MATH 16B

CH.4 - SOLUTIONS

$\underline{4.3}$ - (16) $y = x^2e^x - 2xe^x + 2e^x = (x^2-2x+2)e^x$

$y' = \underline{(x^2-2x+2)e^x + (2x-2)e^x} = \left[(x^2-2x+2)+(2x-2)\right]e^x = \boxed{x^2e^x}$

(18) $g(x) = e^{x^3}$ AT $(-1, 1/e)$

$g'(x) = e^{x^3}\cdot 3x^2$ so $\underline{m} = g'(-1) = e^{-1}\cdot 3(-1)^2 = \underline{\frac{3}{e}}$

$\boxed{y - \frac{1}{e} = \frac{3}{e}(x+1)}$ OR $\boxed{y = \frac{3}{e}x + \frac{4}{e}}$

(26) $e^{xy} + x^2 - y^2 = 10$ DIFFERENTIATING WITH RESPECT TO X GIVES

$e^{xy}(xy' + 1\cdot y) + 2x - 2yy' = 0$

$\underline{xe^{xy}y'} + ye^{xy} + 2x \underline{-2yy'} = 0$ $(xe^{xy} - 2y)y' = -2x - ye^{xy}$

$$y' = \boxed{\frac{-2x - ye^{xy}}{xe^{xy} - 2y}} = \boxed{\frac{2x + ye^{xy}}{2y - xe^{xy}}}$$

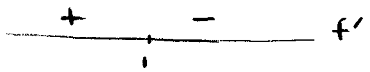
(28) $f(x) = (1+2x)e^{4x}$

$f'(x) = (1+2x)(e^{4x}\cdot 4) + 2e^{4x} = e^{4x}\left[4+8x+2\right] = (6+8x)e^{4x}$

$f''(x) = (6+8x)(e^{4x}\cdot 4) + 8e^{4x} = e^{4x}\left[24+32x+8\right] = \boxed{(32+32x)e^{4x}} = \boxed{32(1+x)e^{4x}}$

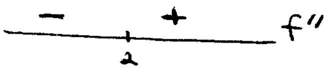
(34) $f(x) = xe^{-x}$

1) $f'(x) = x(e^{-x}(-1)) + 1\cdot e^{-x} = e^{-x}(-x+1) = (1-x)e^{-x}$



$\boxed{f(1) = \frac{1}{e} \text{ IS A REL. (AND ABSOLUTE) MAX.}}$

2) $f''(x) = (1-x)(e^{-x}(-1)) + (-1)e^{-x} = e^{-x}(-1+x-1) = (x-2)e^{-x}$

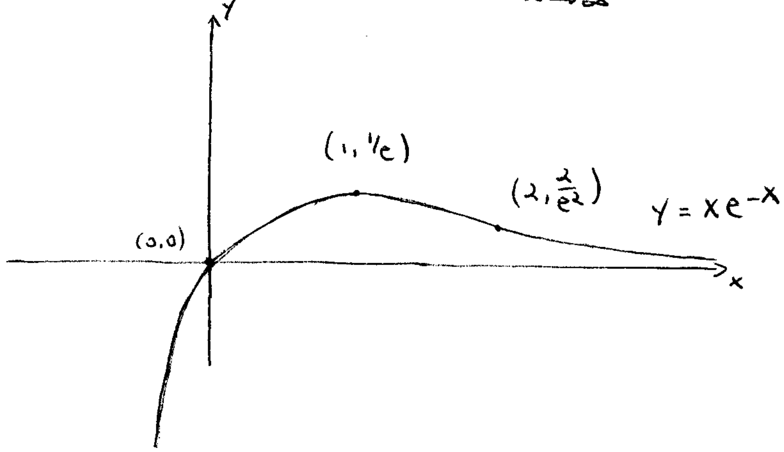


$\boxed{\text{POINT OF INFLECTION: } (2, \frac{2}{e^2})}$

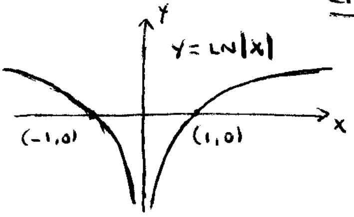
3) $\underline{\text{Y-INT.}}$: $x = 0$ GIVES $y = \underline{0}$

$\underline{\text{X-INT.}}$: $y = 0$ GIVES $xe^{-x} = 0$ SO $x = \underline{0}$

$\underline{\text{HORIZ. ASYMPTOTE}}$: X-AXIS (SINCE $\lim_{x\to\infty} xe^{-x} = 0$)



4.4 - ⑭



(THE GRAPH OF $Y = \ln X$, TOGETHER WITH ITS REFLECTION IN THE Y-AXIS)

