MATH 16B

SOLUTIONS – Sec. 6.2

(5) $\int \ln 2x\,dx$ Let $u = \ln 2x$, $dv = dx$
$du = \frac{2}{2x}\,dx = \frac{1}{x}\,dx$, $v = x$

$$= x\ln 2x - \int x\cdot\frac{1}{x}\,dx = x\ln 2x - \int 1\,dx = \boxed{x\ln 2x - x + C}$$

(15) $\int t\ln(t+1)\,dt$ Let $u = \ln(t+1)$, $dv = t\,dt$
$du = \frac{1}{t+1}\,dt$, $v = \frac{t^2}{2}$

(OR use $v = \frac{1}{2}(t^2-1)$; see bottom of next page)

$$= \frac{t^2}{2}\ln(t+1) - \frac{1}{2}\int \frac{t^2}{t+1}\,dt = \frac{t^2}{2}\ln(t+1) - \frac{1}{2}\int\left(t - 1 + \frac{1}{t+1}\right)dt$$

$$\begin{array}{r|l} & t-1 \\ t+1 & t^2 \\ & t^2+t \\ & -t \\ & -t-1 \\ & 1 \end{array}$$

$$= \boxed{\frac{t^2}{2}\ln(t+1) - \frac{1}{2}\left[\frac{t^2}{2} - t + \ln(t+1)\right] + C}$$

(since $t+1>0$, we don't need abs. values around $t+1$)

OR Let $x = t+1$, so $t = x-1$ and $dt = dx$:

$\int t\ln(t+1)\,dt = \int (x-1)\ln x\,dx$ Now let $u = \ln x$, $dv = (x-1)\,dx$
$du = \frac{1}{x}\,dx$, $v = \frac{x^2}{2} - x$

$$= \left(\frac{x^2}{2} - x\right)\ln x - \int\left(\frac{x^2}{2} - x\right)\cdot\frac{1}{x}\,dx = \left(\frac{x^2}{2} - x\right)\ln x - \int\left(\frac{x}{2} - 1\right)dx$$

$$= \left(\frac{x^2}{2} - x\right)\ln x - \left[\frac{x^2}{4} - x\right] + C = \boxed{\left(\frac{(t+1)^2}{2} - (t+1)\right)\ln(t+1) - \left[\frac{(t+1)^2}{4} - (t+1)\right] + C}$$

(19) $\int x(\ln x)^2\,dx$ Let $u = (\ln x)^2$, $dv = x\,dx$
$du = 2(\ln x)\cdot\frac{1}{x}\,dx$, $v = \frac{x^2}{2}$

$$= \frac{x^2}{2}(\ln x)^2 - \int(\ln x)\cdot\frac{1}{x}\cdot x^2\,dx = \frac{x^2}{2}(\ln x)^2 - \int x\ln x\,dx$$

Let $u = \ln x$, $dv = x\,dx$
$du = \frac{1}{x}\,dx$, $v = \frac{x^2}{2}$

$$= \frac{x^2}{2}(\ln x)^2 - \left[\frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx\right]$$

$$= \boxed{\frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C}$$

(OR Let $t = \ln x$, $x = e^t$, $dx = e^t\,dt$ to get $\int t^2 e^{2t}\,dt$, and then use vertical I by P)

(34) $\int_0^1 \ln(1+2x)\,dx$ Let $t = 1+2x$, $dt = 2\,dx$ if $x=0$, $t=1$; $x=1$, $t=3$

$$= \frac{1}{2}\int_0^1 \ln(1+2x)\cdot 2\,dx = \frac{1}{2}\int_1^3 \ln t\,dt$$

Now let $u = \ln t$, $dv = dt$
$du = \frac{1}{t}\,dt$, $v = t$

$$= \frac{1}{2}\left[t\ln t - \int \cancel{t}\cdot\frac{1}{\cancel{t}}\,dt\right]_1^3 = \frac{1}{2}\Big[t\ln t - t\Big]_1^3 = \frac{1}{2}\Big((3\ln 3 - 3) - (1\cdot 0 - 1)\Big) = \boxed{\frac{3}{2}\ln 3 - 1}$$

OR $\int_0^1 \ln(1+2x)\,dx$ Let $u = \ln(1+2x)$, $dv = dx$
$du = \frac{2}{1+2x}\,dx$, $v = x$

