MATH 16B

Sec. 6.2 – Solutions

(12) $\int x^2 e^{x^3}\,dx$ Let $u = x^3$, $du = 3x^2\,dx$

$$= \frac{1}{3}\int e^{x^3}\cdot 3x^2\,dx = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{x^3} + C}$$

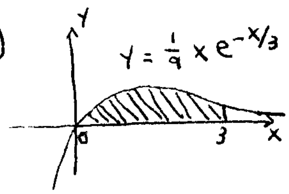
(14) $\int \frac{x}{e^x}\,dx = \int x e^{-x}\,dx$ Let $\dfrac{u = x,\ dv = e^{-x}dx}{du = dx \quad v = -e^{-x}}$

$$= -xe^{-x} - \int -e^{-x}\,dx = \boxed{-xe^{-x} - e^{-x} + C}$$

(18) $\int \frac{1}{x(\ln x)^3}\,dx$ Let $u = \ln x$, $du = \frac{1}{x}\,dx$

$$= \int \frac{1}{(\ln x)^3}\cdot\frac{1}{x}\,dx = \int \frac{1}{u^3}\,du = \int u^{-3}\,du = -\frac{1}{2}u^{-2} + C = \boxed{-\frac{1}{2}(\ln x)^{-2} + C}$$

(50c)



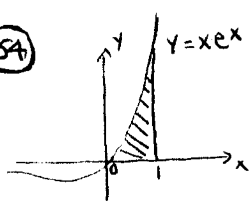
$A = \int_0^3 \frac{1}{9}xe^{-x/3}\,dx$ Let $\dfrac{u = \frac{1}{9}x,\ dv = e^{-x/3}dx}{du = \frac{1}{9}dx \quad v = -3e^{-x/3}}$

$$= \left[-\frac{1}{3}xe^{-x/3} - \left(-\frac{1}{3}\right)\int e^{-x/3}\,dx\right]_0^3$$

$$= \left[-\frac{1}{3}xe^{-x/3} + \frac{1}{3}\left(-3e^{-x/3}\right)\right]_0^3$$

$$= (-e^{-1} - e^{-1}) - (0 - 1) = \boxed{1 - 2e^{-1}} = \boxed{1 - \frac{2}{e}}$$

(54)



a) $\underline{A} = \int_0^1 xe^x\,dx$ Let $\dfrac{u = x,\ dv = e^x\,dx}{du = dx \quad v = e^x}$

$$= \left[xe^x - \int e^x\,dx\right]_0^1 = \left[xe^x - e^x\right]_0^1 = (e - e) - (0 - 1) = \boxed{1}$$

b) $\underline{V} = \pi\int_0^1 (xe^x)^2\,dx = \pi\int_0^1 x^2e^{2x}\,dx$

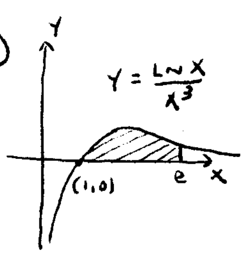
u		dv
x^2	+	$e^{2x}\,dx$
$2x$	−	$\frac{1}{2}e^{2x}$
2	+	$\frac{1}{4}e^{2x}$
0		$\frac{1}{8}e^{2x}$

$$= \pi\left[\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x}\right]_0^1$$

$$= \pi\left(\left(\frac{1}{2}e^2 - \frac{1}{2}e^2 + \frac{1}{4}e^2\right) - \left(0 - 0 + \frac{1}{4}\right)\right)$$

$$= \pi\left(\frac{1}{4}e^2 - \frac{1}{4}\right) = \boxed{\frac{\pi}{4}(e^2 - 1)}$$

(52)



