MATH 16B

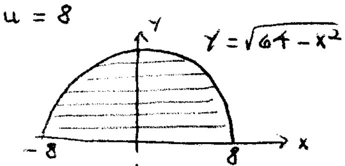
SOLUTIONS – TAN SHEET AND SEC. 5.5

(9) $\displaystyle\int_{-\pi/8}^{\pi/8} \sin^5\frac{x}{3}\,dx = \boxed{0}$ since $f(x)=\sin^5\frac{x}{3}$ is odd:

$f(-x) = \left(\sin\left(-\frac{x}{3}\right)\right)^5 = \left(-\sin\frac{x}{3}\right)^5 = -\left(\sin\frac{x}{3}\right)^5 = -\sin^5\frac{x}{3} = -f(x)$

(10) $\displaystyle\int_{-2}^{2} x^2\sqrt{64-x^6}\,dx$ Let $u=x^3$, $du = 3x^2\,dx$ If $x=-2$, $u=-8$; $x=2$, $u=8$

$\displaystyle = \frac{1}{3}\int_{-2}^{2}\sqrt{64-x^6}\cdot 3x^2\,dx = \frac{1}{3}\int_{-8}^{8}\sqrt{64-u^2}\,du = \frac{1}{3}\left[\frac{1}{2}(\pi\cdot 8^2)\right] = \boxed{\frac{32\pi}{3}}$



(TOP HALF OF A CIRCLE)

(12) $\displaystyle\overline{P} = \frac{1}{10-0}\int_0^{10} 6.1\,e^{t/80}\,dt = \frac{1}{10}(6.1)\int_0^{10} e^{t/80}\,dt = .61\left[80\,e^{t/80}\right]_0^{10} = \boxed{48.8\,(e^{1/8}-1)\text{ BILLION}}$

$\approx$ 6.5 BILLION

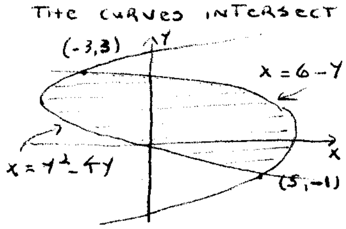
(13) A) AVERAGE COST PER UNIT $= \dfrac{C(1000)}{1000} = \dfrac{30{,}000 + 20{,}000}{1{,}000} = \boxed{\$50}$

B) $\displaystyle\overline{C} = \frac{1}{1000-0}\int_0^{1000}(30000+20x)\,dx = \frac{1}{1000}\left[30{,}000x + 10x^2\right]_0^{1000} = \frac{1}{1000}\left[30{,}000(1{,}000) + 10(1{,}000)^2\right]$

$= 30{,}000 + 10{,}000 = \boxed{\$40{,}000}$ (THIS IS THE AVERAGE VALUE OF $C(x)$ FOR $0 \le x \le 1000$.)

(16) $x = 6-y^2$, $x = y^2-4y$

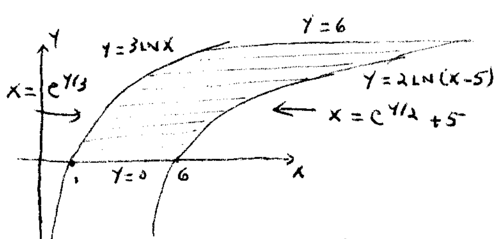
THE CURVES INTERSECT WHERE $6-y^2 = y^2-4y$, $0 = 2y^2-4y-6$, $y^2-2y-3=0$, $(y-3)(y+1)=0$, $y=3$ OR $y=-1$



$\displaystyle A = \int_{-1}^{3}\left((6-y^2)-(y^2-4y)\right)dy = \int_{-1}^{3}(6-2y^2+4y)\,dy$

$\displaystyle = \left[6y - \frac{2}{3}y^3 + 2y^2\right]_{-1}^{3} = \left(18-\frac{2}{3}(27)+18\right) - \left(-6+\frac{2}{3}+2\right) = \boxed{\frac{64}{3}}$

(17)



$y = 3\ln x$, $\ln x = \frac{y}{3}$, $x = e^{y/3}$

$y = 2\ln(x-5)$, $\ln(x-5) = \frac{y}{2}$, $x-5 = e^{y/2}$, $x = e^{y/2}+5$

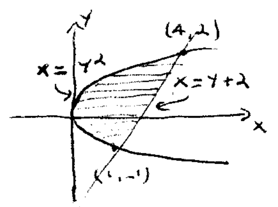
$\displaystyle A = \int_0^6 \left((e^{y/2}+5) - e^{y/3}\right)dy$

$\displaystyle = \left[2e^{y/2} + 5y - 3e^{y/3}\right]_0^6$

$= (2e^3 + 30 - 3e^2) - (2+0-3) = \boxed{2e^3 - 3e^2 + 31}$

5.5 – (27) $x = y^2$, $x = y+2$

THE CURVES INTERSECT WHERE $y^2 = y+2$, $y^2-y-2=0$, $(y-2)(y+1)=0$, $y=2$ OR $y=-1$

