MATH 16B SOLUTIONS – SEC. 6.3

(19) $\int \frac{1}{2x^2+x}\,dx$ $\quad \frac{1}{2x^2+x} = \frac{1}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1}$

$1 = A(2x+1) + Bx$

$x=0$: $1 = A$

$x=-1/2$: $1 = -\frac{1}{2}B$ $\quad B=-2$

$$= \int \left(\frac{1}{x} - \frac{2}{2x+1}\right)dx = \boxed{\ln|x| - \ln|2x+1| + C}$$

$$= \boxed{\ln\left|\frac{x}{2x+1}\right| + C}$$

(23) $\int \frac{5-x}{2x^2+x-1}\,dx$ $\quad \frac{5-x}{2x^2+x-1} = \frac{5-x}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$

$5-x = A(x+1) + B(2x-1)$

$x=-1$: $6 = -3B$ $\quad B=-2$

$x=1/2$: $9/2 = 3/2\,A$ $\quad A=3$

$$= \int\left(\frac{3}{2x-1} + \frac{-2}{x+1}\right)dx = \frac{3}{2}\int \frac{2}{2x-1}\,dx - 2\int\frac{1}{x+1}\,dx$$

$$= \boxed{\frac{3}{2}\ln|2x-1| - 2\ln|x+1| + C}$$

(25) $\int \frac{x^2+12x+12}{x^3-4x}\,dx$ $\quad \frac{x^2+12x+12}{x^3-4x} = \frac{x^2+12x+12}{x(x^2-4)} = \frac{x^2+12x+12}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$

$x^2+12x+12 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$

$x=0$: $12 = -4A$ $\quad A=-3$

$x=2$: $40 = 8B$ $\quad B=5$

$x=-2$: $-8 = 8C$ $\quad C=-1$

$$= \int\left(-\frac{3}{x} + \frac{5}{x-2} + \frac{-1}{x+2}\right)dx$$

$$= \boxed{-3\ln|x| + 5\ln|x-2| - \ln|x+2| + C}$$

(31) $\int \frac{3x^2+3x+1}{x(x^2+2x+1)}\,dx$ $\quad \frac{3x^2+3x+1}{x(x^2+2x+1)} = \frac{3x^2+3x+1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$3x^2+3x+1 = A(x+1)^2 + Bx(x+1) + Cx$

$x=0$: $1 = A$

$x=-1$: $1 = -C$ $\quad C=-1$

COEFF. OF x^2: $3 = A+B$ $\quad$ so $3 = 1+B$ $\quad B=2$

$$= \int\left(\frac{1}{x} + \frac{2}{x+1} + \frac{-1}{(x+1)^2}\right)dx$$

$u = x+1$, $du = dx$

$$= \ln|x| + 2\ln|x+1| - \int\frac{1}{(x+1)^2}\,dx = \boxed{\ln|x| + 2\ln|x+1| + (x+1)^{-1} + C}$$

(37) $\int_0^1 \frac{x^3}{x^2-2}\,dx$

$$\begin{array}{r} x \\ x^2-2 \,\big)\, x^3 \\ \underline{x^3-2x} \\ 2x \end{array}$$

$$= \int_0^1\left(x + \frac{2x}{x^2-2}\right)dx = \left[\frac{x^2}{2} + \ln|x^2-2|\right]_0^1 = \left(\frac{1}{2} + \cancel{\ln 1}^{\,0}\right) - (0 + \ln 2) = \boxed{\frac{1}{2} - \ln 2}$$

(13) $\int \frac{1}{x^2-1}\,dx$ $\quad \frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$1 = A(x+1) + B(x-1)$

$x=1$: $1 = 2A$ $\quad A = 1/2$

$x=-1$: $1 = -2B$ $\quad B=-1/2$

$$= \int\left(\frac{1/2}{x-1} - \frac{1/2}{x+1}\right)dx$$

$$= \frac{1}{2}\int\left(\frac{1}{x-1} - \frac{1}{x+1}\right)dx = \boxed{\frac{1}{2}\Big[\ln|x-1| - \ln|x+1|\Big] + C} = \boxed{\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C}$$

28) $$\int \frac{4x^2+2x-1}{x^3+x^2}\,dx \qquad \frac{4x^2+2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$4x^2+2x-1 = Ax(x+1) + B(x+1) + Cx^2$$
$$= (A+C)x^2 + (A+B)x + B$$

