{x-1} + \sqrt{x+}$ ...

63. $y = \left(\dfrac{6-5x}{x^2-1}\right)^2$

In Exercises 65–70, find an ... graph of the function at t... utility to graph the functic... viewing window.

Function

65. $f(t) = \dfrac{36}{(3-t)^2}$

66. $s(x) = \dfrac{1}{\sqrt{x^2-3x+4}}$

67. $f(t) = (t^2-9)\sqrt{t+}$ ...

68. $y = \dfrac{2x}{\sqrt{x+1}}$

69. $f(x) = \dfrac{x+1}{\sqrt{2x-3}}$

70. $y = \dfrac{x}{\sqrt{25+x^2}}$

71. ***Compound Interest*** ... with an annual inte... compounded monthly.

$$A = 1000\left(1 + \frac{r}{12}\right.$$ ...

Find the rates of ch... (a) $r = 0.08$, (b) $r = 0$ ...

72. ***Environment*** An e... average daily level P o... per million can be mo...

$$P = 0.25\sqrt{0.5n^2} ...$$

where n is the numbe... thousands. Find the ra... increasing when the pc...

51. $g(t) = \dfrac{1}{t^2 - 2}$

52. $g(x) = \dfrac{3}{\sqrt[3]{x^3 - 1}}$

53. $f(x) = x(3x - 9)^3$

54. $f(x) = x^3(x - 4)^2$

55. $y = x\sqrt{2x + 3}$

56. $y = t\sqrt{t + 1}$

57. $y = t^2\sqrt{t - 2}$

58. $y = \sqrt{x}(x - 2)^2$

59. $f(x) = \sqrt{\dfrac{3 - 2x}{4x}}$

60. $g(t) = \dfrac{3t^2}{\sqrt{t^2 + 2t - 1}}$

61. $f(x) = \sqrt{x^2 + 1} - \sqrt{x^2 - 1}$

62. $y = \sqrt{x - 1} + \sqrt{x + 1}$

63. $y = \left(\dfrac{6 - 5x}{x^2 - 1}\right)^2$

64. $y = \left(\dfrac{4x^2}{3 - x}\right)^3$

In Exercises 65–70, find an equation of the tangent line to the graph of the function at the given point. Then use a graphing utility to graph the function and the tangent line in the same viewing window.

Function	*Point*
65. $f(t) = \dfrac{36}{(3 - t)^2}$	$(0, 4)$
66. $s(x) = \dfrac{1}{\sqrt{x^2 - 3x + 4}}$	$(3, \frac{1}{2})$
67. $f(t) = (t^2 - 9)\sqrt{t + 2}$	$(-1, -8)$
68. $y = \dfrac{2x}{\sqrt{x + 1}}$	$(3, 3)$
69. $f(x) = \dfrac{x + 1}{\sqrt{2x - 3}}$	$(2, 3)$
70. $y = \dfrac{x}{\sqrt{25 + x^2}}$	$(0, 0)$

71. ***Compound Interest*** You deposit \$1000 in an account with an annual interest rate of r (in decimal form) compounded monthly. At the end of 5 years, the balance is

$$A = 1000\left(1 + \frac{r}{12}\right)^{60}.$$

Find the rates of change of A with respect to r when (a) $r = 0.08$, (b) $r = 0.10$, and (c) $r = 0.12$.

72. ***Environment*** An environmental study indicates that the average daily level P of a certain pollutant in the air in parts per million can be modeled by the equation

$$P = 0.25\sqrt{0.5n^2 + 5n + 25}$$

where n is the number of residents of the community in thousands. Find the rate at which the level of pollutant is increasing when the population of the community is 12,000.

73. ***Biology*** The number N of bacteria in a culture after t days is modeled by

$$N = 400\left[1 - \frac{3}{(t^2 + 2)^2}\right].$$

Complete the table. What can you conclude?

t	0	1	2	3	4
dN/dt					

74. ***Depreciation*** The value V of a machine t years after it is purchased is inversely proportional to the square root of $t + 1$. The initial value of the machine is \$10,000.

(a) Write V as a function of t.

(b) Find the rate of depreciation when $t = 1$.

(c) Find the rate of depreciation when $t = 3$.

75. ***Depreciation*** Repeat Exercise 74 given that the value of the machine t years after it is purchased is inversely proportional to the cube root of $t + 1$.

76. ***Credit Card Rate*** The average annual rate r (in percent form) for commercial bank credit cards from 1994 through 2002 can be modeled by

$$r = \sqrt{-0.14239t^4 + 3.939t^3 - 39.0835t^2 + 161.037t + 22.13}$$

where $t = 4$ corresponds to 1994. *(Source: Federal Reserve Bulletin)*

(a) Find the derivative of this model. Which differentiation rule(s) did you use?

(b) Use a graphing utility to graph the derivative. Use the interval $4 \le t \le 12$.

(c) Use the *trace* feature to find the years during which the finance rate was changing the most.

(d) Use the *trace* feature to find the years during which the finance rate was changing the least.

True or False? In Exercises 77 and 78, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

77. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}$.

78. If y is a differentiable function of u, u is a differentiable function of v, and v is a differentiable function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}.$$

PREREQUISITE REVIEW 2.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, solve the equation.

1. $-16t^2 + 24t = 0$
2. $-16t^2 + 80t + 224 = 0$
3. $-16t^2 + 128t + 320 = 0$
4. $-16t^2 + 9t + 1440 = 0$

In Exercises 5–8, find dy/dx.

5. $y = x^2(2x + 7)$
6. $y = (x^2 + 3x)(2x^2 - 5)$
7. $y = \dfrac{x^2}{2x + 7}$
8. $y = \dfrac{x^2 + 3x}{2x^2 - 5}$

In Exercises 9 and 10, find the domain and range of f.

9. $f(x) = x^2 - 4$
10. $f(x) = \sqrt{x - 7}$

EXERCISES 2.6

In Exercises 1–14, find the second derivative of the function.

1. $f(x) = 5 - 4x$
2. $f(x) = 3x - 1$
3. $f(x) = x^2 + 7x - 4$
4. $f(x) = 3x^2 + 4x$
5. $g(t) = \frac{1}{3}t^3 - 4t^2 + 2t$
6. $f(x) = 4(x^2 - 1)^2$
7. $f(t) = \dfrac{3}{4t^2}$
8. $g(t) = t^{-1/3}$
9. $f(x) = 3(2 - x^2)^3$
10. $f(x) = x\sqrt[3]{x}$
11. $f(x) = \dfrac{x + 1}{x - 1}$
12. $g(t) = -\dfrac{4}{(t + 2)^2}$
13. $y = x^2(x^2 + 4x + 8)$
14. $h(s) = s^3(s^2 - 2s + 1)$

In Exercises 15–20, find the third derivative of the function.

15. $f(x) = x^5 - 3x^4$
16. $f(x) = x^4 - 2x^3$
17. $f(x) = 5x(x + 4)^3$
18. $f(x) = (x - 1)^2$
19. $f(x) = \dfrac{3}{16x^2}$
20. $f(x) = \dfrac{1}{x}$

In Exercises 21–26, find the given value.

	Function	*Value*
21.	$g(t) = 5t^4 + 10t^2 + 3$	$g''(2)$
22.	$f(x) = 9 - x^2$	$f''(-\sqrt{5})$
23.	$f(x) = \sqrt{4 - x}$	$f'''(-5)$
24.	$f(t) = \sqrt{2t + 3}$	$f'''(\frac{1}{2})$
25.	$f(x) = x^2(3x^2 + 3x - 4)$	$f'''(-2)$
26.	$g(x) = 2x^3(x^2 - 5x + 4)$	$g'''(0)$

In Exercises 27–32, find the higher-order derivative.

	Given	*Derivative*
27.	$f'(x) = 2x^2$	$f''(x)$
28.	$f''(x) = 20x^3 - 36x^2$	$f'''(x)$
29.	$f''(x) = (2x - 2)/x$	$f'''(x)$
30.	$f'''(x) = 2\sqrt{x - 1}$	$f^{(4)}(x)$
31.	$f^{(4)}(x) = (x + 1)^2$	$f^{(6)}(x)$
32.	$f(x) = x^3 - 2x$	$f''(x)$

In Exercises 33–40, find the second derivative and solve the equation $f''(x) = 0$.

33. $f(x) = x^3 - 9x^2 + 27x - 27$
34. $f(x) = 3x^3 - 9x + 1$
35. $f(x) = (x + 3)(x - 4)(x + 5)$
36. $f(x) = (x + 2)(x - 2)(x + 3)(x - 3)$
37. $f(x) = x\sqrt{x^2 - 1}$
38. $f(x) = x\sqrt{4 - x^2}$
39. $f(x) = \dfrac{x}{x^2 + 3}$
40. $f(x) = \dfrac{x}{x^2 + 1}$

41. ***Velocity and Acceleration*** A ball is propelled straight upward from ground level with an initial velocity of 144 feet per second.
    (a) Write the position function of the ball.
    (b) Write the velocity and acceleration functions.
    (c) When is the ball at its highest point? How high is this point?
    (d) How fast is the ball traveling when it hits the ground? How is this speed related to the initial velocity?

42. ***Velocity and Acceleration*** A brick becomes dislodged from the top of the Empire State Building (at a height of 1250 feet) and falls to the sidewalk below.

(a) Write the position function of the brick.

(b) Write the velocity and acceleration functions.

(c) How long does it take the brick to hit the sidewalk?

(d) How fast is the brick traveling when it hits the sidewalk?

43. ***Velocity and Acceleration*** The velocity (in feet per second) of an automobile starting from rest is modeled by

$$\frac{ds}{dt} = \frac{90t}{t + 10}.$$

Create a table showing the velocity and acceleration at 10-second intervals during the first minute of travel. What can you conclude?

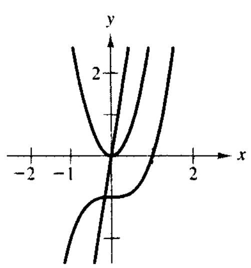
44. ***Stopping Distance*** A car is traveling at a rate of 66 feet per second (45 miles per hour) when the brakes are applied. The position function for the car is given by $s = -8.25t^2 + 66t$, where s is measured in feet and t is measured in seconds. Create a table showing the position, velocity, and acceleration for each given value of t. What can you conclude?

In Exercises 45 and 46, use a graphing utility to graph f, f', and f'' in the same viewing window. What is the relationship among the degree of f and the degrees of its successive derivatives? In general, what is the relationship among the degree of a polynomial function and the degrees of its successive derivatives?