$A = \int_1^e \frac{\ln x}{x^3}\,dx$ Let $\dfrac{u = \ln x,\ dv = x^{-3}dx}{du = \frac{1}{x}dx \quad v = -\frac{1}{2}x^{-2}}$

$$= \left[-\frac{1}{2}x^{-2}(\ln x) - \left(-\frac{1}{2}\right)\int x^{-3}\,dx\right]_1^e$$

$$= \left[-\frac{\ln x}{2x^2} + \frac{1}{2}\left(-\frac{1}{2}x^{-2}\right)\right]_1^e = \left[-\frac{\ln x}{2x^2} - \frac{1}{4x^2}\right]_1^e = \left(-\frac{1}{2e^2} - \frac{1}{4e^2}\right) - \left(0 - \frac{1}{4}\right)$$

$$= \boxed{\frac{1}{4} - \frac{3}{4e^2}} = \boxed{\frac{e^2 - 3}{4e^2}}$$

6.2 - 17

$$\int \frac{e^{1/\tau}}{\tau^2}\,d\tau \qquad \text{LET } u = \frac{1}{\tau},\quad du = -\frac{1}{\tau^2}\,d\tau$$

$$= -\int e^{1/\tau}\left(-\frac{1}{\tau^2}\right)d\tau = -\int e^u\,du = -e^u + C = \boxed{-e^{1/\tau} + C}$$

31

$$\int_1^e x^5 \ln x\,dx \qquad \text{LET } u = \ln x,\quad dv = x^5\,dx$$

$$du = \frac{1}{x}\,dx,\quad v = x^6/6$$

$$= \left[\frac{1}{6}x^6 \ln x - \frac{1}{6}\int x^5\,dx\right]_1^e = \left[\frac{1}{6}x^6 \ln x - \frac{1}{6}\left(\frac{1}{6}x^6\right)\right]_1^e = \left(\frac{e^6}{6}\cdot \underset{\ln e = 1}{1} - \frac{e^6}{36}\right) - \left(\frac{1}{6}\cdot \underset{\ln 1 = 0}{0} - \frac{1}{36}\right) = \boxed{\frac{5e^6+1}{36}}$$

8.5 - 19

$$\int \csc 2x\,dx \qquad \text{LET } \tau = 2x,\quad d\tau = 2\,dx$$

$$= \frac{1}{2}\int \csc\tau\,d\tau = \frac{1}{2}\ln|\csc\tau - \cot\tau| + C = \boxed{\frac{1}{2}\ln|\csc 2x - \cot 2x| + C}$$

31

$$\int e^{\sin x}\cos x\,dx \qquad \text{LET } u = \sin x,\quad du = \cos x\,dx$$

$$= \int e^u\,du = e^u + C = \boxed{e^{\sin x} + C}$$

33

$$\int (\sin 2x + \cos 2x)^2\,dx = \int(\sin^2 2x + 2\sin 2x\cos 2x + \cos^2 2x)\,dx$$

$$= \int(1 + 2\sin 2x\cos 2x)\,dx = x + \int \sin 2x\cdot 2\cos 2x\,dx \qquad \text{LET } u = \sin 2x,\quad du = 2\cos 2x\,dx$$

$$= x + \int u\,du = x + \frac{1}{2}u^2 + C = \boxed{x + \frac{1}{2}\sin^2 2x + C}$$

OR use $\int(1 + 2\sin 2x\cos 2x)\,dx = \int(1 + \sin 4x)\,dx$

$(\sin 2\theta = 2\sin\theta\cos\theta)$

$$= \boxed{x - \frac{1}{4}\cos 4x + C}$$

41

$$\int_{\pi/2}^{2\pi/3} \sec^2\frac{x}{2}\,dx \qquad \text{LET } \tau = \frac{x}{2},\quad d\tau = \frac{1}{2}\,dx$$

IF $x = \frac{\pi}{2},\ \tau = \frac{\pi}{4}$; $x = \frac{2\pi}{3},\ \tau = \frac{\pi}{3}$

$$= 2\int_{\pi/2}^{2\pi/3} \sec^2\frac{x}{2}\cdot\frac{1}{2}\,dx = 2\int_{\pi/4}^{\pi/3}\sec^2\tau\,d\tau = 2\Big[\tan\tau\Big]_{\pi/4}^{\pi/3} = 2\left(\tan\frac{\pi}{3} - \tan\frac{\pi}{4}\right)$$

$$= \boxed{2(\sqrt{3} - 1)}$$

MATH 16B   Sec. 8.5 – SOLUTIONS

(30) $\int \sec\frac{x}{2}\,dx$ LET $u=\frac{x}{2}$, $du=\frac{1}{2}dx$

$= 2\int \sec\frac{x}{2}\cdot\left(\frac{1}{2}\right)dx = 2\int \sec u\,du = 2\ln|\sec u+\tan u| + C = \boxed{2\ln\left|\sec\frac{x}{2}+\tan\frac{x}{2}\right| + C}$