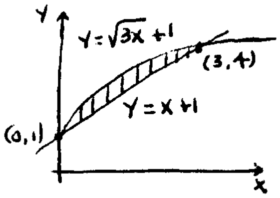


$\displaystyle A = \int_{-1}^{2}(y+2-y^2)\,dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$

$= \left(2+4-\frac{8}{3}\right) - \left(\frac{1}{2}-2+\frac{1}{3}\right) = \boxed{\frac{9}{2}}$

Sec. 5.5 – SOLUTIONS

(18) $\sqrt{3x} + \cancel{1} = x + \cancel{1}$ $\sqrt{3x} = x$ $3x = x^2$ $0 = x^2 - 3x$ $x(x-3) = 0$ $\underline{x=0}$ or $\underline{x=3}$



$$A = \int_0^3 \left((\sqrt{3x}+1) - (x+1) \right) dx = \int_0^3 \sqrt{3x}\, dx - \int_0^3 x\, dx$$

$\underline{u = 3x}$, $du = 3\,dx$; if $x=0$, $u=0$; $x=3$, $u=9$

$$= \frac{1}{3}\int_0^9 \sqrt{u}\, du - \left[\frac{x^2}{2}\right]_0^3 = \frac{1}{3}\int_0^9 u^{1/2}\, du - \left(\frac{9}{2} - 0\right)$$

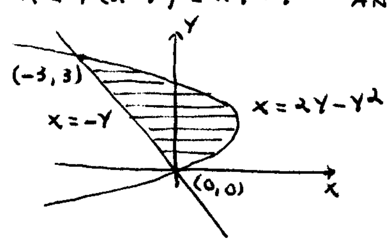
$$= \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_0^9 - \frac{9}{2} = \frac{2}{9}(27) - \frac{9}{2} = 6 - \frac{9}{2} = \boxed{\frac{3}{2}}$$

OR use $A = \int_1^4 \left((y-1) - \frac{(y-1)^2}{3} \right) dy = \int_0^3 \left(u - \frac{u^2}{3} \right) du$ $\underline{u = y-1}$, $du = dy$; if $y=1$, $u=0$; $y=4$, $u=3$

$$= \left[\frac{u^2}{2} - \frac{u^3}{9}\right]_0^3 = \left(\frac{9}{2} - 3\right) - 0 = \boxed{\frac{3}{2}}$$

(28) $x = y(2-y) = 2y - y^2$ and $x = -y$ intersect where $2y - y^2 = -y$ $3y - y^2 = 0$

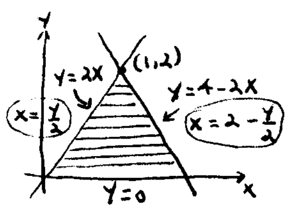
$y(3-y) = 0$ $\underline{y=0}$ or $\underline{y=3}$



$$A = \int_0^3 \left((2y - y^2) - (-y) \right) dy = \int_0^3 (3y - y^2)\, dy$$

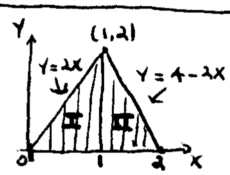
$$= \left[\frac{3}{2}y^2 - \frac{y^3}{3}\right]_0^3 = \left(\frac{27}{2} - 9\right) - 0 = \boxed{\frac{9}{2}}$$

(31)



$$A = \int_0^2 \left(\left(2 - \frac{y}{2}\right) - \frac{y}{2} \right) dy = \int_0^2 (2 - y)\, dy = \left[2y - \frac{y^2}{2}\right]_0^2 = (4-2) - 0 = \boxed{2}$$

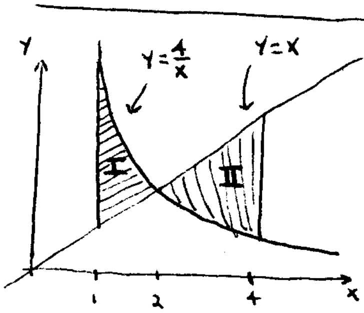
OR



$$A = \int_0^1 2x\, dx + \int_1^2 (4 - 2x)\, dx = \left[x^2\right]_0^1 + \left[4x - x^2\right]_1^2$$

$$= (1 - 0) + (8 - 4) - (4 - 1) = 1 + 4 - 3 = \boxed{2}$$

(33)



$\frac{4}{x} = x$ $4 = x^2$ $x = \pm 2$ so $\underline{x = 2}$ (since $1 \le x \le 4$)

$$A = \int_1^2 \left(\frac{4}{x} - x\right) dx + \int_2^4 \left(x - \frac{4}{x}\right) dx$$

$$= \left[4\ln x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 4\ln x\right]_2^4$$

$$= (4\ln 2 - 2) - (4\underbrace{\ln 1}_{0} - \tfrac{1}{2}) + (8 - 4\ln 4) - (2 - 4\ln 2)$$

$$= 8\ln 2 - 4\ln 4 + 4 + \frac{1}{2}$$

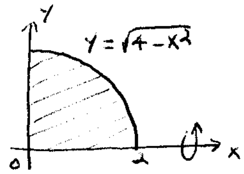
$$= 8\ln 2 - 4(2\ln 2) + \frac{9}{2} = \boxed{\frac{9}{2}}$$

