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complex as these fo
to enable you to use th
e seem formidable.
ry equation, you can us
r instance, the graphs o
n in Figure 3.35.

PREREQUISITE REVIEW 3.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, write a formula for the written statement.

1. The sum of one number and half a second number is 12.
2. The product of one number and twice another is 24.
3. The area of a rectangle is 24 square units.
4. The distance between two points is 10 units.

In Exercises 5–10, find the critical numbers of the function.

5. $y = x^2 + 6x - 9$
6. $y = 2x^3 - x^2 - 4x$
7. $y = 5x + \dfrac{125}{x}$
8. $y = 3x + \dfrac{96}{x^2}$
9. $y = \dfrac{x^2 + 1}{x}$
10. $y = \dfrac{x}{x^2 + 9}$

EXERCISES 3.4

In Exercises 1–6, find two positive numbers satisfying the given requirements.

1. The sum is 110 and the product is a maximum.
2. The sum is S and the product is a maximum.
3. The sum of the first and twice the second is 36 and the product is a maximum.
4. The sum of the first and twice the second is 100 and the product is a maximum.
5. The product is 192 and the sum is a minimum.
6. The product is 192 and the sum of the first plus three times the second is a minimum.
7. What positive number x minimizes the sum of x and its reciprocal?
8. The difference of two numbers is 50. Find the two numbers such that their product is a minimum.

In Exercises 9 and 10, find the length and width of a rectangle that has the given perimeter and a maximum area.

9. Perimeter: 100 meters
10. Perimeter: P units

In Exercises 11 and 12, find the length and width of the rectangle that has the given area and a minimum perimeter.

11. Area: 64 square feet
12. Area: A square centimeters

13. ***Maximum Area*** A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?

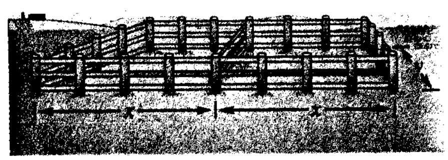
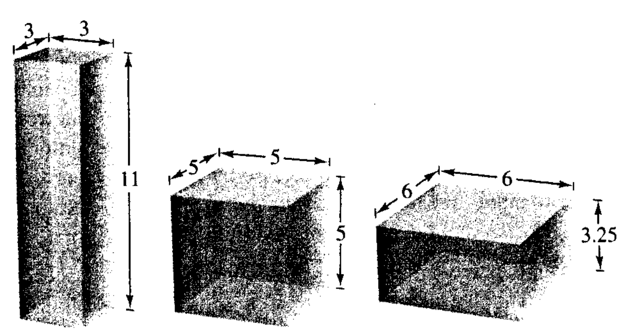


Figure for 13

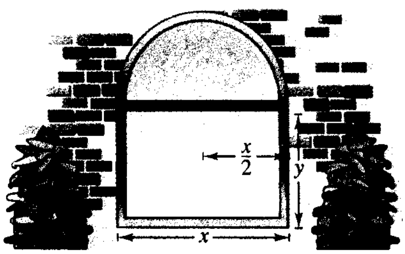
14. ***Area*** A dairy farmer plans to enclose a rectangular pasture adjacent to a river. To provide enough grass for the herd, the pasture must contain 180,000 square meters. No fencing is required along the river. What dimensions will use the smallest amount of fencing?

15. ***Maximum Volume***
    (a) Verify that each of the rectangular solids shown in the figure has a surface area of 150 square inches.
    (b) Find the volume of each solid.
    (c) Determine the dimensions of a rectangular solid (with a square base) of maximum volume if its surface area is 150 square inches.

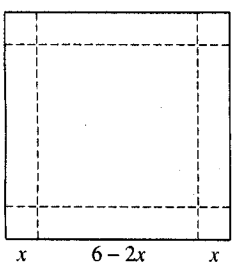


16. *Maximum Volume* Determine the dimensions of a rectangular solid (with a square base) with maximum volume if its surface area is 337.5 square centimeters.

17. *Maximum Area* A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.

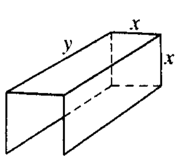


18. *Volume* An open box is to be made from a six-inch by six-inch square piece of material by cutting equal squares from the corners and turning up the sides (see figure). Find the volume of the largest box that can be made.