4.5 - ⑥ $f(x) = \ln 2X \qquad f'(x) = \frac{2}{2X} = \boxed{\frac{1}{X}}$

(OR $f(x) = \ln 2 + \ln X$, SO $f'(x) = 0 + \frac{1}{X} = \boxed{\frac{1}{X}}$)

⑭ $Y = \frac{\ln X}{X^2} \qquad Y' = \frac{X^2 \cdot \frac{1}{X} - \ln X \cdot 2X}{(X^2)^2} = \frac{X - 2X\ln X}{X^4} = \frac{X(1-2\ln X)}{X^4} = \boxed{\frac{1-2\ln X}{X^3}}$

㉖ $f(x) = \ln \frac{1+e^x}{1-e^x} = \ln(1+e^x) - \ln(1-e^x)$

$f'(x) = \boxed{\frac{e^x}{1+e^x} - \frac{-e^x}{1-e^x}} = \frac{e^x(1-e^x) + e^x(1+e^x)}{(1+e^x)(1-e^x)} = \boxed{\frac{2e^x}{1-e^{2x}}}$

(54) $4XY + \ln(X^2Y) = 7 \qquad 4XY + \ln(X^2) + \ln Y = 7$

$4XY + 2\ln X + \ln Y = 7$ DIFFERENTIATING WITH RESPECT TO X GIVES

$\underline{4XY'} + 4 \cdot Y + \frac{2}{X} + \frac{Y'}{\underline{Y}} = 0$

$(4X + \frac{1}{Y})Y' = -4Y - \frac{2}{X} \qquad Y' = \boxed{\frac{-4Y - \frac{2}{X}}{4X + \frac{1}{Y}}} = \boxed{-\frac{4XY^2 + 2Y}{4X^2Y + X}}$

(70) $Y = X\ln X$

1) $Y' = X \cdot \frac{1}{X} + 1 \cdot \ln X = 1 + \ln X = 0$ IF $\ln X = -1$ SO $\underline{X} = e^{-1} = \underline{\frac{1}{e}}$:

$0 \quad - \quad \frac{1}{e} \quad + \quad Y'$

$\uparrow \quad \uparrow$

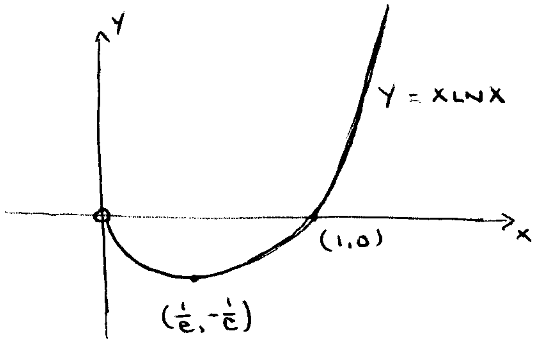
$Y'(\frac{1}{e^2}) = -1 \qquad Y'(1) = 1$

$\boxed{Y(\frac{1}{e}) = -\frac{1}{e} \text{ IS A REL. (AND ABS.) MIN.}}$

2) $Y'' = \frac{1}{X} \qquad 0 \quad + \quad Y''$ $\boxed{\text{POINTS OF INFLECTION: NONE}}$

3) $\underline{\text{Y-INT.}}$: NONE (SINCE 0 IS NOT IN THE DOMAIN)

$\underline{\text{X-INT.}}$: $Y = 0$ GIVES $X\ln X = 0$, SO ~~$X=0$~~ OR $\ln X = 0$, $X = \underline{1}$



(THE GRAPH APPROACHES (0,0) AS $X \to 0^+$, SINCE $\lim_{X \to 0^+} X\ln X = 0$)

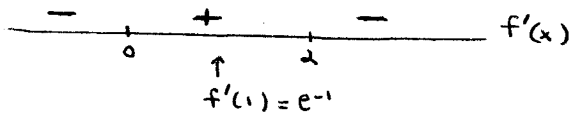
CH. 4 SOLUTIONS

4.1 – ⑳ c ㉒ f ㉔ b

4.3 – ㉝ $f(x) = x^2 e^{-x}$

1) $f'(x) = x^2(e^{-x}(-1)) + 2xe^{-x} = xe^{-x}\left[-x+2\right] = x(2-x)e^{-x}$

CRITICAL NUMBERS: 0, 2



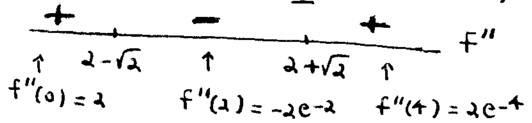
$$\boxed{\begin{array}{l} f(0)=0 \text{ IS A REL. MIN.} \\ f(2)=\frac{4}{e^2} \text{ IS A REL. MAX.} \end{array}}$$

