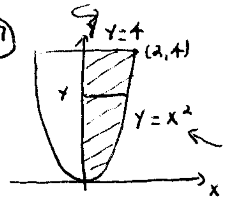
MATH 16B SOLUTIONS - SEC. 5.7 AND 6.1

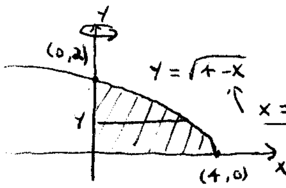
5.7 - 17



$x = \sqrt{y}$

$$V = \int_0^4 \pi (f(y))^2\, dy = \int_0^4 \pi (\sqrt{y})^2\, dy = \pi \int_0^4 y\, dy = \pi \left[\frac{y^2}{2}\right]_0^4 = \pi\left(\frac{16}{2} - 0\right) = \boxed{8\pi}$$

23



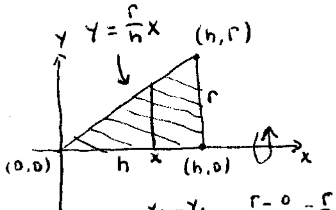
$x = 4 - y^2$

$$V = \int_0^2 \pi (f(y))^2\, dy = \int_0^2 \pi (4 - y^2)^2\, dy = \pi \int_0^2 (16 - 8y^2 + y^4)\, dy$$

$$= \pi \left[16y - 8 \cdot \frac{y^3}{3} + \frac{y^5}{5}\right]_0^2 = \pi\left(32 - \frac{64}{3} + \frac{32}{5}\right)$$

$$= \pi \cdot 32 \left(1 - \frac{2}{3} + \frac{1}{5}\right) = 32\pi\left(\frac{1}{3} + \frac{1}{5}\right) = \boxed{\frac{256\pi}{15}}$$

27



$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{r - 0}{h - 0} = \frac{r}{h}$

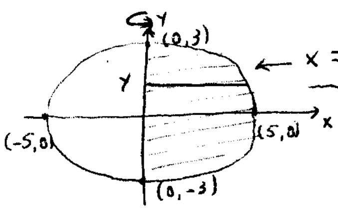
$$V = \int_0^h \pi (f(x))^2\, dx = \int_0^h \pi \left(\frac{r}{h} x\right)^2 dx = \pi \int_0^h \frac{r^2}{h^2} x^2\, dx$$

$$= \frac{\pi r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h = \frac{\pi r^2}{\cancel{h^2}} \cdot \frac{h^{\cancel{3}}}{3} = \boxed{\frac{1}{3}\pi r^2 h}$$

29

$9x^2 + 25y^2 = 225$ so $\frac{x^2}{25} + \frac{y^2}{9} = 1$, $\frac{x^2}{25} = 1 - \frac{y^2}{9} = \frac{9 - y^2}{9}$, $x^2 = \frac{25}{9}(9 - y^2)$,

$x = \pm \frac{5}{3}\sqrt{9 - y^2}$



$x = \frac{5}{3}\sqrt{9 - y^2}$

$$V = \int_{-3}^3 \pi (f(y))^2\, dy = \int_{-3}^3 \pi \left(\frac{5}{3}\sqrt{9 - y^2}\right)^2 dy = \int_{-3}^3 \pi \cdot \frac{25}{9}(9 - y^2)\, dy$$

$$= \frac{25\pi}{9}\int_{-3}^3 (9 - y^2)\, dy = \frac{25\pi}{9} \cdot 2 \int_0^3 (9 - y^2)\, dy = \frac{50\pi}{9}\left[9y - \frac{y^3}{3}\right]_0^3$$

(even)

$$= \frac{50\pi}{9}(27 - 9) = \frac{50\pi}{\cancel{9}} \cdot \overset{2}{\cancel{18}} = \boxed{100\pi}$$

