MATH 16B SOLUTIONS – TAN SHEET

(1) $\int_0^3 x\sqrt{9-x^2}\,dx$ LET $u = 9-x^2$, $du = -2x\,dx$ IF $x=0,\ u=9$; $x=3,\ u=0$

$$= \left(-\frac{1}{2}\right)\int_0^3 \sqrt{9-x^2}\,(-2)\,x\,dx = -\frac{1}{2}\int_9^0 \sqrt{u}\,du = \frac{1}{2}\int_0^9 u^{1/2}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^9 = \frac{1}{3}\left(9^{3/2}-0^{3/2}\right) = \frac{1}{3}(3^3) = 3^2 = \boxed{9}$$

(2) $\int_e^{e^2} \frac{1}{x\ln x}\,dx$ LET $u=\ln x$, $du = \frac{1}{x}\,dx$ IF $x=e,\ u=\ln e=1$; $x=e^2,\ u=\ln e^2=2$

$$= \int_e^{e^2} \frac{1}{\ln x}\cdot\frac{1}{x}\,dx = \int_1^2 \frac{1}{u}\,du = \left[\ln u\right]_1^2 = \ln 2 - \overset{0}{\ln 1} = \boxed{\ln 2}$$

(3) $\int_1^e \frac{60}{x(3\ln x+2)^2}\,dx$ LET $u=3\ln x+2$, $du=\frac{3}{x}\,dx$ IF $x=1,\ u=3\cdot 0+2=2$; $x=e,\ u=3\cdot 1+2=5$

$$= \frac{60}{3}\int_1^e \frac{1}{(3\ln x+2)^2}\cdot\frac{3}{x}\,dx = 20\int_2^5 \frac{1}{u^2}\,du = 20\int_2^5 u^{-2}\,du = 20\left[-\frac{1}{u}\right]_2^5 = 20\left(-\frac{1}{5}-\left(-\frac{1}{2}\right)\right) = 20\left(\frac{3}{10}\right) = \boxed{6}$$

(4) $\int_4^{36} \frac{5}{x+2\sqrt{x}}\,dx = \int_4^{36}\frac{5}{\sqrt{x}(\sqrt{x}+2)}\,dx$ LET $u=\sqrt{x}+2$, $du = \frac{1}{2\sqrt{x}}\,dx$ IF $x=4,\ u=4$; $x=36,\ u=8$

$$= 5\cdot 2\int_4^{36}\frac{1}{\sqrt{x}+2}\cdot\frac{1}{2\sqrt{x}}\,dx = 10\int_4^8 \frac{1}{u}\,du = 10\left[\ln u\right]_4^8 = 10(\ln 8 - \ln 4) = \boxed{10\ln 2}$$

(5) $\int_0^{\pi/3} \frac{18\sec\theta\tan\theta}{(2\sec\theta-1)^3}\,d\theta$ LET $u=2\sec\theta-1$, $du = 2\sec\theta\tan\theta\,d\theta$ IF $\theta=0,\ u=2\cdot 1-1=1$; $\theta=\pi/3,\ u=2\cdot 2-1=3$ $\left(\sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1,\ \sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{1/2} = 2\right)$

$$= \frac{18}{2}\int_0^{\pi/3}\frac{1}{(2\sec\theta-1)^3}\cdot 2\sec\theta\tan\theta\,d\theta = 9\int_1^3 \frac{1}{u^3}\,du = 9\int_1^3 u^{-3}\,du = 9\left[-\frac{1}{2}u^{-2}\right]_1^3 = -\frac{9}{2}\left(\frac{1}{9}-1\right) = -\frac{9}{2}\left(-\frac{8}{9}\right) = \boxed{4}$$

(6) $\int_0^{\ln 3}\frac{36}{e^{2x}+9}\,dx = \int_0^{\ln 3}\frac{36}{e^{2x}+9}\cdot\frac{e^{-2x}}{e^{-2x}}\,dx = \int_0^{\ln 3}\frac{36e^{-2x}}{1+9e^{-2x}}\,dx$ LET $u=1+9e^{-2x}$, $du=-18e^{-2x}\,dx$