2) $f'(x) = (2x - x^2)e^{-x}$, so

$f''(x) = (2x-x^2)(e^{-x}(-1)) + (2-2x)(e^{-x}) = e^{-x}(-2x+x^2+2-2x)$

$= (x^2-4x+2)e^{-x}$, so $f''(x)=0$ IFF $x^2-4x+2=0$ IFF

$x^2 - 4x + \underline{4} = -2 + \underline{4}$ IFF $(x-2)^2 = 2$ IFF $x-2 = \pm\sqrt{2}$ IFF $\underline{x = 2 \pm \sqrt{2}}$

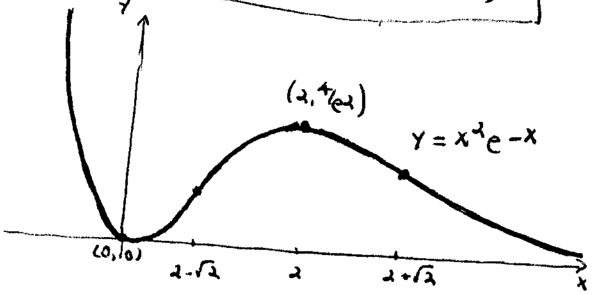


$$\boxed{\begin{array}{ll} \text{PTS OF INFLECTION}: & (2-\sqrt{2},\ (6-4\sqrt{2})e^{-2+\sqrt{2}}) \\ & \text{AND } (2+\sqrt{2},\ (6+4\sqrt{2})e^{-2-\sqrt{2}}) \end{array}}$$

3) Y-INT.: $x=0$ GIVES $y=\underline{0}$

X-INT.: $y=0$ GIVES $x^2e^{-x}=0$ SO $x=\underline{0}$

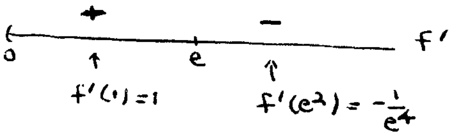
HORIZ. ASYMPTOTE: X-AXIS (SINCE $\lim_{x\to\infty} x^2e^{-x}=0$)



4.5 – ㊾ $f(x) = \frac{\ln x}{x}$

1) $f'(x) = \dfrac{x\cdot\frac{1}{x} - \ln x\cdot 1}{x^2} = \dfrac{1-\ln x}{x^2}$, so

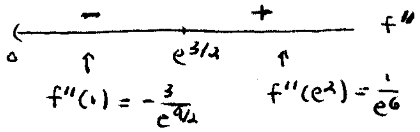
$f'(x)=0$ IF $\ln x = 1$ AND $\underline{x = e}$



$$\boxed{f(e) = \frac{1}{e} \text{ IS A REL. (AND ABS.) MAX.}}$$

2) $f''(x) = \dfrac{x^2(-\frac{1}{x}) - (1-\ln x)\cdot 2x}{(x^2)^2} = \dfrac{-x-(1-\ln x)\cdot 2x}{x^4} = \dfrac{-3x+2x\ln x}{x^4} = \dfrac{\cancel{x}(-3+2\ln x)}{\cancel{x}^{\cancel{4}3}} = \dfrac{2\ln x - 3}{x^3}$

so $f''(x)=0$ IF $2\ln x = 3$, $\ln x = 3/2$, $\underline{x = e^{3/2}}$



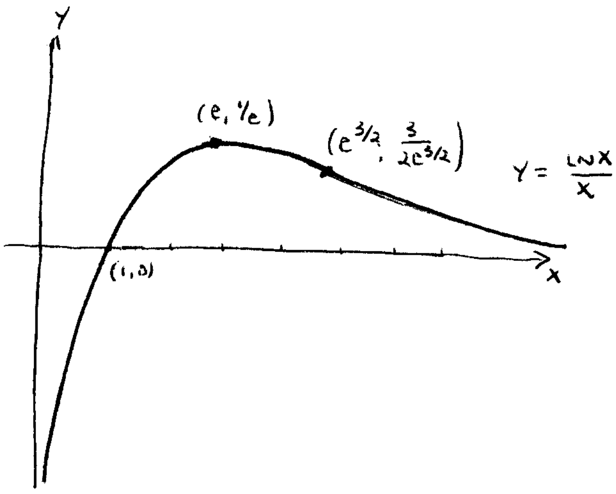
$$\boxed{\text{POINT OF INFLECTION}: \left(e^{3/2}, \frac{3}{2e^{3/2}}\right)}$$

3) Y-INT.: NONE (SINCE $f(0)$ IS UNDEFINED)

X-INT.: $y=0$ GIVES $\ln x = 0$ SO $x = \underline{1}$

VERT. ASYMP.: Y-AXIS (SINCE $\lim_{x\to 0^+} \frac{\ln x}{x} = -\infty$)