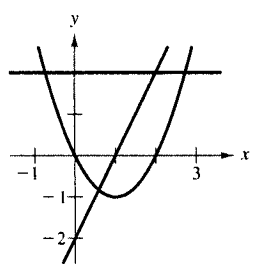
45. $f(x) = x^2 - 6x + 6$ **46.** $f(x) = 3x^3 - 9x$

In Exercises 47 and 48, the graphs of f, f', and f'' are shown on the same set of coordinate axes. Which is which? Explain your reasoning.

47.



48.



49. ***Data Analysis*** The table shows the median prices y (in thousands of dollars) of new privately owned U.S. homes in the South for 1995 to 2002. *(Source: U.S. Census Bureau)*

t	5	6	7	8	9
y	124.5	126.2	129.6	135.8	145.9

t	10	11	12
y	148.0	155.4	163.4

A model for the data is

$$y = -0.0828t^3 + 2.443t^2 - 17.06t + 158.7$$

where t is the year, with $t = 5$ corresponding to 1995.

(a) Use a graphing utility to graph the model and plot the data in the same viewing window.

(b) Find the first and second derivatives of the function.

(c) Show that the price of homes was increasing from 1995 to 2002.

(d) Find the year when the price was increasing at the greatest rate.

(e) Explain the relationship among your answers for parts (b), (c), and (d).

50. ***Projectile Motion*** An object is thrown upward from the top of a 64-foot building with an initial velocity of 48 feet per second.

(a) Write the position function of the object.

(b) Find the velocity and acceleration functions.

(c) When will the object hit the ground?

(d) When is the velocity of the object zero?

(e) How high does the object go?

(f) Use a graphing utility to graph the position, velocity, and acceleration functions in the same viewing window. Write a short paragraph that describes the relationship among these functions.

True or False? In Exercises 51–56, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

51. If $y = f(x)g(x)$, then $y' = f'(x)g'(x)$.

52. If $y = (x + 1)(x + 2)(x + 3)(x + 4)$, then $\dfrac{d^5y}{dx^5} = 0$.

53. If $f'(c)$ and $g'(c)$ are zero and $h(x) = f(x)g(x)$, then $h'(c) = 0$.

54. If $f(x)$ is an nth-degree polynomial, then $f^{(n+1)}(x) = 0$.

55. The second derivative represents the rate of change of the first derivative.

56. If the velocity of an object is constant, then its acceleration is zero.

57. ***Finding a Pattern*** Develop a general rule for $[xf(x)]^{(n)}$ where f is a differentiable function of x.

2.7

Implicit and Expli

So far in the text, functic in the **explicit form** $y =$ given in terms of the otl

$y = 3x - 5, \quad s =$

are each written in expl and w, explicitly. Many implied by a given equa

EXAMPLE 1 F

Find dy/dx for the equa

$xy = 1.$

SOLUTION In this equ to find dy/dx is first to

$$xy = 1$$
$$y = \frac{1}{x}$$
$$= x^{-1}$$
$$\frac{dy}{dx} = -x^{-2}$$
$$= -\frac{1}{x^2}$$

TRY IT 1

Find dy/dx for the eq

The procedure shc write the given functior you are unable to solve dy/dx in the equation

$x^2 - 2y^3 + 4y =$

where it is very difficul can use a procedure cal

PREREQUISITE REVIEW 2.7

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, solve the equation for y.

1. $x - \dfrac{y}{x} = 2$

2. $\dfrac{4}{x-3} = \dfrac{1}{y}$

3. $xy - x + 6y = 6$

4. $12 + 3y = 4x^2 + x^2y$

5. $x^2 + y^2 = 5$

6. $x = \pm\sqrt{6 - y^2}$

In Exercises 7–10, evaluate the expression at the given point.

7. $\dfrac{3x^2 - 4}{3y^2}$, $(2, 1)$

8. $\dfrac{x^2 - 2}{1 - y}$, $(0, -3)$

9. $\dfrac{5x}{3y^2 - 12y + 5}$, $(-1, 2)$

10. $\dfrac{1}{y^2 - 2xy + x^2}$, $(4, 3)$

EXERCISES 2.7

In Exercises 1–12, find dy/dx.

1. $5xy = 1$

2. $\frac{1}{2}x^2 - y = 6x$

3. $y^2 = 1 - x^2$, $0 \le x \le 1$

4. $4x^2y - \dfrac{3}{y} = 0$

5. $x^2y^2 - 4y = 1$

6. $xy^2 + 4xy = 10$

7. $4y^2 - xy = 2$

8. $2xy^3 - x^2y = 2$

9. $\dfrac{2y - x}{y^2 - 3} = 5$

10. $\dfrac{xy - y^2}{y - x} = 1$

11. $\dfrac{x + y}{2x - y} = 1$

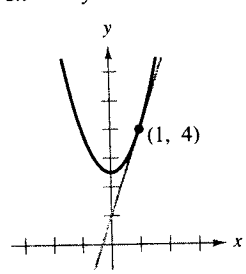
12. $\dfrac{2x + y}{x - 5y} = 1$

In Exercises 13–24, find dy/dx by implicit differentiation and evaluate the derivative at the given point.

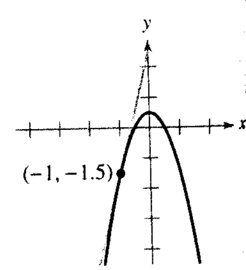
	Equation	*Point*
13.	$x^2 + y^2 = 49$	$(0, 7)$
14.	$x^2 - y^2 = 16$	$(4, 0)$
15.	$y + xy = 4$	$(-5, -1)$
16.	$x^2 - y^3 = 3$	$(2, 1)$
17.	$x^3 - xy + y^2 = 4$	$(0, -2)$
18.	$x^2y + y^2x = -2$	$(2, -1)$
19.	$x^3y^3 - y = x$	$(0, 0)$
20.	$x^3 + y^3 = 2xy$	$(1, 1)$
21.	$x^{1/2} + y^{1/2} = 9$	$(16, 25)$
22.	$\sqrt{xy} = x - 2y$	$(4, 1)$
23.	$x^{2/3} + y^{2/3} = 5$	$(8, 1)$
24.	$(x + y)^3 = x^3 + y^3$	$(-1, 1)$

In Exercises 25–30, find the slope of the graph at the given point.

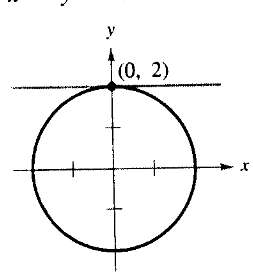
25. $3x^2 - 2y + 5 = 0$



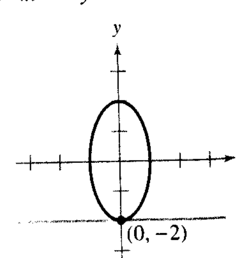
26. $4x^2 + 2y - 1 = 0$



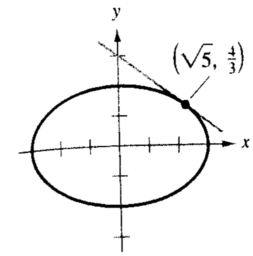
27. $x^2 + y^2 = 4$



28. $4x^2 + y^2 = 4$



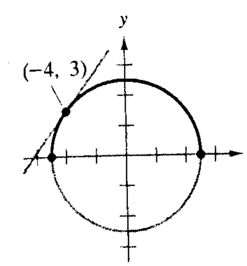
29. $4x^2 + 9y^2 = 36$



In Exercises 31–34, find $dy/$
explicit functions are shown
results are equivalent. Use th
the tangent line at the label
analytically by evaluating $dy/$

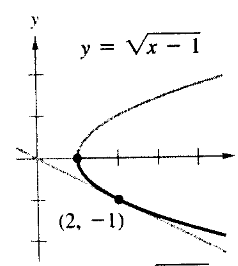
31. $x^2 + y^2 = 25$

$y = \sqrt{25 - x^2}$



$y = -\sqrt{25 - x^2}$

33. $x - y^2 - 1 = 0$



$y = -\sqrt{x - 1}$

In Exercises 35–40, find equ
graph at the given points. U
equation and the tangent lin

Equation

35. $x^2 + y^2 = 169$

earlier sections. You will

Point

2, − 1)

0, 0)

1, 1)

16, 25)

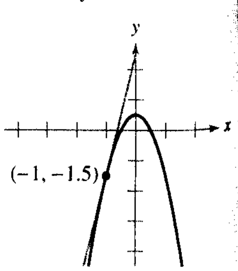
4, 1)

8, 1)

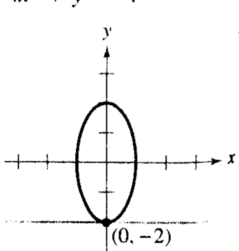
− 1, 1)

f the graph at the given point.

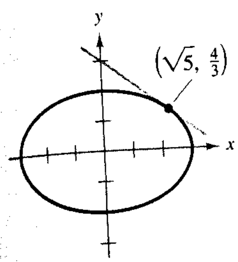
6. $4x^2 + 2y - 1 = 0$



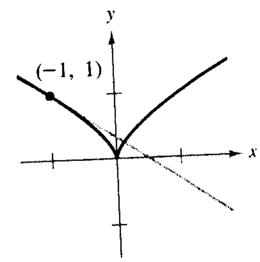
8. $4x^2 + y^2 = 4$



29. $4x^2 + 9y^2 = 36$

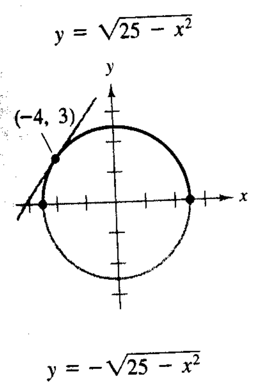


30. $x^2 - y^3 = 0$

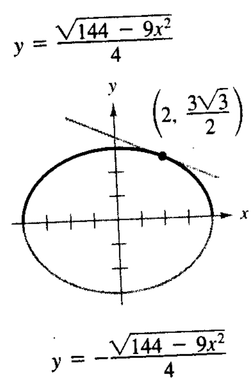


In Exercises 31–34, find dy/dx implicitly and explicitly (the explicit functions are shown on the graph) and show that the results are equivalent. Use the graph to estimate the slope of the tangent line at the labeled point. Then verify your result analytically by evaluating dy/dx at the point.

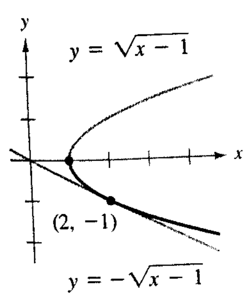
31. $x^2 + y^2 = 25$



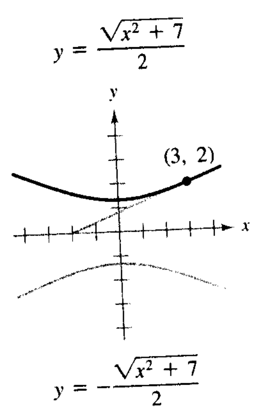
32. $9x^2 + 16y^2 = 144$



33. $x - y^2 - 1 = 0$



34. $4y^2 - x^2 = 7$



In Exercises 35–40, find equations of the tangent lines to the graph at the given points. Use a graphing utility to graph the equation and the tangent lines in the same viewing window.