19. *Volume* An open box is to be made from a two-foot by three-foot rectangular piece of material by cutting equal squares from the corners and turning up the sides. Find the volume of the largest box that can be made in this manner.

20. *Minimum Surface Area* A net enclosure for golf practice is open at one end (see figure). The volume of the enclosure is $83\frac{1}{3}$ cubic meters. Find the dimensions that require the smallest amount of netting.



21. *Gardening* A home gardener estimates that if she plants 16 apple trees, the average yield will be 80 apples per tree. But because of the size of the garden, for each additional tree planted the yield will decrease by four apples per tree. How many trees should be planted to maximize the total yield of apples? What is the maximum yield?

22. *Area* A rectangular page is to contain 36 square inches of print. The margins at the top and bottom and on each side are to be $1\frac{1}{2}$ inches. Find the dimensions of the page that will minimize the amount of paper used.

23. *Area* A rectangular page is to contain 30 square in of print. The margins at the top and bottom of the page to be 2 inches wide. The margins on each side are to 1 inch wide. Find the dimensions of the page such that least amount of paper is used.

24. *Maximum Area* A rectangle is bounded by the x- y-axes and the graph of

$$y = \frac{6 - x}{2}$$

(see figure). What length and width should the recta have so that its area is a maximum?

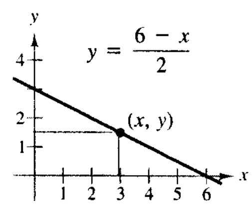


Figure for 24

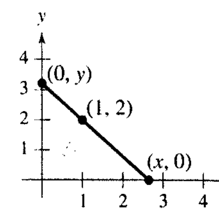


Figure for 25

25. *Minimum Length* A right triangle is formed in the f quadrant by the x- and y-axes and a line through the po (1, 2) (see figure).

(a) Write the length L of the hypotenuse as a function of

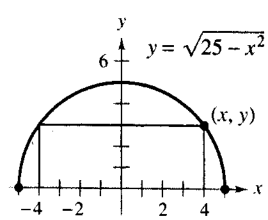
(b) Use a graphing utility to approximate x graphica such that the length of the hypotenuse is a minimum

(c) Find the vertices of the triangle such that its area is minimum.

26. *Maximum Area* A rectangle is bounded by the x-a and the semicircle

$$y = \sqrt{25 - x^2}$$

(see figure). What length and width should the rectang have so that its area is a maximum?



27. *Area* Find the dimensions of the largest rectangle that ca be inscribed in a semicircle of radius r. (See Exercise 26.)

28. *Volume* You are designing a soft drink container that h the shape of a right circular cylinder. The container supposed to hold 12 fluid ounces (1 fluid ounce is appro imately 1.80469 cubic inches). Find the dimensions th will use a minimum amount of construction material.

29. *Volume* Find the volume of the largest right circul cylinder that can be inscribed in a sphere of radius r (se figure on next page).

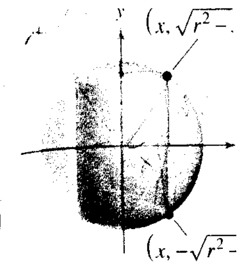


Figure for 29

30. *Maximum Volume* circular cone that can t

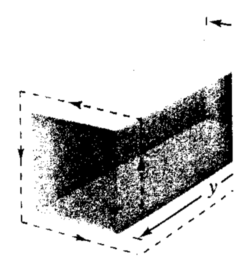
In Exercises 31 and 32, fir function that are closest to t

Function

31. $f(x) = x^2 + 1$

32. $f(x) = x^2$

33. *Maximum Volume* a postal service can ha' girth of 108 inches. F with maximum volu dimensions are x by x l



34. *Minimum Surface A* two hemispheres to the total volume of the soli of the cylinder that proc

35. *Minimum Area* Tl and a square is 16. Fi square that produce a r

36. *Minimum Area* Tl eral triangle and a squa triangle and square tha

37. *Minimum Time* Yo nearest point on the coa 3 miles down the coast can row at a rate of 2 n rate of 4 miles per ho should you row in orde