6.1 - 19

$$\int \frac{x^2}{x-1}\, dx = \int \left(x + 1 + \frac{1}{x-1}\right) dx$$

$$\begin{array}{r} x + 1 \\ x - 1 \enclose{longdiv}{x^2} \\ \underline{x^2 - x} \\ x \\ \underline{x - 1} \\ 1 \end{array}$$

$$= \frac{x^2}{2} + x + \int \frac{1}{x-1}\, dx \qquad u = x - 1,\ du = dx$$

$$= \boxed{\frac{x^2}{2} + x + \ln|x-1| + C}$$

OR Let $u = x - 1$, $x = u + 1$, $dx = du$ to get

$$\int \frac{x^2}{x-1}\, dx = \int \frac{(u+1)^2}{u}\, du = \int \frac{u^2 + 2u + 1}{u}\, du = \int \left(\frac{u^2}{u} + \frac{2u}{u} + \frac{1}{u}\right) du$$

$$= \int \left(u + 2 + \frac{1}{u}\right) du = \frac{u^2}{2} + 2u + \ln|u| + C = \boxed{\frac{(x-1)^2}{2} + 2(x-1) + \ln|x-1| + C}$$

(Show this is equivalent to the first answer.)

39

$$\int_0^4 \sqrt{2x+1}\, dx \qquad \text{Let } u = 2x + 1,\ du = 2\, dx \qquad \text{if } x = 0,\ u = 1;\ x = 4,\ u = 9$$

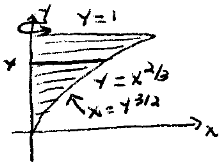
$$= \frac{1}{2}\int_0^4 \sqrt{2x+1} \cdot 2\, dx = \frac{1}{2}\int_1^9 \sqrt{u}\, du = \frac{1}{2}\int_1^9 u^{1/2}\, du = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{3} u^{3/2}\right]_1^9 = \frac{1}{3}\left(9^{3/2} - 1^{3/2}\right) = \frac{1}{3}(27 - 1) = \boxed{\frac{26}{3}}$$

OR Let $u = \sqrt{2x+1}$, so $x = \frac{1}{2}(u^2 - 1)$, $dx = \frac{1}{2}(2u)\, du = u\, du$ if $x = 0, u = 1$; $x = 4, u = 3$

$$\int_0^4 \sqrt{2x+1}\, dx = \int_1^3 u \cdot u\, du = \int_1^3 u^2\, du = \left[\frac{u^3}{3}\right]_1^3 = \frac{1}{3}(27 - 1) = \boxed{\frac{26}{3}}$$

MATH 16B $\qquad$ SOLUTIONS – SEC. 5.7 AND 6.1

5.7 – 21



$$V = \int_0^1 \pi (f(y))^2\,dy = \pi \int_0^1 (y^{3/2})^2\,dy = \pi \int_0^1 y^3\,dy = \pi \left[\frac{y^4}{4}\right]_0^1 = \frac{\pi}{4}(1-0) = \boxed{\frac{\pi}{4}}$$

6.1 – 5

$$\int \frac{2t-1}{t^2-t+2}\,dt \qquad \text{Let } u = t^2-t+2,\quad du = (2t-1)\,dt$$

$$= \boxed{\ln|t^2-t+2| + C} = \boxed{\ln(t^2-t+2) + C}$$

(since $t^2-t+2 = (t-\frac{1}{2})^2 + \frac{7}{4} > 0$ for all t)

9

$$\int \frac{12x+2}{3x^2+x}\,dx \qquad \text{Let } u = 3x^2+x,\quad du = (6x+1)\,dx$$

$$= 2\int \frac{6x+1}{3x^2+x}\,dx = \boxed{2\ln|3x^2+x| + C}$$

11

$$\int \frac{1}{(5x+1)^3}\,dx \qquad \text{Let } u = 5x+1,\quad du = 5\,dx$$

$$= \frac{1}{5}\int \frac{1}{(5x+1)^3}\cdot 5\,dx = \frac{1}{5}\int \frac{1}{u^3}\,du = \frac{1}{5}\int u^{-3}\,du = \frac{1}{5}\left(-\frac{1}{2}u^{-2}\right) + C$$

$$= \boxed{-\frac{1}{10}(5x+1)^{-2} + C}$$

15

$$\int \frac{e^{3x}}{1-e^{3x}}\,dx \qquad \text{Let } u = 1-e^{3x},\quad du = -3e^{3x}\,dx$$

$$= \left(-\frac{1}{3}\right)\int \frac{-3e^{3x}}{1-e^{3x}}\,dx = \boxed{-\frac{1}{3}\ln|1-e^{3x}| + C}$$