	Equation	*Points*
35.	$x^2 + y^2 = 169$	$(5, 12)$ and $(-12, 5)$
36.	$x^2 + y^2 = 9$	$(0, 3)$ and $(2, \sqrt{5})$
37.	$y^2 = 5x^3$	$(1, \sqrt{5})$ and $(1, -\sqrt{5})$
38.	$4xy + x^2 = 5$	$(1, 1)$ and $(5, -1)$
39.	$x^3 + y^3 = 8$	$(0, 2)$ and $(2, 0)$
40.	$y^2 = \dfrac{x^3}{4 - x}$	$(2, 2)$ and $(2, -2)$

Demand In Exercises 41–44, find the rate of change of x with respect to p.

41. $p = 0.006x^4 + 0.02x^2 + 10, \quad x \geq 0$

42. $p = 0.002x^4 + 0.01x^2 + 5, \quad x \geq 0$

43. $p = \sqrt{\dfrac{200 - x}{2x}}, \quad 0 < x \leq 200$

44. $p = \sqrt{\dfrac{500 - x}{2x}}, \quad 0 < x \leq 500$

45. ***Production*** Let x represent the units of labor and y the capital invested in a manufacturing process. When 135,540 units are produced, the relationship between labor and capital can be modeled by $100x^{0.75}y^{0.25} = 135{,}540$.

(a) Find the rate of change of y with respect to x when $x = 1500$ and $y = 1000$.

(b) The model used in the problem is called the *Cobb-Douglas production function*. Graph the model on a graphing utility and describe the relationship between labor and capital.

46. ***Health: U.S. AIDS Epidemic*** The numbers (in millions) of cases y of AIDS reported in the years 1994 to 2001 can be modeled by

$$y^2 + 4436 = -4.2460t^4 + 146.821t^3 - 1728.00t^2 + 7456.6t$$

where $t = 4$ corresponds to 1994. *(Source: U.S. Centers for Disease Control and Prevention)*

(a) Use a graphing utility to graph the model and describe the results.

(b) Use the graph to determine the year during which the number of reported cases decreasing most rapidly.

(c) Complete the table to confirm your estimate.

t	4	5	6	7	8	9	10	11
y								
y'								

PREREQUISITE REVIEW 2.8

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, write a formula for the given quantity.

1. Area of a circle
2. Volume of a sphere
3. Surface area of a cube
4. Volume of a cube
5. Volume of a cone
6. Area of a triangle

In Exercises 7–10, find dy/dx by implicit differentiation.

7. $x^2 + y^2 = 9$
8. $3xy - x^2 = 6$
9. $x^2 + 2y + xy = 12$
10. $x + xy^2 - y^2 = xy$

EXERCISES 2.8

In Exercises 1–4, find the given values of dy/dt and dx/dt.

	Equation	*Find*	*Given*
1.	$y = x^2 - \sqrt{x}$	(a) $\dfrac{dy}{dt}$	$x = 4, \dfrac{dx}{dt} = 8$
		(b) $\dfrac{dx}{dt}$	$x = 16, \dfrac{dy}{dt} = 12$
2.	$y = x^2 - 3x$	(a) $\dfrac{dy}{dt}$	$x = 3, \dfrac{dx}{dt} = 2$
		(b) $\dfrac{dx}{dt}$	$x = 1, \dfrac{dy}{dt} = 5$
3.	$xy = 4$	(a) $\dfrac{dy}{dt}$	$x = 8, \dfrac{dx}{dt} = 10$
		(b) $\dfrac{dx}{dt}$	$x = 1, \dfrac{dy}{dt} = -6$
4.	$x^2 + y^2 = 25$	(a) $\dfrac{dy}{dt}$	$x = 3, y = 4, \dfrac{dx}{dt} = 8$
		(b) $\dfrac{dx}{dt}$	$x = 4, y = 3, \dfrac{dy}{dt} = -2$

5. ***Area*** The radius r of a circle is increasing at a rate of 2 inches per minute. Find the rates of change of the area when (a) $r = 6$ inches and (b) $r = 24$ inches.

6. ***Volume*** The radius r of a sphere is increasing at a rate of 2 inches per minute. Find the rates of change of the volume when (a) $r = 6$ inches and (b) $r = 24$ inches.

7. ***Area*** Let A be the area of a circle of radius r that is changing with respect to time. If dr/dt is constant, is dA/dt constant? Explain your reasoning.

8. ***Volume*** Let V be the volume of a sphere of radius r that is changing with respect to time. If dr/dt is constant, is dV/dt constant? Explain your reasoning.

9. ***Volume*** A spherical balloon is inflated with gas at a rate of 20 cubic feet per minute. How fast is the radius of the balloon changing at the instant the radius is (a) 1 foot and (b) 2 feet?

10. ***Volume*** The radius r of a right circular cone is increasing at a rate of 2 inches per minute. The height h of the cone is related to the radius by $h = 3r$. Find the rates of change of the volume when (a) $r = 6$ inches and (b) $r = 24$ inches.

11. ***Cost, Revenue, and Profit*** A company that manufactures sport supplements calculates that its costs and revenue can be modeled by the equations

$$C = 125{,}000 + 0.75x \quad \text{and} \quad R = 250x - \frac{1}{10}x^2$$

where x is the number of units of sport supplements produced in 1 week. If production in one particular week is 1000 units and is increasing at a rate of 150 units per week, find:

(a) the rate at which the cost is changing.

(b) the rate at which the revenue is changing.

(c) the rate at which the profit is changing.

12. ***Cost, Revenue, and Profit*** A company that manufactures pet toys calculates that its costs and revenue can be modeled by the equations

$$C = 75{,}000 + 1.05x \quad \text{and} \quad R = 500x - \frac{x^2}{25}$$

where x is the number of toys produced in 1 week. If production in one particular week is 5000 toys and is increasing at a rate of 250 toys per week, find:

(a) the rate at which the cost is changing.

(b) the rate at which the revenue is changing.

(c) the rate at which the profit is changing.

13. ***Expanding Cube*** a rate of 3 centimeter changing when each centimeters?

14. ***Expanding Cube*** at a rate of 3 centimet area changing when (b) 10 centimeters?

15. ***Moving Point*** A $y = x^2$ such that $dx/$ dy/dt for each value

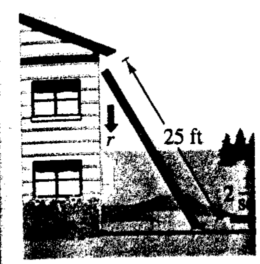
(a) $x = -3$ (b) x

16. ***Moving Point*** A $y = 1/(1 + x^2)$ such t Find dy/dt for each v

(a) $x = -2$ (b) x

17. ***Speed*** A 25-foot la figure). The base of house at a rate of 2 f the ladder moving do (b) 15 feet, and (c) 24

Figure for 17



18. ***Speed*** A boat is p winch is 12 feet abov winch pulls the rope speed of the boat whe to the speed of the b dock?

19. ***Air Traffic Contro*** airplanes at the same fly at right angles to from the point and h other is 200 miles 600 miles per hour.

(a) At what rate i changing?

(b) How much time the airplanes on

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13. ***Expanding Cube*** All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is (a) 1 centimeter and (b) 10 centimeters?

14. ***Expanding Cube*** All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the surface area changing when each edge is (a) 1 centimeter and (b) 10 centimeters?

15. ***Moving Point*** A point is moving along the graph of $y = x^2$ such that dx/dt is 2 centimeters per minute. Find dy/dt for each value of x.

(a) $x = -3$ (b) $x = 0$ (c) $x = 1$ (d) $x = 3$

16. ***Moving Point*** A point is moving along the graph of $y = 1/(1 + x^2)$ such that dx/dt is 2 centimeters per minute. Find dy/dt for each value of x.

(a) $x = -2$ (b) $x = 2$ (c) $x = 0$ (d) $x = 10$

17. ***Speed*** A 25-foot ladder is leaning against a house (see figure). The base of the ladder is pulled away from the house at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when the base is (a) 7 feet, (b) 15 feet, and (c) 24 feet from the house?

Figure for 17

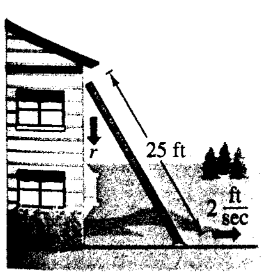
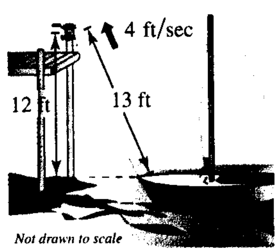


Figure for 18



18. ***Speed*** A boat is pulled by a winch on a dock, and the winch is 12 feet above the deck of the boat (see figure). The winch pulls the rope at a rate of 4 feet per second. Find the speed of the boat when 13 feet of rope is out. What happens to the speed of the boat as it gets closer and closer to the dock?

19. ***Air Traffic Control*** An air traffic controller spots two airplanes at the same altitude converging to a point as they fly at right angles to each other. One airplane is 150 miles from the point and has a speed of 450 miles per hour. The other is 200 miles from the point and has a speed of 600 miles per hour.

(a) At what rate is the distance between the planes changing?

(b) How much time does the controller have to get one of the airplanes on a different flight path?

20. ***Speed*** An airplane flying at an altitude of 6 miles passes directly over a radar antenna (see figure). When the airplane is 10 miles away ($s = 10$), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the airplane?

Figure for 20

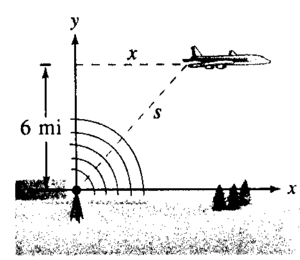
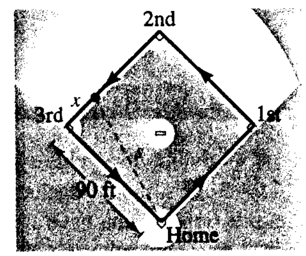


Figure for 21



21. ***Athletics*** A (square) baseball diamond has sides that are 90 feet long (see figure). A player 26 feet from third base is running at a speed of 30 feet per second. At what rate is the player's distance from home plate changing?