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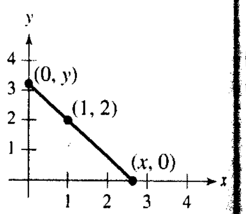

Figure for 25

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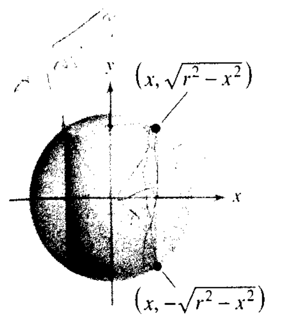


Figure for 29

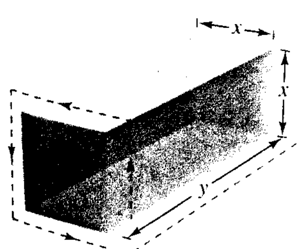
Figure for 30

30. ***Maximum Volume*** Find the volume of the largest right circular cone that can be inscribed in a sphere of radius r.

In Exercises 31 and 32, find the points on the graph of the function that are closest to the given point.

	Function	*Point*
31.	$f(x) = x^2 + 1$	$(0, 4)$
32.	$f(x) = x^2$	$\left(2, \frac{1}{2}\right)$

33. ***Maximum Volume*** A rectangular package to be sent by a postal service can have a maximum combined length and girth of 108 inches. Find the dimensions of the package with maximum volume. Assume that the package's dimensions are x by x by y (see figure).



34. ***Minimum Surface Area*** A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 12 cubic inches. Find the radius of the cylinder that produces the minimum surface area.

35. ***Minimum Area*** The combined perimeter of a circle and a square is 16. Find the dimensions of the circle and square that produce a minimum total area.

36. ***Minimum Area*** The combined perimeter of an equilateral triangle and a square is 10. Find the dimensions of the triangle and square that produce a minimum total area.

37. ***Minimum Time*** You are in a boat 2 miles from the nearest point on the coast. You are to go to point Q, located 3 miles down the coast and 1 mile inland (see figure). You can row at a rate of 2 miles per hour and you can walk at a rate of 4 miles per hour. Toward what point on the coast should you row in order to reach point Q in the least time?

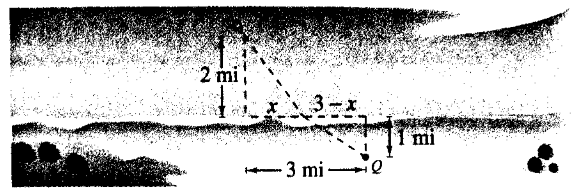
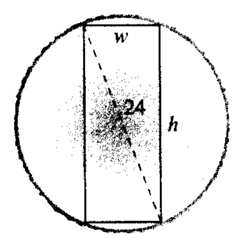


Figure for 37

38. ***Maximum Area*** An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter running track. Find the dimensions that will make the area of the rectangular region as large as possible.

39. ***Farming*** A strawberry farmer will receive \$4 per bushel of strawberries during the first week of harvesting. Each week after that, the value will drop \$0.10 per bushel. The farmer estimates that there are approximately 120 bushels of strawberries in the fields, and that the crop is increasing at a rate of four bushels per week. When should the farmer harvest the strawberries to maximize their value? How many bushels of strawberries will yield the maximum value? What is the maximum value of the strawberries?

40. ***Beam Strength*** A wooden beam has a rectangular cross section of height h and width w (see figure). The strength S of the beam is directly proportional to its width and the square of its height. What are the dimensions of the strongest beam that can be cut from a round log of diameter 24 inches? (*Hint:* $S = kh^2w$, where k is the proportionality constant.)



41. ***Maximum Area*** Use a graphing utility to graph the primary equation and its first derivative to find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius 10.

42. ***Area*** Four feet of wire is to be used to form a square and a circle.

(a) Express the sum of the areas of the square and the circle as a function A of the side of the square x.

(b) What is the domain of A?

(c) Use a graphing utility to graph A on its domain.

(d) How much wire should be used for the square and how much for the circle in order to enclose the smallest total area? the greatest total area?

PREREQUISITE REVIEW 3.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, evaluate the expression for $x = 150$.