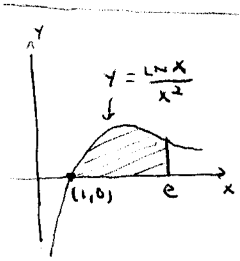
$$= \left[x\ln(1+2x) - \int\frac{2x}{1+2x}\,dx\right]_0^1 = \left[x\ln(1+2x) - \int\left(1 - \frac{1}{1+2x}\right)dx\right]_0^1$$

$$\begin{array}{r|l} & 1 \\ 2x+1 & 2x \\ & 2x+1 \\ & -1 \end{array}$$

$$= \left[x\ln(1+2x) - x + \frac{1}{2}\int\frac{2}{1+2x}\,dx\right]_0^1 = \left[x\ln(1+2x) - x + \frac{1}{2}\ln(1+2x)\right]_0^1$$

$$= \left(\ln 3 - 1 + \frac{1}{2}\ln 3\right) - (0 - 0 + 0) = \boxed{\frac{3}{2}\ln 3 - 1}$$

(38)



$A = \int_1^e \frac{\ln x}{x^2}\,dx$ Let $u = \ln x$, $dv = x^{-2}\,dx$
$du = \frac{1}{x}\,dx$, $v = -\frac{1}{x}$

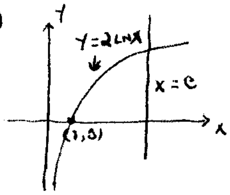
$$= \left[-\frac{\ln x}{x} - \int -\frac{1}{x^2}\,dx\right]_1^e = \left[-\frac{\ln x}{x} - \frac{1}{x}\right]_1^e$$

$$= \left(-\frac{1}{e} - \frac{1}{e}\right) - (0 - 1) = \boxed{1 - \frac{2}{e}}$$

(since $\ln e = 1$ and $\ln 1 = 0$)

MATH 168 SOLUTIONS - 6.2 AND 8.5

6.2 - 53



a) $A = \int_1^e 2\ln x\,dx = 2\int_1^e \ln x\,dx$ LET $u = \ln x,\ dv = dx$, $du = \frac{1}{x}dx,\ v = x$

$= 2\left[x\ln x - \int x\cdot\frac{1}{x}dx\right]_1^e = 2\left[x\ln x - x\right]_1^e = 2(e\cdot 1 - e - (1\cdot 0 - 1)) = \boxed{2}$

b) $V = \int_1^e \pi(2\ln x)^2 dx = 4\pi\int_1^e (\ln x)^2 dx$ LET $u = (\ln x)^2,\ dv = dx$, $du = 2(\ln x)\cdot\frac{1}{x}dx,\ v = x$

$= 4\pi\left[x(\ln x)^2 - 2\int \ln x\,dx\right]_1^e$ ← LET $u = \ln x,\ dv = dx$ AS IN PART a): $du = \frac{1}{x}dx,\ v = x$

$= 4\pi\left[x(\ln x)^2 - 2(x\ln x - x)\right]_1^e = 4\pi(e\cdot 1 - 2(e\cdot 1 - e) - (1\cdot 0 - 2(0-1))) = \boxed{4\pi(e-2)}$

8.5 - 30 $\int e^{-x}\tan e^{-x}\,dx$ LET $u = e^{-x}$, $du = -e^{-x}dx$

$= -\int \tan e^{-x}(-e^{-x})\,dx = -\int \tan u\,du = -(-\ln|\cos u|) + C = \boxed{\ln|\cos e^{-x}| + C}$

$= -\ln|\sec u| + C = \boxed{-\ln|\sec e^{-x}| + C}$

35 $\int x\cos x\,dx$ LET $u = x,\ dv = \cos x\,dx$, $du = dx,\ v = \sin x$

$= x\sin x - \int \sin x\,dx = x\sin x - (-\cos x) + C = \boxed{x\sin x + \cos x + C}$

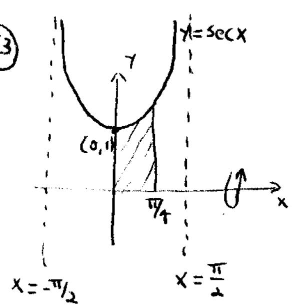
37 $\int x\sec^2 x\,dx$ LET $u = x,\ dv = \sec^2 x\,dx$, $du = dx,\ v = \tan x$

$= x\tan x - \int \tan x\,dx = \boxed{x\tan x - \ln|\sec x| + C} = \boxed{x\tan x + \ln|\cos x| + C}$

52 $A = \int_0^\pi x\sin x\,dx$ LET $u = x,\ dv = \sin x\,dx$, $du = dx,\ v = -\cos x$

$= \left[-x\cos x - \int -\cos x\,dx\right]_0^\pi = \left[-x\cos x + \sin x\right]_0^\pi = (\underbrace{-\pi(-1)}_{\cos\pi} + 0) - (\underbrace{0\cdot 1}_{\cos 0} + 0) = \boxed{\pi}$

53



$V = \int_0^{\pi/4}\pi(\sec x)^2 dx = \pi\int_0^{\pi/4}\sec^2 x\,dx = \pi\left[\tan x\right]_0^{\pi/4}$