IF $x=0,\ u=1+9\cdot 1=10$; $x=\ln 3,\ u=1+9(e^{\ln 3})^{-2} = 1+9\cdot 3^{-2} = 1+1=2$

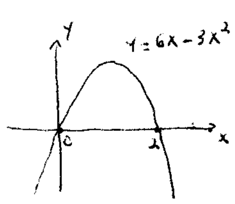
$$= \frac{36}{-18}\int_0^{\ln 3}\frac{-18e^{-2x}}{1+9e^{-2x}}\,dx = -2\int_{10}^2\frac{1}{u}\,du = 2\int_2^{10}\frac{1}{u}\,du = 2\left[\ln u\right]_2^{10}$$

$$= 2(\ln 10 - \ln 2) = \boxed{2\ln 5}$$

(7) $\int_{-3}^3 x^7\sqrt[3]{x^{10}+4}\,dx = \boxed{0}$ SINCE $f(x)=x^7\sqrt[3]{x^{10}+4}$ IS ODD BECAUSE $f(-x) = (-x)^7\sqrt[3]{(-x)^{10}+4} = -x^7\sqrt[3]{x^{10}+4} = -f(x)$

(OR LET $u=x^2$, $du=2x\,dx$ TO GET $\frac{1}{2}\int_9^9 u^3\sqrt[3]{u^5+4}\,du = \boxed{0}$)

(8)



$6x-3x^2 = 3x(2-x) > 0$ FOR $0<x<2$ AND $6x-3x^2<0$ ON $(-\infty,0)$ AND $(2,\infty)$, SO

$$\int_{-1}^3 |6x-3x^2|\,dx = \int_{-1}^0 -(6x-3x^2)\,dx + \int_0^2 (6x-3x^2)\,dx + \int_2^3 -(6x-3x^2)\,dx$$

$$= \left[x^3-3x^2\right]_{-1}^0 + \left[3x^2-x^3\right]_0^2 + \left[x^3-3x^2\right]_2^3$$

$$= (0-(-1-3)) + ((12-8)-0) + ((27-27)-(8-12))$$

$$= 4+4+4 = \boxed{12}$$

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TAN – (11)

$$\bar{f} = \frac{1}{3-0}\int_0^3 \frac{60t}{(t^2+1)^2}\,dt \qquad \text{LET } u = t^2+1,\ du = 2t\,dt \qquad \text{IF } t=0,\ u=1;\ t=3,\ u=10$$

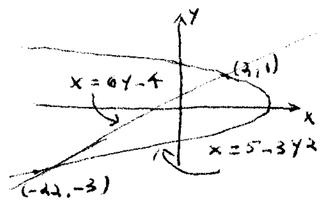
$$= \frac{1}{3}\cdot\frac{60}{2}\int_0^3 \frac{1}{(t^2+1)^2}\cdot 2t\,dt = 10\int_1^{10}\frac{1}{u^2}\,du = 10\int_1^{10}u^{-2}\,du = 10\left[-\frac{1}{u}\right]_1^{10}$$

$$= 10\left(-\frac{1}{10}-(-1)\right) = 10\cdot\frac{9}{10} = \boxed{9\ \text{FT/MIN}}$$

REMARK: NOTICE THAT DISTANCE = (AVERAGE SPEED) x (TIME) = 9 · 3 = 27 FT.

(14) $y = \frac{1}{6}(x+4)$, $x = 5-3y^2$ SO $x = 6y-4$ AND $x = 5-3y^2$;

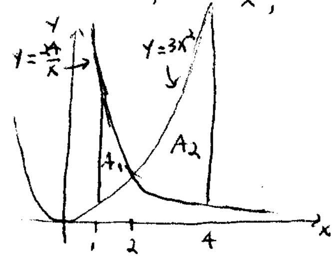
$6y-4 = 5-3y^2$ GIVES $3y^2+6y-9=0$, $y^2+2y-3=0$, $(y+3)(y-1)=0$, $y=-3$ OR $y=1$



$$A = \int_{-3}^{1}\left((5-3y^2)-(6y-4)\right)dy = \int_{-3}^{1}(9-3y^2-6y)\,dy = \left[9y-y^3-3y^2\right]_{-3}^{1}$$

$$= (9-1-3)-\left(-27-(-27)-27\right) = 5+27 = \boxed{32}$$

(15) $y = 3x^2$, $y = \frac{24}{x}$, $x=1$, $x=4$ $\qquad 3x^2 = \frac{24}{x}$ GIVES $3x^3 = 24$, $x^3=8$, $x=2$