22. ***Advertising Costs*** A retail sporting goods store estimates that weekly sales S and weekly advertising costs x are related by the equation $S = 2250 + 50x + 0.35x^2$. The current weekly advertising costs are \$1500, and these costs are increasing at a rate of \$125 per week. Find the current rate of change of weekly sales.

23. ***Environment*** An accident at an oil drilling platform is causing a circular oil slick. The slick is 0.08 foot thick, and when the radius is 750 feet, the radius of the slick is increasing at the rate of 0.5 foot per minute. At what rate (in cubic feet per minute) is oil flowing from the site of the accident?

24. ***Profit*** A company is increasing the production of a product at the rate of 25 units per week. The demand and cost functions for the product are given by $p = 50 - 0.01x$ and $C = 4000 + 40x - 0.02x^2$. Find the rate of change of the profit with respect to time when the weekly sales are $x = 800$ units. Use a graphing utility to graph the profit function, and use the *zoom* and *trace* features of the graphing utility to verify your result.

25. ***Sales*** The profit for a product is increasing at a rate of \$6384 per week. The demand and cost functions for the product are given by $p = 6000 - 0.4x^2$ and $C = 2400x + 5200$. Find the rate of change of sales with respect to time when the weekly sales are $x = 44$ units.

26. ***Cost*** The annual cost (in millions of dollars) for a government agency to seize $p\%$ of an illegal drug is given by

$$C = \frac{528p}{100 - p}, \qquad 0 \le p < 100.$$

The agency's goal is to increase p by 5% per year. Find the rates of change of the cost when (a) $p = 30\%$ and (b) $p = 60\%$. Use a graphing utility to graph C. What happens to the graph of C as p approaches 100?

PREREQUISITE REVIEW 3.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, solve the equation.

1. $x^2 = 8x$

2. $15x = \frac{5}{8}x^2$

3. $\frac{x^2 - 25}{x^3} = 0$

4. $\frac{2x}{\sqrt{1 - x^2}} = 0$

In Exercises 5–8, find the domain of the expression.

5. $\frac{x + 3}{x - 3}$

6. $\frac{2}{\sqrt{1 - x}}$

7. $\frac{2x + 1}{x^2 - 3x - 10}$

8. $\frac{3x}{\sqrt{9 - 3x^2}}$

In Exercises 9–12, evaluate the expression when $x = -2$, 0, and 2.

9. $-2(x + 1)(x - 1)$

10. $4(2x + 1)(2x - 1)$

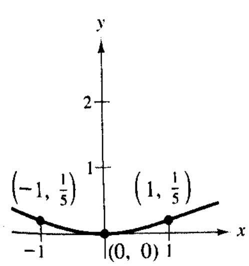
11. $\frac{2x + 1}{(x - 1)^2}$

12. $\frac{-2(x + 1)}{(x - 4)^2}$

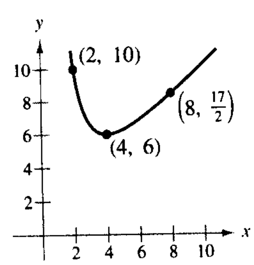
EXERCISES 3.1

In Exercises 1–4, evaluate the derivative of the function at the indicated points on the graph.

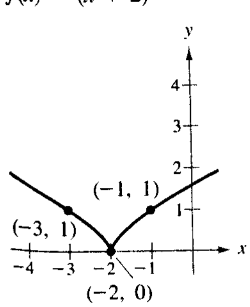
1. $f(x) = \frac{x^2}{x^2 + 4}$



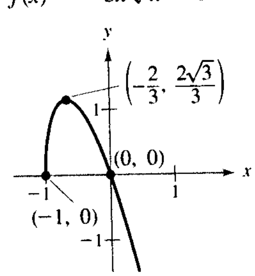
2. $f(x) = x + \frac{32}{x^2}$



3. $f(x) = (x + 2)^{2/3}$

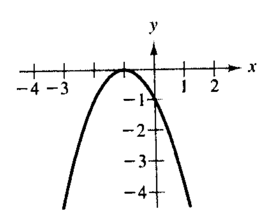


4. $f(x) = -3x\sqrt{x + 1}$

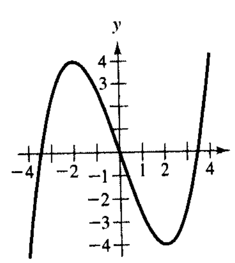


In Exercises 5–8, use the derivative to identify the open intervals on which the function is increasing or decreasing. Verify your result with the graph of the function.

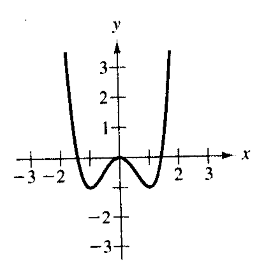
5. $f(x) = -(x + 1)^2$



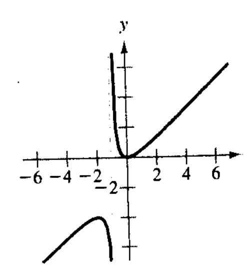
6. $f(x) = \frac{x^3}{4} - 3x$



7. $f(x) = x^4 - 2x^2$



8. $f(x) = \frac{x^2}{x + 1}$



In Exercises 9–18, find the critical numbers and the open intervals on which the function is increasing or decreasing. Sketch the graph of the function.

9. $f(x) = 2x - 3$ **10.** $f(x) = 5 - 3x$

11. $g(x) = -(x - 1)^2$ **12.** $g(x) = (x + 2)^2$

13. $y = x^2 - 5x$ **14.** $y = -x^2 + 2x$

15. $y = x^3 - 6x^2$ **16.** $y = (x - 2)^3$

17. $f(x) = \sqrt{x^2 - 1}$ **18.** $f(x) = \sqrt{4 - x^2}$

In Exercises 19–28, find the critical numbers and the open intervals on which the function is increasing or decreasing. Then use a graphing utility to graph the function.

19. $f(x) = -2x^2 + 4x + 3$ **20.** $f(x) = x^2 + 8x + 10$

21. $y = 3x^3 + 12x^2 + 15x$ **22.** $y = x^3 - 3x + 2$

23. $f(x) = x\sqrt{x + 1}$ **24.** $h(x) = x\sqrt[3]{x - 1}$

25. $f(x) = x^4 - 2x^3$ **26.** $f(x) = \frac{1}{4}x^4 - 2x^2$

27. $f(x) = \dfrac{x}{x^2 + 4}$ **28.** $f(x) = \dfrac{x^2}{x^2 + 4}$

In Exercises 29–34, find the critical numbers and the open intervals on which the function is increasing or decreasing. (*Hint:* Check for discontinuities.) Sketch the graph of the function.

29. $f(x) = \dfrac{2x}{16 - x^2}$

30. $f(x) = \dfrac{x}{x + 1}$

31. $y = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$

32. $y = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

33. $y = \begin{cases} 3x + 1, & x \le 1 \\ 5 - x^2, & x > 1 \end{cases}$

34. $y = \begin{cases} -x^3 + 1, & x \le 0 \\ -x^2 + 2x, & x > 0 \end{cases}$

35. ***Cost*** The ordering and transportation cost C (in hundreds of dollars) for an automobile dealership is modeled by

$$C = 10\left(\frac{1}{x} + \frac{x}{x + 3}\right), \quad 1 \le x$$

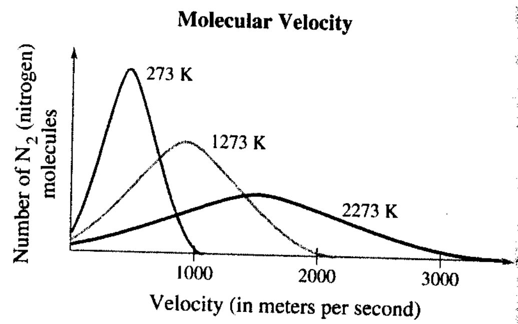
where x is the number of automobiles ordered.

(a) Find the intervals on which C is increasing or decreasing.

(b) Use a graphing utility to graph the cost function.

(c) Use the *trace* feature to determine the order sizes for which the cost is \$900. Assuming that the revenue function is increasing for $x \ge 0$, which order size would you use? Explain your reasoning.

36. ***Chemistry: Molecular Velocity*** Plots of the rel numbers of N_2 (nitrogen) molecules that have a g velocity at each of three temperatures (in degrees Ke are shown in the figure. Identify the differences in the age velocities (indicated by the peaks of the curves) fo three temperatures, and describe the intervals on which velocity is increasing and decreasing for each of the t temperatures. (*Source: Adapted from Zum Chemistry, Sixth Edition*)



Position Function In Exercises 37 and 38, the posi function gives the height s (in feet) of a ball, where the time measured in seconds. Find the time interval on which the is rising and the interval on which it is falling.

37. $s = 96t - 16t^2, \quad 0 \le t \le 6$

38. $s = -16t^2 + 64t, \quad 0 \le t \le 4$

39. ***Law Degrees*** The number y of law degrees conferred the United States from 1970 to 2000 can be modeled by

$$y = 2.743t^3 - 171.55t^2 + 3462.3t + 15{,}265,$$
$$0 \le t \le 30$$

where t is the time in years, with $t = 0$ corresponding 1970. (*Source: U.S. National Center for Educati Statistics*)

(a) Use a graphing utility to graph the model. Then grap ically estimate the years during which the model increasing and the years during which it is decreasin

(b) Use the test for increasing and decreasing functions verify the result of part (a).

40. ***Profit*** The profit P made by a cinema from selling x bag of popcorn can be modeled by

$$P = 2.36x - \frac{x^2}{25{,}000} - 3500, \quad 0 \le x \le 50{,}000.$$

(a) Find the intervals on which P is increasing an decreasing.

(b) If you owned the cinema, what price would you charg to obtain a maximum profit for popcorn? Explain you reasoning.

3.2

elative Extrema

ou have used the deriv creasing or decreasing. nction changes from in nction has a **relative e** ive **extrema** of a functi e function. For instanc trema—the left point inimum.

Definition of Relati

Let f be a function d

1. $f(c)$ is a **relative** containing c such

2. $f(c)$ is a **relative** containing c such

If $f(c)$ is a relative e *occur* at $x = c$.

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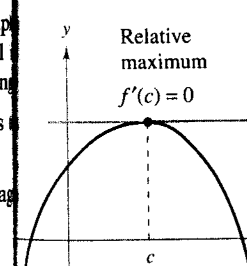


FIGURE 3.11

Occurrence of Rela

If f has a relative mi critical number of f.

PREREQUISITE REVIEW 3.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, solve the equation $f'(x) = 0$.