1. $\left|-\dfrac{300}{x} + 3\right|$

2. $\left|-\dfrac{600}{5x} + 2\right|$

3. $\left|\dfrac{(20x^{-1/2})/x}{-10x^{-3/2}}\right|$

4. $\left|\dfrac{(4000/x^2)/x}{-8000x^{-3}}\right|$

In Exercises 5–10, find the marginal revenue, marginal cost, or marginal profit.

5. $C = 650 + 1.2x + 0.003x^2$

6. $P = 0.01x^2 + 11x$

7. $R = 14x - \dfrac{x^2}{2000}$

8. $R = 3.4x - \dfrac{x^2}{1500}$

9. $P = -0.7x^2 + 7x - 50$

10. $C = 1700 + 4.2x + 0.001x^3$

EXERCISES 3.5

In Exercises 1–4, find the number of units x that produces a maximum revenue R.

1. $R = 800x - 0.2x^2$

2. $R = 48x^2 - 0.02x^3$

3. $R = 400x - x^2$

4. $R = 30x^{2/3} - 2x$

In Exercises 5–8, find the number of units x that produces the minimum average cost per unit $\overline{C}$.

5. $C = 1.25x^2 + 25x + 8000$

6. $C = 0.001x^3 + 5x + 250$

7. $C = 2x^2 + 255x + 5000$

8. $C = 0.02x^3 + 55x^2 + 1250$

In Exercises 9–12, find the price per unit p that produces the maximum profit P.

	Cost Function	*Demand Function*
9.	$C = 100 + 30x$	$p = 90 - x$
10.	$C = 0.5x + 600$	$p = \dfrac{60}{\sqrt{x}}$
11.	$C = 8000 + 50x + 0.03x^2$	$p = 70 - 0.001x$
12.	$C = 35x + 500$	$p = 50 - 0.1\sqrt{x}$

Average Cost In Exercises 13 and 14, use the cost function to find the production level for which the average cost is a minimum. For this production level, show that the marginal cost and average cost are equal. Use a graphing utility to graph the average cost function and verify your results.

13. $C = 2x^2 + 5x + 18$

14. $C = x^3 - 6x^2 + 13x$

15. ***Maximum Profit*** A commodity has a demand function modeled by

$$p = 100 - 0.5x^2$$

and a total cost function modeled by $C = 40x + 37.5$.

(a) What price yields a maximum profit?

(b) When the profit is maximized, what is the average cost per unit?

16. ***Maximum Profit*** How would the answer to Exercise 15 change if the marginal cost rose from \$40 per unit to \$50 per unit? In other words, rework Exercise 15 using the cost function $C = 50x + 37.5$.

Maximum Profit In Exercises 17 and 18, find the amount s of advertising that maximizes the profit P. (s and P are measured in thousands of dollars.) Find the point of diminishing returns.

17. $P = -2s^3 + 35s^2 - 100s + 200$

18. $P = -0.1s^3 + 6s^2 + 400$

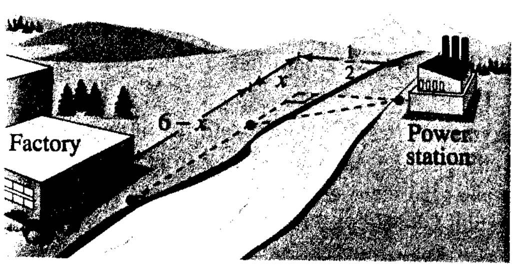
19. ***Maximum Profit*** The cost per unit of producing a type of radio is \$60. The manufacturer charges \$90 per unit for orders of 100 or less. To encourage large orders, however, the manufacturer reduces the charge by \$0.10 per radio for each order in excess of 100 units. For instance, an order of 101 radios would be \$89.90 per radio, an order of 102 radios would be \$89.80 per radio, and so on. Find the largest order the manufacturer should allow to obtain a maximum profit.

20. ***Maximum Profit*** A real estate office handles a 50-unit apartment complex. When the rent is \$580 per month, all units are occupied. For each \$40 increase in rent, however, an average of one unit becomes vacant. Each occupied unit requires an average of \$45 per month for service and repairs. What rent should be charged to obtain a maximum profit?

21. ***Maximum Revenue*** When a wholesaler sold a product at \$40 per unit, sales were 300 units per week. After a price increase of \$5, however, the average number of units sold dropped to 275 per week. Assuming that the demand function is linear, what price per unit will yield a maximum total revenue?