(since $\ln 4 = \ln 2^2 = 2\ln 2$)

MATH 16B

Sec. 5.7 – Solutions

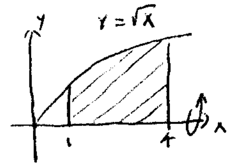
1



$$V = \int_0^2 \pi\left(\sqrt{4-x^2}\right)^2 dx = \pi\int_0^2 (4-x^2)\,dx = \pi\left[4x - \frac{x^3}{3}\right]_0^2 = \pi\left(8-\frac{8}{3}\right) = \boxed{\frac{16\pi}{3}}$$

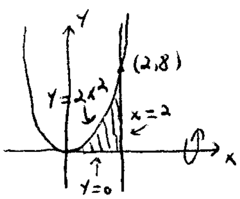
(NOTICE THAT THIS SOLID IS HALF OF A SOLID SPHERE WITH RADIUS 2.)

3



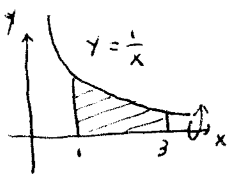
$$V = \int_1^4 \pi(\sqrt{x})^2\,dx = \pi\int_1^4 x\,dx = \pi\left[\frac{x^2}{2}\right]_1^4 = \frac{\pi}{2}(16-1) = \boxed{\frac{15\pi}{2}}$$

13



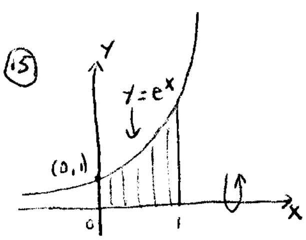
$$V = \int_0^2 \pi(2x^2)^2\,dx = \pi\int_0^2 4x^4\,dx = 4\pi\left[\frac{x^5}{5}\right]_0^2 = 4\pi\left(\frac{32}{5}\right) = \boxed{\frac{128\pi}{5}}$$

14



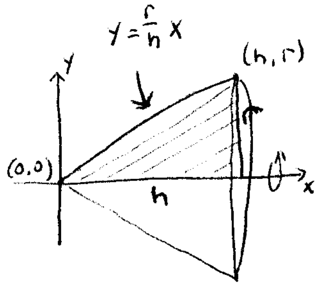
$$V = \int_1^3 \pi\left(\frac{1}{x}\right)^2 dx = \pi\int_1^3 \frac{1}{x^2}\,dx = \pi\left[-\frac{1}{x}\right]_1^3 = \pi\left(-\frac{1}{3}-(-1)\right) = \boxed{\frac{2\pi}{3}}$$

15



$$V = \int_0^1 \pi(e^x)^2\,dx = \pi\int_0^1 e^{2x}\,dx = \pi\left[\frac{1}{2}e^{2x}\right]_0^1 = \boxed{\frac{\pi}{2}(e^2-1)}$$

27



$$V = \int_0^h \pi\left(\frac{r}{h}x\right)^2 dx = \pi\int_0^h \frac{r^2}{h^2}x^2\,dx = \frac{\pi r^2}{h^2}\int_0^h x^2\,dx$$

$$= \frac{\pi r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h = \frac{\pi r^2}{\cancel{h^2}}\cdot\frac{h^{\cancel{3}}}{3} = \boxed{\frac{1}{3}\pi r^2 h}$$ FOR A RIGHT CIRCULAR CONE.

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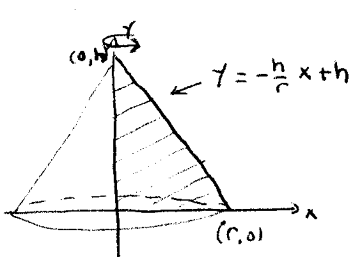
eg



$y = -\frac{h}{r}x + h$

$\frac{h}{r}x = h - y$

$x = \frac{r}{h}(h-y) = r\left(1-\frac{1}{h}y\right)$

$$V = \int_0^h \pi\left(r\left(1-\frac{1}{h}y\right)\right)^2 dy = \pi\int_0^h r^2\left(1-\frac{1}{h}y\right)^2 dy$$

$$= \pi r^2\int_0^h \left(1-\frac{2}{h}y+\frac{1}{h^2}y^2\right)dy$$

$$= \pi r^2\left[y - \frac{1}{h}y^2 + \frac{1}{h^2}\cdot\frac{y^3}{3}\right]_0^h$$

$$= \pi r^2\left(\cancel{h} - \cancel{h} + \frac{1}{3}h\right)$$

$$= \boxed{\frac{1}{3}\pi r^2 h}$$