1. $f(x) = 4x^4 - 2x^2 + 1$
2. $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 10x$
3. $f(x) = 5x^{4/5} - 4x$
4. $f(x) = \frac{1}{2}x^2 - 3x^{5/3}$
5. $f(x) = \dfrac{x+4}{x^2+1}$
6. $f(x) = \dfrac{x-1}{x^2+4}$

In Exercises 7–10, use $g(x) = -x^5 - 2x^4 + 4x^3 + 2x - 1$ to determine the sign of the derivative.

7. $g'(-4)$
8. $g'(0)$
9. $g'(1)$
10. $g'(3)$

In Exercises 11 and 12, decide whether the function is increasing or decreasing on the given interval.

11. $f(x) = 2x^2 - 11x - 6, \quad (3, 6)$
12. $f(x) = x^3 + 2x^2 - 4x - 8, \quad (-2, 0)$

EXERCISES 3.2

In Exercises 1–4, use a table similar to that in Example 1 to find all relative extrema of the function.

1. $f(x) = -2x^2 + 4x + 3$
2. $f(x) = x^2 + 8x + 10$
3. $f(x) = x^2 - 6x$
4. $f(x) = -4x^2 + 4x + 1$

In Exercises 5–12, find all relative extrema of the function.

5. $g(x) = 6x^3 - 15x^2 + 12x$
6. $g(x) = \frac{1}{5}x^5 - x$
7. $h(x) = -(x+4)^3$
8. $h(x) = 2(x-3)^3$
9. $f(x) = x^3 - 6x^2 + 15$
10. $f(x) = x^4 - 32x + 4$
11. $f(x) = x^4 - 2x^3$
12. $f(x) = x^4 - 12x^3$

In Exercises 13–18, use a graphing utility to graph the function. Then find all relative extrema of the function.

13. $f(x) = (x-1)^{2/3}$
14. $f(t) = (t-1)^{1/3}$
15. $g(t) = t - \dfrac{1}{2t^2}$
16. $f(x) = x + \dfrac{1}{x}$
17. $f(x) = \dfrac{x}{x+1}$
18. $h(x) = \dfrac{4}{x^2+1}$

In Exercises 19–26, find the absolute extrema of the function on the closed interval.

	Function	*Interval*
19.	$f(x) = 2(3 - x)$	$[-1, 2]$
20.	$f(x) = \frac{1}{3}(2x + 5)$	$[0, 5]$
21.	$f(x) = 5 - 2x^2$	$[0, 3]$
22.	$f(x) = x^2 + 2x - 4$	$[-1, 1]$
23.	$f(x) = x^3 - 3x^2$	$[-1, 3]$
24.	$f(x) = x^3 - 12x$	$[0, 4]$
25.	$h(s) = \dfrac{1}{3-s}$	$[0, 2]$
26.	$h(t) = \dfrac{t}{t-2}$	$[3, 5]$

In Exercises 27–30, find the absolute extrema of the function on the closed interval. Use a graphing utility to verify your results.

	Function	*Interval*
27.	$f(x) = 3x^{2/3} - 2x$	$[-1, 2]$
28.	$g(t) = \dfrac{t^2}{t^2+3}$	$[-1, 1]$
29.	$h(t) = (t-1)^{2/3}$	$[-7, 2]$
30.	$g(x) = 4\left(1 + \dfrac{1}{x} + \dfrac{1}{x^2}\right)$	$[-4, 5]$

In Exercises 31–34, use a graphing utility to find graphically the absolute extrema of the function on the closed interval.

	Function	*Interval*
31.	$f(x) = 0.4x^3 - 1.8x^2 + x - 3$	$[0, 5]$
32.	$f(x) = 3.2x^5 + 5x^3 - 3.5x$	$[0, 1]$
33.	$f(x) = \frac{4}{3}x\sqrt{3-x}$	$[0, 3]$
34.	$f(x) = 4\sqrt{x} - 2x + 1$	$[0, 6]$

Exercises 35–38, find the absolute extrema of the function on e interval $[0, \infty)$.

5. $f(x) = \dfrac{4x}{x^2 + 1}$

6. $f(x) = \dfrac{8}{x + 1}$

7. $f(x) = \dfrac{2x}{x^2 + 4}$

8. $f(x) = 8 - \dfrac{4x}{x^2 + 1}$

Exercises 39 and 40, find the maximum value of $|f''(x)|$ on the losed interval. (You will use this skill in Section 6.5 to estimate he error in the Trapezoidal Rule.)

Function	*Interval*
9. $f(x) = x^3(3x^2 - 10)$	$[0, 1]$
0. $f(x) = \dfrac{1}{x^2 + 1}$	$[0, 3]$

n Exercises 41 and 42, find the maximum value of $|f^{(4)}(x)|$ on the losed interval. (You will use this skill in Section 6.5 to estimate he error in Simpson's Rule.) Use a graphing utility to verify your nswer.

Function	*Interval*
41. $f(x) = 15x^4 - \left(\dfrac{2x - 1}{2}\right)^6$	$[0, 1]$
42. $f(x) = \dfrac{1}{x^2}$	$[1, 2]$

43. ***Cost*** A retailer has determined the cost C for ordering and storing x units of a product to be modeled by

$$C = 3x + \frac{20{,}000}{x}, \quad 0 < x \le 200.$$

The delivery truck can bring at most 200 units per order. Find the order size that will minimize the cost. Use a graphing utility to verify your result.

44. ***Profit*** The quantity demanded x for a product is inversely proportional to the cube of the price p for $p > 1$. When the price is \$10 per unit, the quantity demanded is eight units. The initial cost is \$100 and the cost per unit is \$4. What price will yield a maximum profit?

45. ***Profit*** When soft drinks were sold for \$0.80 per can at football games, approximately 6000 cans were sold. When the price was raised to \$1.00 per can, the quantity demanded dropped to 5600. The initial cost is \$5000 and the cost per unit is \$0.40. Assuming that the demand function is linear, use the *table* feature of a graphing utility to determine the price that will yield a maximum profit.

46. ***Medical Science*** Coughing forces the trachea (windpipe) to contract, which in turn affects the velocity of the air through the trachea. The velocity of the air during coughing can be modeled by

$$v = k(R - r)r^2, \quad 0 \le r < R$$

where k is a constant, R is the normal radius of the trachea, and r is the radius during coughing. What radius r will produce the maximum air velocity?

47. ***Population*** The resident population P (in millions) of the United States from 1790 to 2000 can be modeled by

$$P = 0.00000583t^3 + 0.005003t^2 + 0.13775t + 4.658,$$
$$-10 \le t \le 200$$

where $t = 0$ corresponds to 1800. *(Source: U.S. Census Bureau)*

(a) Make a conjecture about the maximum and minimum populations in the U.S. from 1790 to 2000.

(b) Analytically find the maximum and minimum populations over the interval.

(c) Write a brief paragraph comparing your conjecture with your results in part (b).

48. ***Biology: Fertility Rates*** The graph of the United States fertility rate shows the number of births per 1000 women in their lifetime according to the birth rate in that particular year. *(Source: U.S. National Center for Health Statistics)*

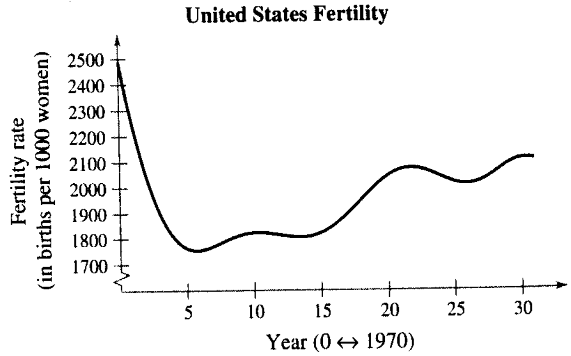
(a) Around what year was the fertility rate the highest, and to how many births per 1000 women did this rate correspond?

(b) During which time periods was the fertility rate increasing most rapidly? Most slowly?

(c) During which time periods was the fertility rate decreasing most rapidly? Most slowly?

(d) Give some possible real-life reasons for fluctuations in the fertility rate.

United States Fertility



3.3 C

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c
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■ F

Concavity

You already know that locat decreases is helpful in dete locating the intervals on whi graph of f is curving upwa upward or downward is def function.

Definition of Concavity

Let f be differentiable o

1. **concave upward** on

2. **concave downward**

From Figure 3.20, you concavity.

1. A curve that is concave u

2. A curve that is concave c

This visual test for concavit determine concavity without that you can use the second same way that you use the f increasing or decreasing.

Test for Concavity

Let f be a function whos

1. If $f''(x) > 0$ for all x

2. If $f''(x) < 0$ for all x

PREREQUISITE REVIEW 3.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find the second derivative of the function.

1. $f(x) = 4x^4 - 9x^3 + 5x - 1$
2. $g(s) = (s^2 - 1)(s^2 - 3s + 2)$
3. $g(x) = (x^2 + 1)^4$
4. $f(x) = (x - 3)^{4/3}$
5. $h(x) = \dfrac{4x + 3}{5x - 1}$
6. $f(x) = \dfrac{2x - 1}{3x + 2}$

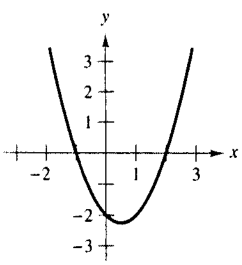
In Exercises 7–10, find the critical numbers of the function.

7. $f(x) = 5x^3 - 5x + 11$
8. $f(x) = x^4 - 4x^3 - 10$
9. $g(t) = \dfrac{16 + t^2}{t}$
10. $h(x) = \dfrac{x^4 - 50x^2}{8}$

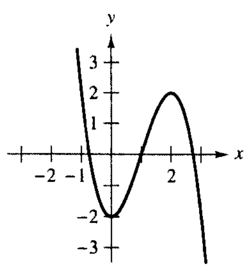
EXERCISES 3.3

In Exercises 1–8, analytically find the intervals on which the graph is concave upward and those on which it is concave downward. Verify your results using the graph of the function.