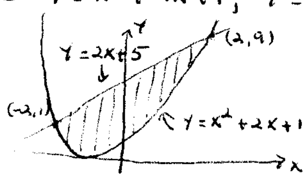
$$A = A_1 + A_2 = \int_1^2\left(\frac{24}{x}-3x^2\right)dx + \int_2^4\left(3x^2-\frac{24}{x}\right)dx$$

$$= \left[24\ln x - x^3\right]_1^2 + \left[x^3 - 24\ln x\right]_2^4$$

$$= (24\ln 2 - 8) - (\underset{0}{24\ln 1} - 1) + (64 - 24\ln 4) - (8 - 24\ln 2)$$

$$= 48\ln 2 - 24\ln 4 + 49 = 48\ln 2 - 24(2\ln 2) + 49 = \boxed{49}$$

5.5 – (2) $y = x^2+2x+1$, $y = 2x+5$ $\qquad$ CURVES INTERSECT WHERE $x^2+2x+1 = 2x+5$, $x^2=4$, $x=\pm 2$

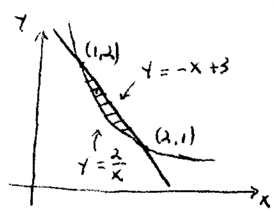


$$A = \int_{-2}^{2}\left((2x+5)-(x^2+2x+1)\right)dx = \int_{-2}^{2}\underset{\text{(EVEN)}}{(4-x^2)}\,dx = 2\int_0^2(4-x^2)\,dx$$

$$= 2\left[4x-\frac{x^3}{3}\right]_0^2 = 2\left(8-\frac{8}{3}-0\right) = 2\cdot\frac{16}{3} = \boxed{\frac{32}{3}}$$

(8) $y = -x+3$, $y = 2x^{-1}$

CURVES INTERSECT WHERE $-x+3 = \frac{2}{x}$, $-x^2+3x=2$, $0 = x^2-3x+2$, $(x-2)(x-1)=0$, $x=1$ OR $x=2$

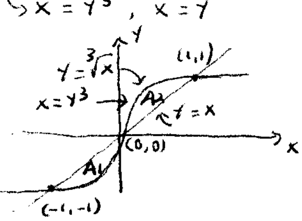


$$A = \int_1^2\left(-x+3-\frac{2}{x}\right)dx = \left[-\frac{x^2}{2}+3x-2\ln x\right]_1^2$$

$$= (-2+6-2\ln 2) - \left(-\frac{1}{2}+3-\underset{0}{2\ln 1}\right) = 4-2\ln 2+\frac{1}{2}-3 = \boxed{\frac{3}{2}-2\ln 2}$$

(17) $y = \sqrt[3]{x}$, $y = x$ $\qquad$ CURVES INTERSECT WHERE $y^3 = y$, $y^3-y=0$, $y(y^2-1)=0$, $y=0$ OR $y^2=1$, $y=\pm 1$

$\hookrightarrow x = y^3$, $x = y$



$$A = A_1 + A_2 = 2A_2 = 2\int_0^1(y-y^3)\,dy = 2\left[\frac{y^2}{2}-\frac{y^4}{4}\right]_0^1 = 2\left(\frac{1}{2}-\frac{1}{4}\right) = 2\cdot\frac{1}{4} = \boxed{\frac{1}{2}}$$

OR

$$A = A_1 + A_2 = 2A_2 = 2\int_0^1(\sqrt[3]{x}-x)\,dx = 2\int_0^1(x^{1/3}-x)\,dx$$

$$= 2\left[\frac{3}{4}x^{4/3}-\frac{x^2}{2}\right]_0^1 = 2\left(\frac{3}{4}-\frac{1}{2}\right) = 2\left(\frac{1}{4}\right) = \boxed{\frac{1}{2}}$$