22. ***Maximum Profit*** Assume that the amount of money deposited in a bank is proportional to the square of the interest rate the bank pays on the money. Furthermore, the bank can reinvest the money at 12% simple interest. Find the interest rate the bank should pay to maximize its profit.

23. ***Minimum Cost*** A power station is on one side of a river that is 0.5 mile wide, and a factory is 6 miles downstream on the other side of the river (see figure). It costs \$6 per foot to run overland power lines and \$8 per foot to run underwater power lines. Write a cost function for running the power lines from the power station to the factory. Use a graphing utility to graph your function. Estimate the value of x that minimizes the cost. Explain your results.



24. ***Minimum Cost*** An offshore oil well is 1 mile off the coast. The oil refinery is 2 miles down the coast. Laying pipe in the ocean is twice as expensive as laying it on land. Find the most economical path for the pipe from the well to the oil refinery.

25. ***Minimum Cost*** A small business uses a minivan to make deliveries. The cost per hour for fuel is $C = v^2/600$, where v is the speed of the minivan (in miles per hour). The driver is paid \$10 per hour. Find the speed that minimizes the cost of a 110-mile trip. (Assume there are no costs other than fuel and wages.)

26. ***Minimum Cost*** Repeat Exercise 25 for a fuel cost per hour of

$$C = \frac{v^2 + 360}{720}$$

and a wage of \$8 per hour.

Elasticity In Exercises 27–32, find the price elasticity of demand for the demand function at the indicated x-value. Is the demand elastic, inelastic, or of unit elasticity at the indicated x-value? Use a graphing utility to graph the revenue function and identify the intervals of elasticity and inelasticity.

	Demand Function	*Quantity Demanded*
27.	$p = 400 - 3x$	$x = 20$
28.	$p = 5 - 0.03x$	$x = 100$
29.	$p = 20 - 0.0002x$	$x = 30$
30.	$p = \dfrac{500}{x + 2}$	$x = 23$
31.	$p = \dfrac{100}{x^2} + 2$	$x = 10$
32.	$p = 100 - \sqrt{0.2x}$	$x = 125$

33. ***Elasticity*** The demand function for a product is given by

$$x = p^2 - 20p + 100.$$

(a) Consider a price of \$2. If the price increases by 5%, what is the corresponding percent change in the quantity demanded?

(b) Average elasticity of demand is defined to be the percent change in quantity divided by the percent change in price. Use the percent in part (a) to find the average elasticity over the interval [2, 2.1].

(c) Find the elasticity for a price of \$2 and compare the result with that in part (b).

(d) Find an expression for the total revenue and find the values of x and p that maximize the total revenue.

34. ***Elasticity*** The demand function for a product is given by

$$p^3 + x^3 = 9.$$

(a) Find the price elasticity of demand when $x = 2$.

(b) Find the values of x and p that maximize the total revenue.

(c) For the value of x found in part (b), show that the price elasticity of demand has unit elasticity.

35. ***Elasticity*** The demand function for a product is given by

$$p = 20 - 0.02x, \quad 0 < x < 1000.$$

(a) Find the price elasticity of demand when $x = 560$.

(b) Find the values of x and p that maximize the total revenue.

(c) For the value of x found in part (b), show that the price elasticity of demand has unit elasticity.

36. ***Minimum Cost*** The
of the components use
modeled by

$$C = 100\left(\frac{200}{x^2} + \frac{}{x +}\right.$$

where C is measured in
order size in hundreds.
the cost. (*Hint:* Use the

37. ***Revenue*** The demand
$x = 600 - 50p$
where the current price
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Use price elasticity of d

38. ***Revenue*** Repeat Exe
$x = 800 - 40p$.

39. ***Demand*** A demand
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40. ***Sales*** The sales S (in
Lowe's for the years 19

$S = 201.556t^2 - 5$

$4 \le t \le 13$

where $t = 4$ correspo
Companies)

(a) During which year
sales increasing mo

(b) During which yea
lowest rate?

(c) Find the rate of in
parts (a) and (b).