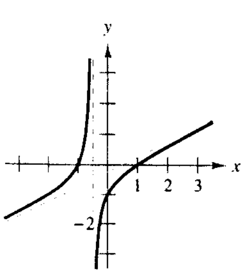
1. $y = x^2 - x - 2$



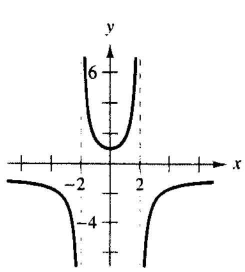
2. $y = -x^3 + 3x^2 - 2$



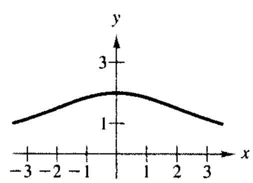
3. $f(x) = \dfrac{x^2 - 1}{2x + 1}$



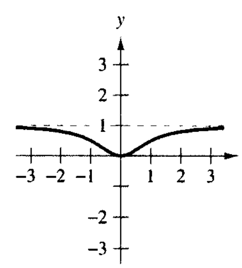
4. $f(x) = \dfrac{x^2 + 4}{4 - x^2}$



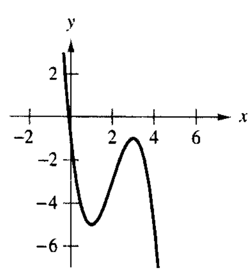
5. $f(x) = \dfrac{24}{x^2 + 12}$



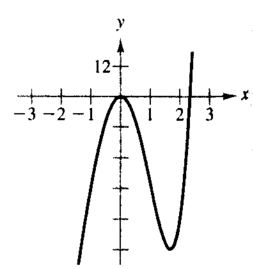
6. $f(x) = \dfrac{x^2}{x^2 + 1}$



7. $y = -x^3 + 6x^2 - 9x - 1$



8. $y = x^5 + 5x^4 - 40x^2$



In Exercises 9–18, find all rel
the Second-Derivative Test w

9. $f(x) = 6x - x^2$

11. $f(x) = x^3 - 5x^2 + 7x$

13. $f(x) = x^{2/3} - 3$

15. $f(x) = \sqrt{x^2 + 1}$

17. $f(x) = \dfrac{x}{x - 1}$

In Exercises 19–22, use a gra
all relative extrema of the fur

19. $f(x) = \frac{1}{2}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2$

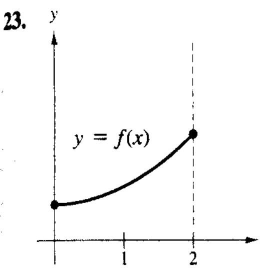
20. $f(x) = -\frac{1}{3}x^5 - \frac{1}{2}x^4 + x$

21. $f(x) = 5 + 3x^2 - x^3$

22. $f(x) = 3x^3 + 5x^2 - 2$

In Exercises 23–26, state th
interval (0, 2).

23.



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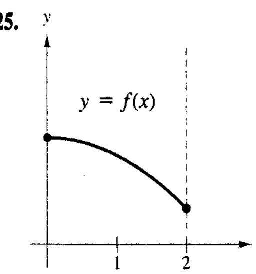


In Exercises 27–34, find the p
the function.

27. $f(x) = x^3 - 9x^2 + 24x$

28. $f(x) = x(6 - x)^2$

29. $f(x) = (x - 1)^3(x - 5)$

30. $f(x) = x^4 - 18x^2 + 5$

31. $g(x) = 2x^4 - 8x^3 + 12$

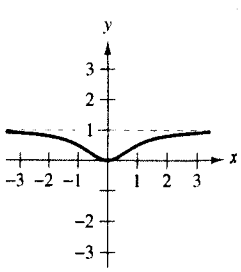
32. $f(x) = -4x^3 - 8x^2 + 3$

33. $h(x) = (x - 2)^3(x - 1)$

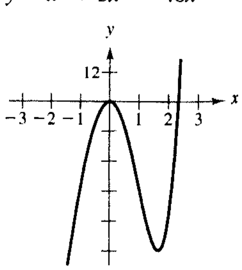
34. $f(t) = (1 - t)(t - 4)(t^2$

rlier sections. You will

$f(x) = \dfrac{x^2}{x^2 + 1}$



$y = x^5 + 5x^4 - 40x^2$



In Exercises 9–18, find all relative extrema of the function. Use the Second-Derivative Test when applicable.

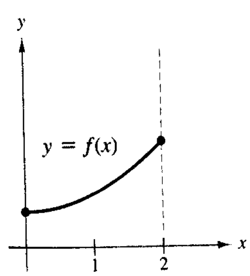
9. $f(x) = 6x - x^2$
10. $f(x) = (x - 5)^2$
11. $f(x) = x^3 - 5x^2 + 7x$
12. $f(x) = x^4 - 4x^3 + 2$
13. $f(x) = x^{2/3} - 3$
14. $f(x) = x + \dfrac{4}{x}$
15. $f(x) = \sqrt{x^2 + 1}$
16. $f(x) = \sqrt{4 - x^2}$
17. $f(x) = \dfrac{x}{x - 1}$
18. $f(x) = \dfrac{x}{x^2 - 1}$

In Exercises 19–22, use a graphing utility to estimate graphically all relative extrema of the function.

19. $f(x) = \frac{1}{2}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2$
20. $f(x) = -\frac{1}{3}x^5 - \frac{1}{2}x^4 + x$
21. $f(x) = 5 + 3x^2 - x^3$
22. $f(x) = 3x^3 + 5x^2 - 2$

In Exercises 23–26, state the signs of $f'(x)$ and $f''(x)$ on the interval $(0, 2)$.

23.



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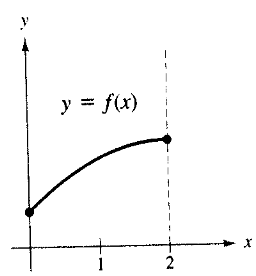

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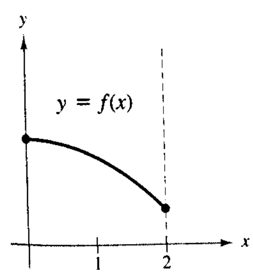

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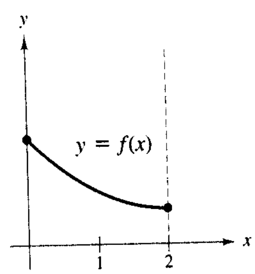


In Exercises 27–34, find the point(s) of inflection of the graph of the function.

27. $f(x) = x^3 - 9x^2 + 24x - 18$
28. $f(x) = x(6 - x)^2$
29. $f(x) = (x - 1)^3(x - 5)$
30. $f(x) = x^4 - 18x^2 + 5$
31. $g(x) = 2x^4 - 8x^3 + 12x^2 + 12x$
32. $f(x) = -4x^3 - 8x^2 + 32$
33. $h(x) = (x - 2)^3(x - 1)$
34. $f(t) = (1 - t)(t - 4)(t^2 - 4)$

In Exercises 35–46, use a graphing utility to graph the function and identify all relative extrema and points of inflection.

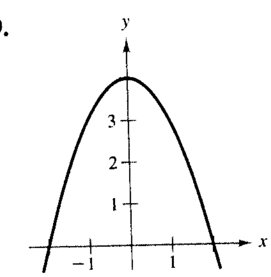
35. $f(x) = x^3 - 12x$
36. $f(x) = x^3 - 3x$
37. $f(x) = x^3 - 6x^2 + 12x$
38. $f(x) = x^3 - \frac{3}{2}x^2 - 6x$
39. $f(x) = \frac{1}{4}x^4 - 2x^2$
40. $f(x) = 2x^4 - 8x + 3$
41. $g(x) = (x - 2)(x + 1)^2$
42. $g(x) = (x - 6)(x + 2)^3$
43. $g(x) = x\sqrt{x + 3}$
44. $g(x) = x\sqrt{9 - x}$
45. $f(x) = \dfrac{4}{1 + x^2}$
46. $f(x) = \dfrac{2}{x^2 - 1}$

In Exercises 47 and 48, sketch a graph of a function f having the given characteristics.

	Function	First Derivative	Second Derivative
47.	$f(2) = 0$	$f'(x) < 0,\ x < 3$	$f''(x) > 0$
	$f(4) = 0$	$f'(3) = 0$	
		$f'(x) > 0,\ x > 3$	
48.	$f(2) = 0$	$f'(x) > 0,\ x < 3$	$f''(x) > 0,\ x \neq 3$
	$f(4) = 0$	$f'(3)$ is undefined.	
		$f'(x) < 0,\ x > 3$	

In Exercises 49 and 50, use the graph to sketch the graph of f'. Find the intervals on which (a) $f'(x)$ is positive, (b) $f'(x)$ is negative, (c) f' is increasing, and (d) f' is decreasing. For each of these intervals, describe the corresponding behavior of f.

49.



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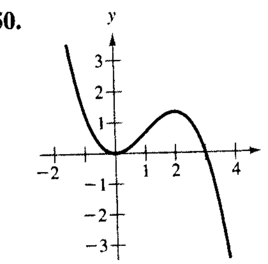


Point of Diminishing Returns In Exercises 51 and 52, identify the point of diminishing returns for the input-output function. For each function, R is the revenue and x is the amount spent on advertising. Use a graphing utility to verify your results.

51. $R = \dfrac{1}{50{,}000}(600x^2 - x^3), \quad 0 \le x \le 400$
52. $R = -\frac{4}{9}(x^3 - 9x^2 - 27), \quad 0 \le x \le 5$

Average Cost In Exercises 53 and 54, you are given the total cost of producing x units. Find the production level that minimizes the average cost per unit. Use a graphing utility to verify your results.

53. $C = 0.5x^2 + 15x + 5000$
54. $C = 0.002x^3 + 20x + 500$

PREREQUISITE REVIEW 3.7

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the vertical and horizontal asymptotes of the graph.

1. $f(x) = \dfrac{1}{x^2}$
2. $f(x) = \dfrac{8}{(x-2)^2}$
3. $f(x) = \dfrac{40x}{x+3}$
4. $f(x) = \dfrac{x^2-3}{x^2-4x+3}$

In Exercises 5–10, determine the open intervals on which the function is increasing or decreasing.

5. $f(x) = x^2 + 4x + 2$
6. $f(x) = -x^2 - 8x + 1$
7. $f(x) = x^3 - 3x + 1$
8. $f(x) = \dfrac{-x^3 + x^2 - 1}{x^2}$
9. $f(x) = \dfrac{x-2}{x-1}$
10. $f(x) = -x^3 - 4x^2 + 3x + 2$

EXERCISES 3.7

In Exercises 1–20, sketch the graph of the function. Choose a scale that allows all relative extrema and points of inflection to be identified on the graph.

1. $y = -x^2 - 2x + 3$
2. $y = 2x^2 - 4x + 1$
3. $y = x^3 - 4x^2 + 6$
4. $y = -\frac{1}{3}(x^3 - 3x + 2)$
5. $y = 2 - x - x^3$
6. $y = x^3 + 3x^2 + 3x + 2$
7. $y = 3x^3 - 9x + 1$
8. $y = -4x^3 + 6x^2$
9. $y = 3x^4 + 4x^3$
10. $y = 3x^4 - 6x^2$
11. $y = x^3 - 6x^2 + 3x + 10$
12. $y = -x^3 + 3x^2 + 9x - 2$
13. $y = x^4 - 8x^3 + 18x^2 - 16x + 5$
14. $y = x^4 - 4x^3 + 16x - 16$
15. $y = x^4 - 4x^3 + 16x$
16. $y = x^5 + 1$
17. $y = x^5 - 5x$
18. $y = (x-1)^5$
19. $y = \begin{cases} x^2 + 1, & x \le 0 \\ 1 - 2x, & x > 0 \end{cases}$
20. $y = \begin{cases} x^2 + 4, & x < 0 \\ 4 - x, & x \ge 0 \end{cases}$

In Exercises 21–32, use a graphing utility to graph the functio[n]. Choose a window that allows all relative extrema and points [of] inflection to be identified on the graph.