$\underline{x=0:}$ $\quad \underline{-1 = B}$

$\underline{x=-1:}$ $\quad \underline{1 = C}$

$\underline{x^2 \text{ coeff.}:}$ $\quad 4 = A+C = A+1$ so $\underline{A=3}$

so $$\int \frac{4x^2+2x-1}{x^3+x^2}\,dx = \int \left(\frac{3}{x} + \frac{-1}{x^2} + \frac{1}{x+1} \right) dx$$

$$= 3\int \frac{1}{x}\,dx - \int x^{-2}\,dx + \int \frac{1}{x+1}\,dx$$

$$= \boxed{3\ln|x| + \frac{1}{x} + \ln|x+1| + C}$$

35) $$\int_1^5 \frac{x-1}{x^2(x+1)}\,dx \qquad \frac{x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$x-1 = Ax(x+1) + B(x+1) + Cx^2 = (A+C)x^2 + (A+B)x + B$$

$\underline{x=0:}$ $\quad \underline{-1 = B}$

$\underline{x=-1:}$ $\quad \underline{-2 = C}$

$\underline{x^2 \text{ coeff.}:}$ $\quad 0 = A+C = A-2$ so $\underline{A=2}$

so $$\int_1^5 \frac{x-1}{x^2(x+1)}\,dx = \int_1^5 \left(\frac{2}{x} - \frac{1}{x^2} - \frac{2}{x+1} \right) dx$$

$$= \left[2\ln|x| + \frac{1}{x} - 2\ln|x+1| \right]_1^5$$

$$= \left(2\ln 5 + \frac{1}{5} - 2\ln 6\right) - \left(\underbrace{2\ln 1}_{0} + 1 - 2\ln 2\right)$$

$$= 2\ln 5 + 2\ln 2 - 2\ln 6 - \frac{4}{5}$$

$$= 2(\ln 5 + \ln 2 - \ln 6) - \frac{4}{5}$$

$$= 2\ln\frac{5\cdot 2}{6} - \frac{4}{5} = \boxed{2\ln\frac{5}{3} - \frac{4}{5}}$$

(33) $\displaystyle\int_4^5 \frac{1}{9-x^2}\,dx \qquad \frac{1}{9-x^2} = \frac{1}{(3-x)(3+x)} = \underline{\frac{A}{3-x} + \frac{B}{3+x}}$

$1 = A(3+x) + B(3-x)$

$\underline{x=3}:\quad 1 = 6A \qquad \underline{A = 1/6}$

$\underline{x=-3}:\quad 1 = 6B \qquad \underline{B = 1/6}$

$$= \int_4^5 \left(\frac{1/6}{3-x} + \frac{1/6}{3+x} \right) dx$$

$$= \frac{1}{6}\int_4^5 \left(\frac{1}{3-x} + \frac{1}{3+x} \right) dx$$

$$= \frac{1}{6}\int_4^5 \left(\frac{-1}{x-3} + \frac{1}{x+3} \right) dx = \frac{1}{6}\Big[-\ln|x-3| + \ln|x+3| \Big]_4^5 = \frac{1}{6}\Big[\ln|x+3| - \ln|x-3| \Big]_4^5$$

$$= \frac{1}{6}\left[\ln\left| \frac{x+3}{x-3} \right| \right]_4^5 = \frac{1}{6}\left(\ln\frac{8}{2} - \ln\frac{7}{1} \right) = \frac{1}{6}(\ln 4 - \ln 7) = \boxed{\frac{1}{6}\ln\frac{4}{7}}$$

(38) a) $\displaystyle\frac{dN}{dt} = \frac{100e^{-.1t}}{(1+4e^{-.1t})^2}$, so $\displaystyle\underline{N = \int \frac{100e^{-.1t}}{(1+4e^{-.1t})^2}\,dt}$ LET $\underline{u = 1+4e^{-.1t}}$, $du = -.4e^{-.1t}\,dt$

$$N = \frac{100}{-.4}\int \frac{1}{(1+4e^{-.1t})^2}(-.4)e^{-.1t}\,dt = -250\int \frac{1}{u^2}\,du = -250\int u^{-2}\,du$$

$$= -250(-u^{-1}) + C = \underline{\frac{250}{1+4e^{-.1t}} + C}$$

$\underline{N(0)} = \dfrac{250}{1+4(1)} + C = 50 + C = \underline{50}$, so $\underline{C=0}$ AND

$$\boxed{N(t) = \frac{250}{1+4e^{-.1t}}}$$

(THIS IS AN EXAMPLE OF <u>LOGISTIC GROWTH</u>; see P. 410.)