(d) Use a graphing util
use the *zoom* and
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41. ***Revenue*** The reven
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$$R = \frac{-18.0 + 24}{1 - 0.16t + }$$

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36. ***Minimum Cost*** The ordering and transportation cost C of the components used in manufacturing a product is modeled by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x+30}\right), \quad x \geq 1$$

where C is measured in thousands of dollars and x is the order size in hundreds. Find the order size that minimizes the cost. (*Hint:* Use the *root* feature of a graphing utility.)

37. ***Revenue*** The demand for a car wash is

$$x = 600 - 50p$$

where the current price is \$5.00. Can revenue be increased by lowering the price and thus attracting more customers? Use price elasticity of demand to determine your answer.

38. ***Revenue*** Repeat Exercise 37 for a demand function of

$$x = 800 - 40p.$$

39. ***Demand*** A demand function is modeled by $x = a/p^m$, where a is a constant and $m > 1$. Show that $\eta = -m$. In other words, show that a 1% increase in price results in an m% decrease in the quantity demanded.

40. ***Sales*** The sales S (in millions of dollars per year) for Lowe's for the years 1994 through 2003 can be modeled by

$$S = 201.556t^2 - 502.29t + 2622.8 + \frac{9286}{t}, \quad 4 \leq t \leq 13$$

where $t = 4$ corresponds to 1994. *(Source: Lowe's Companies)*

(a) During which year, from 1994 to 2003, were Lowe's sales increasing most rapidly?

(b) During which year were the sales increasing at the lowest rate?

(c) Find the rate of increase or decrease for each year in parts (a) and (b).

(d) Use a graphing utility to graph the sales function. Then use the *zoom* and *trace* features to confirm the results in parts (a), (b), and (c).

41. ***Revenue*** The revenue R (in millions of dollars per year) for Papa John's for the years 1994 through 2003 can be modeled by

$$R = \frac{-18.0 + 24.74t}{1 - 0.16t + 0.008t^2}, \quad 4 \leq t \leq 13$$

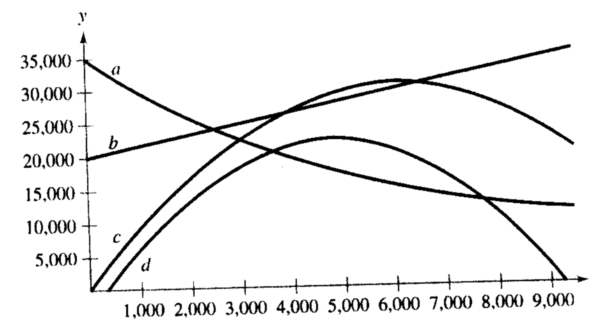
where $t = 4$ corresponds to 1994. *(Source: Papa John's Int'l.)*

(a) During which year, from 1994 to 2003, was Papa John's revenue the greatest? the least?

(b) During which year was the revenue increasing at the greatest rate? decreasing at the greatest rate?

(c) Use a graphing utility to graph the revenue function, and confirm your results in parts (a) and (b).

42. Match each graph with the function it best represents—a demand function, a revenue function, a cost function, or a profit function. Explain your reasoning. (The graphs are labeled a–d.)



BUSINESS CAPSULE


Courtesy of Transperfect Translations

While graduate students, Elizabeth Elting and Phil Shawe co-founded TransPerfect Translations in 1992. They used a rented computer and a \$5000 credit card cash advance to market their service-oriented translation firm, now one of the largest in the country. Currently, they have a network of 4000 certified language specialists in North America, Europe, and Asia, which translates technical, legal, business, and marketing materials. In 2004, the company estimates its gross sales will be \$35 million.

43. ***Research Project*** Choose an innovative product like the one described above. Use your school's library, the Internet, or some other reference source to research the history of the product or service. Collect data about the revenue that the product or service has generated, and find a mathematical model of the data. Summarize your findings.

PREREQUISITE REVIEW 3.8

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–12, find the derivative.

1. $C = 44 + 0.09x^2$

2. $C = 250 + 0.15x$

3. $R = x(1.25 + 0.02\sqrt{x})$

4. $R = x(15.5 - 1.55x)$

5. $P = -0.03x^{1/3} + 1.4x - 2250$

6. $P = -0.02x^2 + 25x - 1000$

7. $A = \frac{1}{4}\sqrt{3}x^2$

8. $A = 6x^2$

9. $C = 2\pi r$

10. $P = 4w$

11. $S = 4\pi r^2$

12. $P = 2x + \sqrt{2x}$

In Exercises 13–16, write a formula for the quantity.