HORIZ. ASYMP.: X-AXIS (SINCE $\lim_{x\to\infty} \frac{\ln x}{x} = 0$)



CH.4 – SOLUTIONS

4.2 – ⑨ $x^{2/3} = \sqrt[3]{e^2} = e^{2/3}$, $(x^{2/3})^3 = (e^{2/3})^3$, $x^2 = e^2$, $\boxed{x = \pm e}$

4.3 – ⑬ $f(x) = \dfrac{2}{(e^x + e^{-x})^3} = 2(e^x + e^{-x})^{-3}$ $\quad \underline{f'(x)} = \boxed{-6(e^x + e^{-x})^{-4} \cdot (e^x + e^{-x}(-1))}$

㉕ $x^2 e^{-x} + 2y^2 - xy = 0 \qquad \dfrac{d}{dx}(x^2 e^{-x} + 2y^2 - xy) = \dfrac{d}{dx}(0)$

$x^2(e^{-x}(-1)) + 2x e^{-x} + \underline{4yy'} \; \underline{-xy'} + (-1)y = 0$

$(4y - x)y' = x^2 e^{-x} - 2x e^{-x} + y \qquad y' = \boxed{\dfrac{x^2 e^{-x} - 2x e^{-x} + y}{4y - x}}$

4.4 – (61) $\dfrac{10}{1 + 4e^{-.01x}} = 2.5$, $\dfrac{10}{2.5} = 1 + 4e^{-.01x}$, $4 = 1 + 4e^{-.01x}$, $4e^{-.01x} = 3$

$e^{-.01x} = \dfrac{3}{4}$, $-.01x = \ln\dfrac{3}{4}$, $x = \boxed{-100 \ln\dfrac{3}{4}} = \boxed{100 \ln\dfrac{4}{3}}$

(69) b) $A = P\left(1 + \dfrac{r}{n}\right)^{nt} = 1000\left(1 + \dfrac{.05}{12}\right)^{12t} = 2000$ so $\left(1 + \dfrac{.05}{12}\right)^{12t} = 2$

$12t \ln\left(1 + \dfrac{.05}{12}\right) = \ln 2$, so $t = \boxed{\dfrac{\ln 2}{12 \ln\left(1 + \frac{.05}{12}\right)} \text{ YRS}} \approx \underline{13.89 \text{ YRS}}$

d) $A = Pe^{rt} = 1000 e^{.05t} = 2000$ so $e^{.05t} = 2$

$.05t = \ln 2$, so $t = \dfrac{\ln 2}{.05} = \boxed{20 \ln 2 \text{ YRS}} \approx \underline{13{,}86 \text{ YRS}}$

4.5 – ⑮ $y = \ln\left(x\sqrt{x^2 - 1}\right) = \ln x + \ln(x^2 - 1)^{1/2} = \ln x + \frac{1}{2}\ln(x^2 - 1)$

$y' = \boxed{\dfrac{1}{x} + \dfrac{1}{2} \cdot \dfrac{2x}{x^2 - 1}} = \boxed{\dfrac{1}{x} + \dfrac{x}{x^2 - 1}}$

㉑ $y = \ln\dfrac{\sqrt{4 + x^2}}{x} = \ln(4 + x^2)^{1/2} - \ln x = \frac{1}{2}\ln(4 + x^2) - \ln x$

$y' = \boxed{\dfrac{1}{2} \cdot \dfrac{2x}{4 + x^2} - \dfrac{1}{x}} = \boxed{\dfrac{x}{4 + x^2} - \dfrac{1}{x}}$

(53) $4x^3 + \ln y^2 + 2y = 2x \qquad \dfrac{d}{dx}(4x^3 + 2\ln y + 2y) = \dfrac{d}{dx}(2x)$

$12x^2 + \underline{2 \cdot \dfrac{y'}{y}} + \underline{2y'} = 2 \qquad 6x^2 + \dfrac{1}{y} \cdot y' + y' = 1$

$\left(\dfrac{1}{y} + 1\right)y' = 1 - 6x^2 \qquad y' = \boxed{\dfrac{1 - 6x^2}{y^{-1} + 1}} = \boxed{\dfrac{y(1 - 6x^2)}{1 + y}}$

(55) $\underline{f(x)} = x \ln\sqrt{x} + 2x = x \cdot \frac{1}{2}\ln x + 2x = \frac{1}{2} x \ln x + 2x$

$\underline{f'(x)} = \frac{1}{2}\cancel{x}\left(\frac{1}{\cancel{x}}\right) + \frac{1}{2}\ln x + 2 = \frac{1}{2}\ln x + \frac{5}{2}$

$\underline{f''(x)} = \boxed{\dfrac{1}{2} \cdot \dfrac{1}{x}} = \boxed{\dfrac{1}{2x}}$