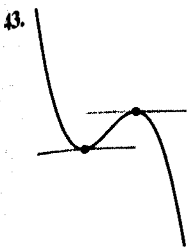
21. $y = \dfrac{x^2+2}{x^2+1}$
22. $y = \dfrac{x}{x^2+1}$
23. $y = 3x^{2/3} - 2x$
24. $y = 3x^{2/3} - x^2$
25. $y = 1 - x^{2/3}$
26. $y = (1-x)^{2/3}$
27. $y = x^{1/3} + 1$
28. $y = x^{-1/3}$
29. $y = x^{5/3} - 5x^{2/3}$
30. $y = x^{4/3}$
31. $y = x\sqrt{x^2-9}$
32. $y = \dfrac{x}{\sqrt{x^2-4}}$

In Exercises 33–42, sketch the graph of the function. Label th[e] intercepts, relative extrema, points of inflection, and asymptote[s]. Then state the domain of the function.

33. $y = \dfrac{5-3x}{x-2}$
34. $y = \dfrac{x^2+1}{x^2-2}$
35. $y = \dfrac{2x}{x^2-1}$
36. $y = \dfrac{x^2-6x+12}{x-4}$
37. $y = x\sqrt{4-x}$
38. $y = x\sqrt{4-x^2}$
39. $y = \dfrac{x-3}{x}$
40. $y = x + \dfrac{32}{x^2}$
41. $y = \dfrac{x^3}{x^3-1}$
42. $y = \dfrac{x^4}{x^4-1}$

In Exercises 43–46, find value graph of $f(x) = ax^3 + bx^2 +$ graph. Then use a graphing uti many correct answers.)

43.



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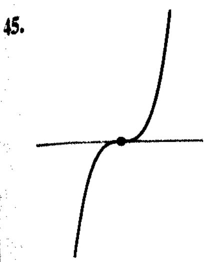
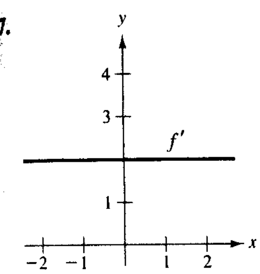


In Exercises 47–50, use the gra of f. (There are many correct a

47.



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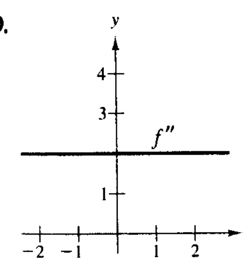


In Exercises 51 and 52, sketch given characteristics. (There ar

51. $f(-2) = 0$

$f(0) = 0$

$f'(x) > 0, \quad -\infty < x <$

$f'(-1) = 0$

$f'(x) < 0, \quad -1 < x < 0$

$f'(0) = 0$

$f'(x) > 0, \quad 0 < x < \infty$

er sections. You will

ility to graph the functio
tive extrema and points o
h.

$y = \dfrac{x}{x^2 + 1}$

$y = 3x^{2/3} - x^2$

$y = (1 - x)^{2/3}$

$y = x^{-1/3}$

$y = x^{4/3}$

$y = \dfrac{x}{\sqrt{x^2 - 4}}$

of the function. Label th
inflection, and asymptote

$y = \dfrac{x^2 + 1}{x^2 - 2}$

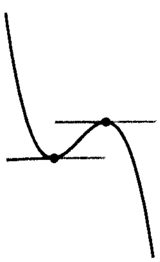
$y = \dfrac{x^2 - 6x + 12}{x - 4}$

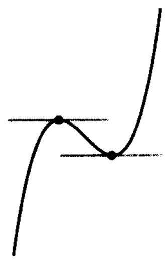
$y = x\sqrt{4 - x^2}$

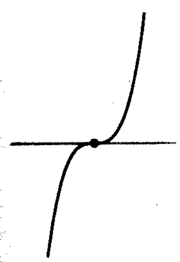
$y = x + \dfrac{32}{x^2}$

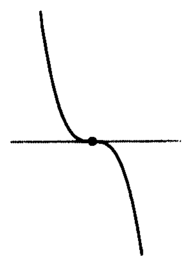
$y = \dfrac{x^4}{x^4 - 1}$

In Exercises 43–46, find values of a, b, c, and d such that the graph of $f(x) = ax^3 + bx^2 + cx + d$ will resemble the given graph. Then use a graphing utility to verify your result. (There are many correct answers.)

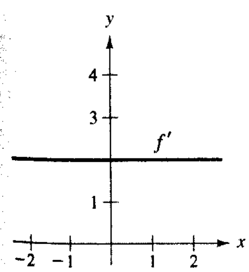
43. 

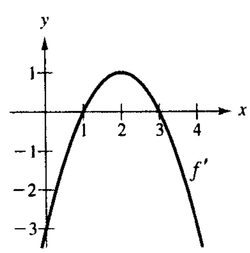
44. 

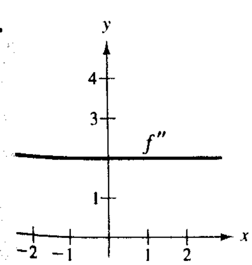
45. 

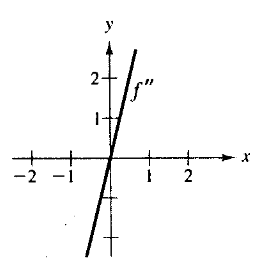
46. 

In Exercises 47–50, use the graph of f' or f'' to sketch the graph of f. (There are many correct answers.)

47. 

48. 

49. 

50. 

In Exercises 51 and 52, sketch a graph of a function f having the given characteristics. (There are many correct answers.)

51. $f(-2) = 0$

$f(0) = 0$

$f'(x) > 0, \quad -\infty < x < -1$

$f'(-1) = 0$

$f'(x) < 0, \quad -1 < x < 0$

$f'(0) = 0$

$f'(x) > 0, \quad 0 < x < \infty$

52. $f(-1) = 0$

$f(3) = 0$

$f'(1)$ is undefined.

$f'(x) < 0, \quad -\infty < x < 1$

$f'(x) > 0, \quad 1 < x < \infty$

$f''(x) < 0, \quad x \neq 1$

$\lim_{x \to \infty} f(x) = 4$

53. ***Cost*** An employee of a delivery company earns \$9 per hour driving a delivery van in an area where gasoline costs \$1.80 per gallon. When the van is driven at a constant speed s (in miles per hour, with $40 \leq s \leq 65$), the van gets $500/s$ miles per gallon.

(a) Find the cost C as a function of s for a 100-mile trip on an interstate highway.

(b) Use a graphing utility to graph the function found in part (a) and determine the most economical speed.

54. ***Profit*** The management of a company is considering three possible models for predicting the company's profits from 2001 through 2006. Model I gives the expected annual profits if the current trends continue. Models II and III give the expected annual profits for various combinations of increased labor and energy costs. In each model, p is the profit (in billions of dollars) and $t = 0$ corresponds to 2001.

Model I: $p = 0.03t^2 - 0.01t + 3.39$

Model II: $p = 0.08t + 3.36$

Model III: $p = -0.07t^2 + 0.05t + 3.38$

(a) Use a graphing utility to graph all three models in the same viewing window.

(b) For which models are profits increasing during the interval from 2001 through 2006?

(c) Which model is the most optimistic? Which is the most pessimistic?

55. ***Meteorology*** The monthly normal temperature T (in degrees Fahrenheit) for Pittsburgh, Pennsylvania can be modeled by

$$T = \frac{23.011 - 1.0t + 0.048t^2}{1 - 0.204t + 0.014t^2}, \quad 1 \leq t \leq 12$$

where t is the month, with $t = 1$ corresponding to January. Use a graphing utility to graph the model and find all absolute extrema. Explain the meaning of those values. *(Source: National Climatic Data Center)*

Writing In Exercises 56 and 57, use a graphing utility to graph the function. Explain why there is no vertical asymptote when a superficial examination of the function may indicate that there should be one.

56. $h(x) = \dfrac{6 - 2x}{3 - x}$

57. $g(x) = \dfrac{x^2 + x - 2}{x - 1}$

PREREQUISITE REVIEW 8.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the derivative of the function.

1. $f(x) = 3x^3 - 2x^2 + 4x - 7$

2. $g(x) = (x^3 + 4)^4$

3. $f(x) = (x - 1)(x^2 + 2x + 3)$

4. $g(x) = \dfrac{2x}{x^2 + 5}$

In Exercises 5 and 6, find the relative extrema of the function.

5. $f(x) = x^2 + 4x + 1$

6. $f(x) = \frac{1}{3}x^3 - 4x + 2$

In Exercises 7–10, solve the trigonometric equation for x where $0 \le x \le 2\pi$.

7. $\sin x = \dfrac{\sqrt{3}}{2}$

8. $\cos x = -\dfrac{1}{2}$

9. $\cos \dfrac{x}{2} = 0$

10. $\sin \dfrac{x}{2} = -\dfrac{\sqrt{2}}{2}$

EXERCISES 8.4

In Exercises 1–26, find the derivative of the function.

1. $y = \frac{1}{2} - 3 \sin x$

2. $y = 5 + \sin x$

3. $y = x^2 - \cos x$

4. $g(t) = \pi \cos t - \dfrac{1}{t^2}$

5. $f(x) = 4\sqrt{x} + 3 \cos x$

6. $f(x) = \sin x + \cos x$

7. $f(t) = t^2 \cos t$

8. $f(x) = (x + 1) \cos x$

9. $g(t) = \dfrac{\cos t}{t}$

10. $f(x) = \dfrac{\sin x}{x}$

11. $y = \tan x + x^2$

12. $y = x + \cot x$

13. $y = e^{x^2} \sec x$

14. $y = e^{-x} \sin x$

15. $y = \cos 3x + \sin^2 x$

16. $y = \csc^2 x - \cos 2x$

17. $y = \sin \pi x$

18. $y = \frac{1}{2} \csc 2x$

19. $y = x \sin \dfrac{1}{x}$

20. $y = x^2 \sin \dfrac{1}{x}$

21. $y = 3 \tan 4x$

22. $y = \tan e^x$

23. $y = 2 \tan^2 4x$

24. $y = -\sin^4 2x$

25. $y = e^{2x} \sin 2x$

26. $y = e^{-x} \cos \dfrac{x}{2}$

In Exercises 27–38, find the derivative of the function and simplify your answer by using the trigonometric identities listed in Section 8.2.