13. Area A of a circle of radius r

14. Area A of a square of side x

15. Volume V of a cube of edge x

16. Volume V of a sphere of radius r

EXERCISES 3.8

In Exercises 1–6, find the differential dy.

1. $y = 3x^2 - 4$

2. $y = 2x^{3/2}$

3. $y = (4x - 1)^3$

4. $y = (1 - 2x^2)^4$

5. $y = \sqrt{x^2 + 1}$

6. $y = \sqrt[3]{6x^2}$

In Exercises 7–10, let $x = 1$ and $\Delta x = 0.01$. Find Δy.

7. $f(x) = 5x^2 - 1$

8. $f(x) = \sqrt{3x}$

9. $f(x) = \dfrac{4}{\sqrt[3]{x}}$

10. $f(x) = \dfrac{x}{x^2 + 1}$

In Exercises 11–14, compare the values of dy and Δy.

11. $y = x^3$ $\quad x = 1$ $\quad \Delta x = dx = 0.1$

12. $y = 1 - 2x^2$ $\quad x = 0$ $\quad \Delta x = dx = -0.1$

13. $y = x^4 + 1$ $\quad x = -1$ $\quad \Delta x = dx = 0.01$

14. $y = 2x^3 + 1$ $\quad x = 2$ $\quad \Delta x = dx = 0.01$

In Exercises 15–20, let $x = 2$ and complete the table for the functio

$dx = \Delta x$	dy	Δy	$\Delta y - dy$	$dy/\Delta y$
1.000				
0.500				
0.100				
0.010				
0.001				

15. $y = x^2$

16. $y = x^5$

17. $y = \dfrac{1}{x^2}$

18. $y = \dfrac{1}{x}$

19. $y = \sqrt[4]{x}$

20. $y = \sqrt{x}$

In Exercises 21–24, find an equation of the tangent line to the function at the given point. Then find the function value and the tangent line value at $f(x + \Delta x)$ and $y(x + \Delta x)$ for $\Delta x = -0.01$ and 0.01.

Function	*Point*
21. $f(x) = 2x^3 - x^2 + 1$	$(-2, -19)$
22. $f(x) = 3x^2 - 1$	$(2, 11)$
23. $f(x) = \dfrac{x}{x^2 + 1}$	$(0, 0)$
24. $f(x) = \sqrt{25 - x^2}$	$(3, 4)$

25. ***Demand*** The demanc by

$$p = 75 - 0.25x.$$

(a) If x changes from change in p? Comp

(b) Repeat part (a) whe

26. ***Biology: Wildlife M*** mission introduces 50 d lands. The population N

$$N = \frac{10(5 + 3t)}{1 + 0.04t}$$

where t is the time in y mate the change in the l

Marginal Analysis In E approximate the change in c to an increase in sales of or approximate the change in c units. Then use a graphing u the *trace* feature to verify yo

Function

27. $C = 0.05x^2 + 4x + 10$

28. $C = 0.025x^2 + 8x + 5$

29. $R = 30x - 0.15x^2$

30. $R = 50x - 1.5x^2$

31. $P = -0.5x^3 + 2500x$ -

32. $P = -x^2 + 60x - 100$

33. ***Marginal Analysis*** monthly sales x of a w \$50, but decreases to Assume that the demar Find the revenue R as a change in revenue for $x = 141$. Make a sketcl

34. ***Marginal Analysis*** demand x for a produ square of the price p. W 2500. Find the revenue mate the change in reve when $x = 3000$. Make

35. ***Marginal Analysis*** units per week when the the price is \$20. The in cost per unit is \$17. A function of the price. I and approximate the increase in sales when dP and ΔP.

25. ***Demand*** The demand function for a product is modeled by

$$p = 75 - 0.25x.$$

(a) If x changes from 7 to 8, what is the corresponding change in p? Compare the values of Δp and dp.

(b) Repeat part (a) when x changes from 70 to 71 units.