21

$$\int x\sqrt{x^2+4}\,dx \qquad \text{Let } u = x^2+4,\quad du = 2x\,dx$$

$$= \frac{1}{2}\int \sqrt{x^2+4}\cdot 2x\,dx = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C$$

$$= \boxed{\frac{1}{3}(x^2+4)^{3/2} + C}$$

MATH 16B SOLUTIONS – SEC. 6.1 AND 8.5

6.1 – (33) $\displaystyle\int \frac{2\sqrt{t}+1}{t}\,dt = \int\left(\frac{2\sqrt{t}}{t}+\frac{1}{t}\right)dt = \int\left(2t^{-1/2}+\frac{1}{t}\right)dt = \boxed{4t^{1/2}+\ln|t|+C}$

$= \boxed{4\sqrt{t}+\ln t + C}$ (CAN DROP THE ABS. VALUES, SINCE $t>0$)

(41) $\displaystyle\int_0^1 3x e^{x^2}\,dx$ LET $u=x^2$, $du = 2x\,dx$ IF $x=0$, $u=0$; $x=1$, $u=1$

$\displaystyle = \frac{3}{2}\int_0^1 e^{x^2}\cdot 2x\,dx = \frac{3}{2}\int_0^1 e^u\,du = \frac{3}{2}\left[e^u\right]_0^1 = \boxed{\frac{3}{2}(e-1)}$

OR LET $u=x^2$, $du = 2x\,dx$ TO GET

$\displaystyle\int_0^1 3x e^{x^2}\,dx = \frac{3}{2}\int_0^1 e^{x^2}\cdot 2x\,dx = \frac{3}{2}\left[e^{x^2}\right]_0^1 = \frac{3}{2}(e^1-e^0) = \boxed{\frac{3}{2}(e-1)}$

8.5 – (9) $\displaystyle\int x\cos x^2\,dx$ LET $u=x^2$, $du = 2x\,dx$

$\displaystyle = \frac{1}{2}\int \cos x^2\cdot 2x\,dx = \frac{1}{2}\int\cos u\,du = \frac{1}{2}\sin u + C = \boxed{\frac{1}{2}\sin x^2 + C}$

(11) $\displaystyle\int \sec^2\frac{x}{2}\,dx$ LET $u=\frac{x}{2}=\frac{1}{2}x$, $du=\frac{1}{2}\,dx$

$\displaystyle = 2\int\sec^2\frac{x}{2}\cdot\left(\frac{1}{2}\right)dx = 2\int\sec^2 u\,du = 2\tan u + C = \boxed{2\tan\frac{x}{2}+C}$

(29) $\displaystyle\int e^x\sin e^x\,dx$ LET $u=e^x$, $du = e^x\,dx$

$\displaystyle = \int \sin e^x\cdot e^x\,dx = \int\sin u\,du = -\cos u + C = \boxed{-\cos e^x + C}$

(14) $\displaystyle\int \csc 2x\cot 2x\,dx$ LET $u=2x$, $du = 2\,dx$

$\displaystyle = \frac{1}{2}\int\csc 2x\cot 2x\cdot 2\,dx = \frac{1}{2}\int\csc u\cot u\,du = \frac{1}{2}(-\csc u)+C = \boxed{-\frac{1}{2}\csc 2x + C}$

OR $\displaystyle\int\csc 2x\cot 2x\,dx = \int\frac{1}{\sin 2x}\cdot\frac{\cos 2x}{\sin 2x}\,dx = \int\frac{\cos 2x}{\sin^2 2x}\,dx$ LET $u=\sin 2x$, $du = \cos 2x\cdot 2\,dx$

$\displaystyle = \frac{1}{2}\int\frac{1}{\sin^2 2x}\cdot\cos 2x\cdot 2\,dx = \frac{1}{2}\int\frac{1}{u^2}\,du = \frac{1}{2}\int u^{-2}\,du = \frac{1}{2}\left(-\frac{1}{u}\right)+C$

$\displaystyle = -\frac{1}{2}\left(\frac{1}{\sin 2x}\right)+C = \boxed{-\frac{1}{2}\csc 2x + C}$