Math 17C
Kouba
Set Theory and Probability Rules

SET THEORY

**DEFINITION:** A set is a collection of objects.

**EXAMPLES:** $S = \{1, 2, 3, 7, 11, 20\}$, $A = \{a, b, r, s, w\}$, $B = \{\text{red, blue, sun, snapple}\}$, $P = \{(X, Y), (A, B, C, D)\}$, $N = \{1, 2, 3, 4, 5, \ldots\}$, $E = \{\}$

**DEFINITION:** Let $A$ and $B$ be sets.

1.) The **union** of sets $A$ and $B$ is $A \cup B = \{x : x \in A \text{ or } x \in B\}$.
2.) The **intersection** of sets $A$ and $B$ is $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

**EXAMPLE:** Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$, $C = \{4, 5\}$. Determine

\[
\begin{align*}
A \cap B &= \{1, 2, 3\} \\
A \cup B &= \{1, 2, 3, 4, 5\} \\
B \cap C &= \{4, 5\} \\
B \cup C &= \{2, 3, 4, 5\} \\
A \cap C &= \{\} \\
A \cup B &= \{1, 2, 3, 4, 5\}
\end{align*}
\]

**DEFINITION:** A sample space, $\Omega$, is the set of all possible outcomes.

**EXAMPLE:** Flip a coin twice and record $H$ or $T$:

Sample Space $\Omega = \{HH, HT, TH, TT\}$

**DEFINITION:** A set $A$ is a subset of set $B$ if each object in set $A$ is also in set $B$. We write $A \subseteq B$.

**EXAMPLE:** Let $A = \{a, b, c\}$ and $B = \{a, b, c, d, e\}$ then $A \subseteq B$.

**FACT:** Every set $A$ is a subset of itself, i.e., $A \subseteq A$.

**FACT:** The empty set, $\{\} = \phi$, is a set containing no objects. The empty set is a subset of every set $A$, i.e., $\phi \subseteq A$.

**DEFINITION:** Let $\Omega$ be a sample space and let $A \subseteq \Omega$. Then $A$ is called an event in $\Omega$. The complement of $A$, written $A^c$, is the set of all objects which are in $\Omega$ but NOT in set $A$.
**EXAMPLE:** If $\Omega = \{HH, HT, TH, TT\}$ and $A = \{HH, HT, TH\}$, then $A \subseteq \Omega$ and $A^c = \{TT\} \subseteq \Omega$.

**PROPERTIES of SETS:**

1.) De Morgan's Laws:
   a.) $(A \cup B)^c = A^c \cap B^c$
   b.) $(A \cap B)^c = A^c \cup B^c$

2.) $(A^c)^c = A$

3.) $\Omega^c = \phi$ and $\phi^c = \Omega$

**PROBABILITY**

**RULES for PROBABILITY:** Let $\Omega$ be a sample space and let sets $A$ and $B$ be events in $\Omega$. Let $n(A)$ represent the number of objects in set $A$. Then the following rules apply in the context of probability:

1.) For equally-likely outcomes the probability of events $A$ and $B$ are defined to be

$$P(A) = \frac{n(A)}{n(\Omega)} \quad \text{and} \quad P(B) = \frac{n(B)}{n(\Omega)}.$$

2.) $0 \leq P(A) \leq 1 \quad \text{and} \quad 0 \leq P(B) \leq 1$

3.) $P(\phi) = \frac{n(\phi)}{n(\Omega)} = \frac{0}{n(\Omega)} = 0 \quad \text{and} \quad P(\Omega) = \frac{n(\Omega)}{n(\Omega)} = 1$

4.) If $A \cap B = \phi$, then $P(A \cup B) = P(A) + P(B)$.

5.) If $A \cap B \neq \phi$, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

6.) $P(A^c) = 1 - P(A)$.

**EXAMPLE:** Let $\Omega = \{HH, HT, TH, TT\}$ (all equally likely outcomes) and let event $A$ be "at least one head" $\rightarrow A = \{HH, HT, TH\}$,
event $B$ be "both are tails" $\rightarrow B = \{TT\}$, and
event $C$ be "a tail on the second flip" $\rightarrow C = \{HT, TT\}$.

What is

a.) $P(A)$

b.) $P(B)$

c.) $P(C)$