$= \pi(1-0) = \boxed{\pi}$ (SINCE $\tan\frac{\pi}{4} = 1$ AND $\tan 0 = 0$)

6.2 - 11 $\int x e^{x^2}dx$ LET $u = x^2$, $du = 2x\,dx$

$= \frac{1}{2}\int e^{x^2}\cdot 2x\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{x^2} + C}$

OR 15 $\int t\ln(t+1)\,dt$ LET $u = \ln(t+1),\ dv = t\,dt$, $du = \frac{1}{t+1}dt,\ v = \frac{1}{2}(t^2-1)$

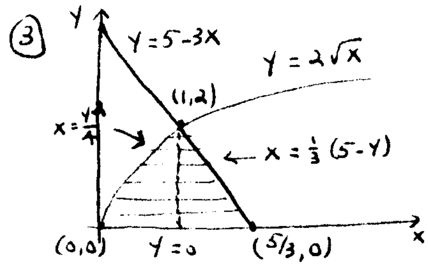
$= \frac{1}{2}(t^2-1)\ln(t+1) - \frac{1}{2}\int\frac{t^2-1}{t+1}dt$

$= \frac{1}{2}(t^2-1)\ln(t+1) - \frac{1}{2}\int(t-1)\,dt = \boxed{\frac{1}{2}(t^2-1)\ln(t+1) - \frac{1}{4}(t-1)^2 + C}$

$y = t-1$, $dy = dt$

MATH 16B SOLUTIONS – WHITE SHEET (SEC. 5.5 AND 5.11)

(3)



$5-3x = 2\sqrt{x}$ $\quad 3x + 2\sqrt{x} - 5 = 0$ $\quad (3\sqrt{x}+5)(\sqrt{x}-1) = 0$

$\sqrt{x} = -5/3$ (NO SOL.) or $\underline{\sqrt{x} = 1}$, $\underline{x = 1}$

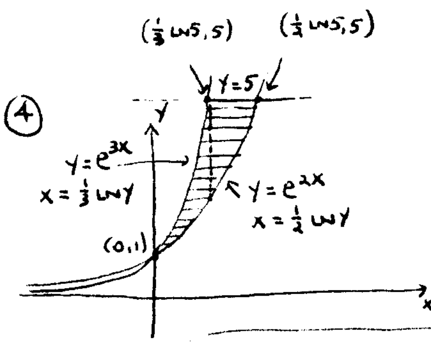
a) $\boxed{A = \int_0^1 2\sqrt{x}\,dx + \int_1^{5/3} (5-3x)\,dx}$

b) $\boxed{A = \int_0^2 \left(\frac{1}{3}(5-y) - \frac{y^2}{4}\right) dy}$

c) $\boxed{V = \int_0^1 \pi(2\sqrt{x})^2\,dx + \int_1^{5/3} \pi(5-3x)^2\,dx}$ (AROUND THE X-AXIS)

d) $\boxed{V = \int_0^2 \pi\left(\left(\frac{1}{3}(5-y)\right)^2 - \left(\frac{y^2}{4}\right)^2\right) dy}$ (AROUND THE Y-AXIS)

(4)



$e^{3x} = 5 \quad 3x = \ln 5 \quad \underline{x = \frac{1}{3}\ln 5}$

$e^{2x} = 5 \quad 2x = \ln 5 \quad \underline{x = \frac{1}{2}\ln 5}$

a) $\boxed{A = \int_0^{\frac{1}{3}\ln 5} (e^{3x} - e^{2x})\,dx + \int_{\frac{1}{3}\ln 5}^{\frac{1}{2}\ln 5} (5 - e^{2x})\,dx}$

b) $\boxed{A = \int_1^5 \left(\frac{1}{2}\ln y - \frac{1}{3}\ln y\right) dy = \int_1^5 \frac{1}{6}\ln y\,dy}$

c) $\boxed{V = \int_0^{\frac{1}{3}\ln 5} \pi\left((e^{3x})^2 - (e^{2x})^2\right) dx + \int_{\frac{1}{3}\ln 5}^{\frac{1}{2}\ln 5} \pi\left(5^2 - (e^{2x})^2\right) dx}$ (AROUND THE X-AXIS)

$= \boxed{\pi \int_0^{\frac{1}{3}\ln 5} (e^{6x} - e^{4x})\,dx + \pi \int_{\frac{1}{3}\ln 5}^{\frac{1}{2}\ln 5} (25 - e^{4x})\,dx}$

d) $V = \boxed{\int_1^5 \pi\left(\left(\frac{1}{2}\ln y\right)^2 - \left(\frac{1}{3}\ln y\right)^2\right) dy}$ (AROUND THE Y-AXIS)

$= \boxed{\pi \cdot \frac{5}{36} \int_1^5 (\ln y)^2\,dy}$