27. $y = \cos^2 x$

28. $y = \frac{1}{4} \sin^2 2x$

29. $y = \cos^2 x - \sin^2 x$

30. $y = \dfrac{x}{2} + \dfrac{\sin 2x}{4}$

31. $y = \ln|\sin x|$

32. $y = -\ln|\cos x|$

33. $y = \ln|\csc x^2 - \cot x^2|$

34. $y = \ln|\sec x + \tan x|$

35. $y = \tan x - x$

36. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

37. $y = \ln(\sin^2 x)$

38. $y = \frac{1}{2}(x \tan x - \sec x)$

In Exercises 39–46, find an equation of the tangent line to the graph of the function at the given point.

	Function	*Point*
39.	$y = \tan x$	$\left(-\dfrac{\pi}{4}, -1\right)$
40.	$y = \sec x$	$\left(\dfrac{\pi}{3}, 2\right)$
41.	$y = \sin 4x$	$(\pi, 0)$
42.	$y = \csc^2 x$	$\left(\dfrac{\pi}{2}, 1\right)$
43.	$y = \dfrac{\cos x}{\sin x}$	$\left(\dfrac{3\pi}{4}, -1\right)$
44.	$y = \sin x \cos x$	$\left(\dfrac{3\pi}{2}, 0\right)$
45.	$y = \ln\lvert\cot x\rvert$	$\left(\dfrac{\pi}{4}, 0\right)$
46.	$y = \sqrt{\sin x}$	$\left(\dfrac{\pi}{6}, \dfrac{\sqrt{2}}{2}\right)$

In Exercises 47 and 48, use implicit differentiation to find dy/dx and evaluate the derivative at the given point.

Function	*Point*
47. $\sin x + \cos 2y = 1$	$\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$
48. $\tan(x + y) = x$	$(0, 0)$

In Exercises 49–52, show that the function satisfies the differential equation.

49. $y = 2 \sin x + 3 \cos x$

$y'' + y = 0$

50. $y = \dfrac{10 - \cos x}{x}$

$xy' + y = \sin x$

51. $y = \cos 2x + \sin 2x$

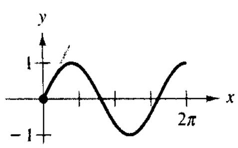
$y'' + 4y = 0$

52. $y = e^{x}(\cos \sqrt{2}x + \sin \sqrt{2}x)$

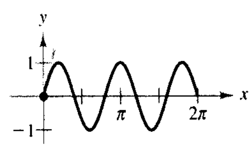
$y'' - 2y' + 3y = 0$

In Exercises 53–58, find the slope of the tangent line to the given sine function at the origin. Compare this value with the number of complete cycles in the interval $[0, 2\pi]$.

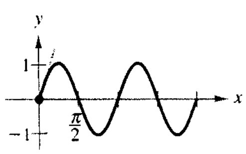
53. $y = \sin \dfrac{5x}{4}$



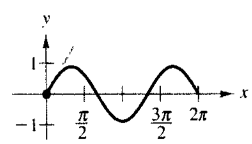
54. $y = \sin \dfrac{5x}{2}$



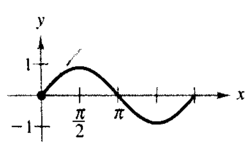
55. $y = \sin 2x$



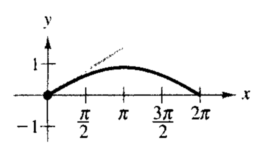
56. $y = \sin \dfrac{3x}{2}$



57. $y = \sin x$



58. $y = \sin \dfrac{x}{2}$



In Exercises 59–64, determine the relative extrema of the function on the interval $(0, 2\pi)$. Use a graphing utility to confirm your result.

59. $y = 2 \sin x + \sin 2x$

60. $y = 2 \sin x + \cos 2x$

61. $y = x - 2 \sin x$

62. $y = e^{-x} \sin x$

63. $y = e^{-x} \cos x$

64. $y = \sec \dfrac{x}{2}$

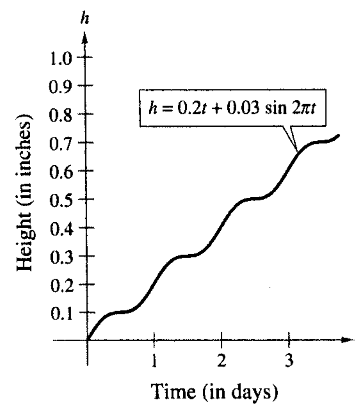
65. ***Biology*** Plants do not grow at constant rates during a normal 24-hour period because their growth is affected by sunlight. Suppose that the growth of a certain plant species in a controlled environment is given by the model

$$h = 0.2t + 0.03 \sin 2\pi t$$

where h is the height of the plant in inches and t is the time in days, with $t = 0$ corresponding to midnight of day 1 (see figure). During what time of day is the rate of growth of this plant

(a) a maximum? (b) a minimum?

Plant Growth



66. ***Meteorology*** The normal average daily temperature in degrees Fahrenheit for a city is given by

$$T = 55 - 21 \cos \frac{2\pi(t - 32)}{365}$$

where t is the time in days, with $t = 1$ corresponding to January 1. Find the expected date of

(a) the warmest day. (b) the coldest day.

67. ***Physics*** An amusement park ride is constructed such that its height h in feet above ground in terms of the horizontal distance x in feet from the starting point can be modeled by

$$h = 50 + 45 \sin \frac{\pi x}{150}, \quad 0 \le x \le 300.$$

(a) Use a graphing utility to graph the model. Be sure to choose an appropriate viewing window.

(b) Determine dh/dx and evaluate for $x = 50$, 150, 200, and 250. Interpret these values of dh/dx.

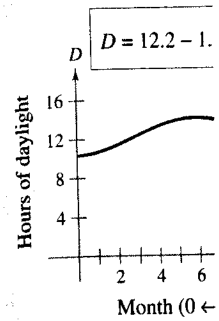
(c) Find the maximum height and the minimum height of the ride.

(d) Find the distance from the starting point at which the ride's rate of change is the greatest.

68. ***Meteorology*** The nu
Orleans can be modele

$D = 12.2 - 1.9 \cos$

where t represents the
January. Find the montl
imum number of dayli
number of daylight hou



69. For $f(x) = \sec^2 x$ and g

70. For $f(x) = \sin^2 x$ and $g($

71. ***Physics*** A 15-centi
the equation

$\theta = 0.2 \cos 8t$

where θ is the angul
radians and t is the tin

(a) Determine the ma

(b) Find the rate of ch

72. ***Tides*** Throughout t
at the end of a dock va
particular day can be

$D = 3.5 + 1.5 \cos$

where $t = 0$ represent

(a) Determine dD/dt.

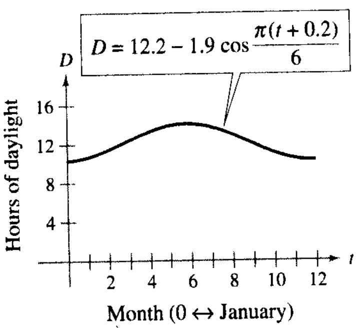
(b) Evaluate dD/dt fo
results.

(c) Find the time(s)
and the time(s) w

68. ***Meteorology*** The number of hours of daylight D in New Orleans can be modeled by

$$D = 12.2 - 1.9\cos\frac{\pi(t+0.2)}{6}, \quad 0 \le t \le 12$$

where t represents the month, with $t = 0$ corresponding to January. Find the month t when New Orleans has the maximum number of daylight hours. What is this maximum number of daylight hours?



69. For $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$, show that $f'(x) = g'(x)$.

70. For $f(x) = \sin^2 x$ and $g(x) = \cos^2 x$, show that $f'(x) = -g'(x)$.

71. ***Physics*** A 15-centimeter pendulum moves according to the equation

$$\theta = 0.2\cos 8t$$

where θ is the angular displacement from the vertical in radians and t is the time in seconds.

(a) Determine the maximum angular displacement.

(b) Find the rate of change of θ when $t = 3$ seconds.

72. ***Tides*** Throughout the day, the depth of water D in meters at the end of a dock varies with the tides. The depth for one particular day can be modeled by

$$D = 3.5 + 1.5\cos\frac{\pi t}{6}, \quad 0 \le t \le 24$$

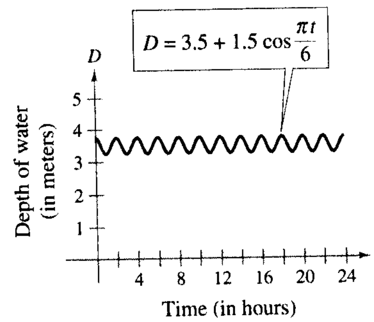
where $t = 0$ represents midnight.

(a) Determine dD/dt.

(b) Evaluate dD/dt for $t = 4$ and $t = 20$ and interpret your results.

(c) Find the time(s) when the water depth is the greatest and the time(s) when the water depth is the least.

(d) What is the greatest depth? What is the least depth? Did you have to use calculus to determine these depths? Explain your reasoning.



In Exercises 73–78, use a graphing utility (a) to graph f and f' on the same coordinate axes over the specified interval, (b) to find the critical numbers of f, and (c) to find the interval(s) on which f' is positive and the interval(s) on which it is negative. Note the behavior of f in relation to the sign of f'.

	Function	*Interval*
73.	$f(t) = t^2 \sin t$	$(0, 2\pi)$
74.	$f(x) = \frac{x}{2} + \cos\frac{x}{2}$	$(0, 4\pi)$
75.	$f(x) = \sin x - \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x$	$(0, \pi)$
76.	$f(x) = x \sin x$	$(0, \pi)$
77.	$f(x) = \sqrt{2x}\sin x$	$(0, 2\pi)$
78.	$f(x) = 4e^{-0.5x}\sin \pi x$	$(0, 4)$

In Exercises 79–84, use a graphing utility to find the relative extrema of the trigonometric function. Let $0 < x < 2\pi$.

79. $f(x) = \dfrac{x}{\sin x}$

80. $f(x) = \dfrac{x^2 - 2}{\sin x} - 5x$

81. $f(x) = \ln x \cos x$

82. $f(x) = \ln x \sin x$

83. $f(x) = \sin(0.1x^2)$

84. $f(x) = \sin\sqrt{x}$

True or False? In Exercises 85–88, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

85. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}$.

86. If $f(x) = \sin^2(2x)$, then $f'(x) = 2(\sin 2x)(\cos 2x)$.

87. If $y = x\sin^3 x$, then $y' = 3x\sin^2 x$.

88. The maximum value of $y = 3\sin x + 2\cos x$ is 5.