26. ***Biology: Wildlife Management*** A state game commission introduces 50 deer into newly acquired state game lands. The population N of the herd can be modeled by

$$N = \frac{10(5 + 3t)}{1 + 0.04t}$$

where t is the time in years. Use differentials to approximate the change in the herd size from $t = 5$ to $t = 6$.

Marginal Analysis In Exercises 27–32, use differentials to approximate the change in cost, revenue, or profit corresponding to an increase in sales of one unit. For instance, in Exercise 27, approximate the change in cost as x increases from 12 units to 13 units. Then use a graphing utility to graph the function, and use the *trace* feature to verify your result.

Function	*x-Value*
27. $C = 0.05x^2 + 4x + 10$	$x = 12$
28. $C = 0.025x^2 + 8x + 5$	$x = 10$
29. $R = 30x - 0.15x^2$	$x = 75$
30. $R = 50x - 1.5x^2$	$x = 15$
31. $P = -0.5x^3 + 2500x - 6000$	$x = 50$
32. $P = -x^2 + 60x - 100$	$x = 25$

33. ***Marginal Analysis*** A retailer has determined that the monthly sales x of a watch is 150 units when the price is \$50, but decreases to 120 units when the price is \$60. Assume that the demand is a linear function of the price. Find the revenue R as a function of x and approximate the change in revenue for a one-unit increase in sales when $x = 141$. Make a sketch showing dR and ΔR.

34. ***Marginal Analysis*** A manufacturer determines that the demand x for a product is inversely proportional to the square of the price p. When the price is \$10, the demand is 2500. Find the revenue R as a function of x and approximate the change in revenue for a one-unit increase in sales when $x = 3000$. Make a sketch showing dR and ΔR.

35. ***Marginal Analysis*** The demand x for a radio is 30,000 units per week when the price is \$25 and 40,000 units when the price is \$20. The initial investment is \$275,000 and the cost per unit is \$17. Assume that the demand is a linear function of the price. Find the profit P as a function of x and approximate the change in profit for a one-unit increase in sales when $x = 28{,}000$. Make a sketch showing dP and ΔP.

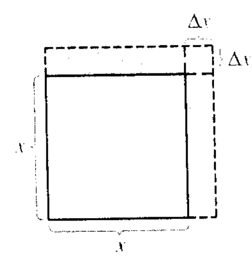
36. ***Marginal Analysis*** The variable cost for the production of a calculator is \$14.25 and the initial investment is \$110,000. Find the total cost C as a function of x, the number of units produced. Then use differentials to approximate the change in the cost for a one-unit increase in production when $x = 50{,}000$. Make a sketch showing dC and ΔC. Explain why $dC = \Delta C$ in this problem.

37. ***Area*** The area A of a square of side x is $A = x^2$.

(a) Compute dA and ΔA in terms of x and Δx.

(b) In the figure, identify the region whose area is dA.

(c) Identify the region whose area is $\Delta A - dA$.



38. ***Area*** The side of a square is measured to be 12 inches, with a possible error of $\frac{1}{64}$ inch. Use differentials to approximate the possible error and the relative error in computing the area of the square.

39. ***Area*** The radius of a circle is measured to be 10 inches, with a possible error of $\frac{1}{8}$ inch. Use differentials to approximate the possible error and the relative error in computing the area of the circle.

40. ***Volume and Surface Area*** The edge of a cube is measured to be 12 inches, with a possible error of 0.03 inch. Use differentials to approximate the possible error and the relative error in computing (a) the volume of the cube and (b) the surface area of the cube.

41. ***Volume*** The radius of a sphere is measured to be 6 inches, with a possible error of 0.02 inch. Use differentials to approximate the possible error and the relative error in computing the volume of the sphere.

42. ***Medical Science*** The concentration C (in milligrams per milliliter) of a drug in a patient's bloodstream t hours after injection into muscle tissue is modeled by

$$C = \frac{3t}{27 + t^3}.$$

Use differentials to approximate the change in the concentration when t changes from $t = 1$ to $t = 1.5$.

True or False? In Exercises 43 and 44, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

43. If $y = x + c$, then $dy = dx$.

44. If $y = ax + b$, then $\Delta y/\Delta x = dy/dx$.