(38) $\int \theta\sec\theta\tan\theta\,d\theta$ LET $u=\theta$, $dV=\sec\theta\tan\theta\,d\theta$

$du=d\theta$ $V=\sec\theta$

$= \theta\sec\theta - \int\sec\theta\,d\theta = \boxed{\theta\sec\theta - \ln|\sec\theta+\tan\theta| + C}$

(44) $\int_0^{\pi/8}\sin 2x\cos 2x\,dx$ LET $u=\sin 2x$, $du=\cos 2x\cdot 2\,dx$ IF $x=0$, $u=\sin 0=0$; $x=\pi/8$, $u=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

$= \frac{1}{2}\int_0^{\pi/8}\sin 2x\,(\cos 2x\cdot 2)\,dx = \frac{1}{2}\int_0^{1/\sqrt{2}}u\,du = \frac{1}{2}\left[\frac{u^2}{2}\right]_0^{\frac{1}{\sqrt{2}}} = \frac{1}{4}\left(\frac{1}{2}-0\right) = \boxed{\frac{1}{8}}$

OR $\int_0^{\pi/8}\sin 2x\cos 2x\,dx = \frac{1}{2}\int_0^{\pi/8}2\sin 2x\cos 2x\,dx$ (since $\sin 2\theta = 2\sin\theta\cos\theta$)

$= \frac{1}{2}\int_0^{\pi/8}\sin 4x\,dx = \frac{1}{2}\left[-\frac{1}{4}\cos 4x\right]_0^{\pi/8}$

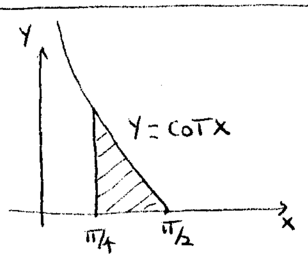
$= -\frac{1}{8}\left(\cos\frac{\pi}{2}-\cos 0\right) = -\frac{1}{8}(0-1) = \boxed{\frac{1}{8}}$

(62) $V=\int_0^3 .9\sin\frac{\pi t}{3}\,dt$ LET $u=\frac{\pi}{3}t$, $du=\frac{\pi}{3}dt$ IF $t=0$, $u=0$; $t=3$, $u=\pi$

$= .9\left(\frac{3}{\pi}\right)\int_0^3\sin\frac{\pi}{3}t\left(\frac{\pi}{3}\right)dt = \frac{2.7}{\pi}\int_0^{\pi}\sin u\,du = \frac{2.7}{\pi}\left[-\cos u\right]_0^{\pi}$

$= \frac{2.7}{\pi}\left(-\cos\pi-(-\cos 0)\right) = \frac{2.7}{\pi}\left(-(-1)+1\right) = \frac{2.7}{\pi}(2) = \boxed{\frac{5.4}{\pi}\text{ LITERS}}$

CH. 8 RE – (114)



$A=\int_{\pi/4}^{\pi/2}\cot x\,dx = \int_{\pi/4}^{\pi/2}\frac{\cos x}{\sin x}\,dx = \left[\ln|\sin x|\right]_{\pi/4}^{\pi/2}$

$= \ln\left(\sin\frac{\pi}{2}\right) - \ln\left(\sin\frac{\pi}{4}\right)$

$= \ln 1 - \ln\left(\frac{1}{\sqrt{2}}\right)$

$= 0 - (-\ln\sqrt{2}) = \boxed{\ln\sqrt{2}} = \boxed{\frac{1}{2}\ln 2}$

8.5 – (26) $\int\frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ LET $u=\sqrt{x}$, $du=\frac{1}{2\sqrt{x}}dx$

$= 2\int\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\,dx = 2\int\sin u\,du = 2(-\cos u) + C = \boxed{-2\cos\sqrt{x} + C}$