b) SINCE $\displaystyle\underline{\lim_{t\to\infty} N(t)} = \lim_{t\to\infty} \frac{250}{1+\frac{4}{e^{.1t}}} = \frac{250}{1+0} = \underline{250}$, (AS $t\to\infty$, $e^{.1t}\to\infty$ so $\frac{4}{e^{.1t}}\to 0$)

THE VIRUS WILL INFECT NO MORE THAN 250 STUDENTS: $\boxed{\text{NO}}$

MATH 16B SOLUTIONS - SEC. 6.3 AND 6.1

6.1 - 28 $\displaystyle\int \frac{x^2}{(x+1)^3}\,dx$

A)

u		dv
x^2	$+$	$(x+1)^{-3}\,dx$
$2x$	$-$	$-\frac{1}{2}(x+1)^{-2}$
2	$+$	$\frac{1}{2}(x+1)^{-1}$
0		$\frac{1}{2}\ln\lvert x+1\rvert$

$$\int \frac{x^2}{(x+1)^3}\,dx = \boxed{-\frac{1}{2}\cdot\frac{x^2}{(x+1)^2} - \frac{x}{x+1} + \ln\lvert x+1\rvert + C}$$

SEE HW SOL. #11 FOR THE SOLUTIONS USING SUBSTITUTION.

B) $$\frac{x^2}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$$x^2 = A(x+1)^2 + B(x+1) + C$$

$x=-1$: $1 = C$

x^2 coeff.: $1 = A$

$x=0$: $0 = A + B + C = B + 2$ so $B = -2$

$$\int\left(\frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{1}{(x+1)^3}\right)dx = \boxed{\ln\lvert x+1\rvert + 2(x+1)^{-1} - \frac{1}{2}(x+1)^{-2} + C}$$

(LET $u = x+1$, $du = dx$)

29 $\displaystyle\int \frac{x}{(3x-1)^2}\,dx$

A)

u		dv
x	$+$	$(3x-1)^{-2}\,dx$
1	$-$	$-\frac{1}{3}(3x-1)^{-1}$
0		$-\frac{1}{9}\ln\lvert 3x-1\rvert$

$v = \int (3x-1)^{-2}\,dx$ LET $\tau = 3x-1$, $d\tau = 3\,dx$

$= \frac{1}{3}\int (3x-1)^{-2}\cdot 3\,dx$

$= \frac{1}{3}\int \tau^{-2}\,d\tau = \frac{1}{3}(-\tau^{-1}) + C$

$= -\frac{1}{3}(3x-1)^{-1} + C$

$$\int \frac{x}{(3x-1)^2}\,dx = \boxed{-\frac{1}{3}x(3x-1)^{-1} + \frac{1}{9}\ln\lvert 3x-1\rvert + C}$$

B) $$\frac{x}{(3x-1)^2} = \frac{A}{3x-1} + \frac{B}{(3x-1)^2}$$

$$x = A(3x-1) + B$$

$x = 1/3$: $1/3 = B$

$x = 0$: $0 = -A + B = -A + 1/3$ so $A = 1/3$

$$\int \frac{x}{(3x-1)^2}\,dx = \int\left(\frac{1/3}{3x-1} + \frac{1/3}{(3x-1)^2}\right)dx = \frac{1}{3}\int\left(\frac{1}{3x-1} + \frac{1}{(3x-1)^2}\right)dx$$

$$= \frac{1}{3}\cdot\frac{1}{3}\int\left(\frac{1}{u} + \frac{1}{u^2}\right)du \qquad u = 3x-1,\ du = 3\,dx$$

$$= \frac{1}{9}\left[\ln\lvert u\rvert - \frac{1}{u}\right] + C$$

$$= \boxed{\frac{1}{9}\left[\ln\lvert 3x-1\rvert - \frac{1}{3x-1}\right] + C}$$