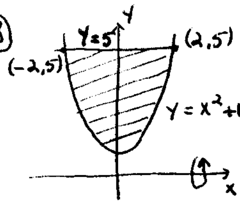
5.7 - (8)



$x^2+1=5 \quad x^2=4 \quad \underline{x=\pm 2}$

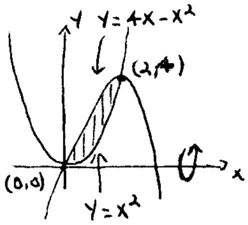
$$\underline{V=\int_{-2}^{2}\pi\left(5^2-(x^2+1)^2\right)dx} = \pi\int_{-2}^{2}\left(25-(x^4+2x^2+1)\right)dx$$

$$= \pi\int_{-2}^{2}(24-x^4-2x^2)\,dx = \pi\cdot 2\int_0^2(24-x^4-2x^2)\,dx$$

(since $y=24-x^4-2x^2$ is even)

$$= 2\pi\left[24x-\frac{x^5}{5}-\frac{2}{3}x^3\right]_0^2 = 2\pi\left(48-\frac{32}{5}-\frac{16}{3}\right) = 2\pi\left(\frac{720-96-80}{15}\right) = \boxed{\frac{1088\pi}{15}}$$

(16)



$x^2=4x-x^2 \quad 2x^2-4x=0 \quad 2x(x-2)=0 \quad \underline{x=0},\ \underline{x=2}$

$$\underline{V=\int_0^2\pi\left((4x-x^2)^2-(x^2)^2\right)dx} = \pi\int_0^2\left(16x^2-8x^3+\cancel{x^4}-\cancel{x^4}\right)dx$$

$$= \pi\left[\frac{16}{3}x^3-2x^4\right]_0^2 = \pi\left(\frac{128}{3}-32\right) = \boxed{\frac{32\pi}{3}}$$

6.1 - (22) $\displaystyle\int\frac{t}{\sqrt{1-t^2}}\,dt$

Let $\underline{u=1-t^2}$

$du=-2t\,dt$

$$=\left(-\frac{1}{2}\right)\int\frac{1}{\sqrt{1-t^2}}(-2)t\,dt = -\frac{1}{2}\int\frac{1}{\sqrt{u}}\,du = -\frac{1}{2}\int u^{-1/2}\,du = -\frac{1}{2}\left(2u^{1/2}\right)+C = \boxed{-(1-t^2)^{1/2}+C}$$

$$= \boxed{-\sqrt{1-t^2}+C}$$

(28) $\displaystyle\int\frac{x^2}{(x+1)^3}\,dx$

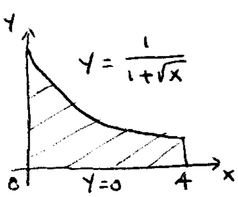
Let $\underline{u=x+1}$, so $x=u-1$

$dx=du$

$$=\int\frac{(u-1)^2}{u^3}\,du = \int\frac{u^2-2u+1}{u^3}\,du = \int\left(\frac{u^2}{u^3}-\frac{2u}{u^3}+\frac{1}{u^3}\right)du = \int\left(\frac{1}{u}-2u^{-2}+u^{-3}\right)du$$

$$= \ln|u|+2u^{-1}-\frac{1}{2}u^{-2}+C = \boxed{\ln|x+1|+2(x+1)^{-1}-\frac{1}{2}(x+1)^{-2}+C}$$

(58)



$$A=\int_0^4\frac{1}{1+\sqrt{x}}\,dx$$

Let $\underline{u=\sqrt{x}}$, so $x=u^2$

$dx=2u\,du$

if $x=0$, $u=0$

$x=4$, $u=2$

$$=\int_0^2\frac{1}{1+u}\cdot 2u\,du = 2\int_0^2\frac{u}{1+u}\,du$$

$$\begin{array}{r} 1 \\ u+1\,\overline{)\,u} \\ \underline{u+1} \\ -1 \end{array}$$

$$= 2\int_0^2\left(1-\frac{1}{1+u}\right)du = 2\left[u-\ln|1+u|\right]_0^2$$

$$= 2\left((2-\ln 3)-(0-\underbrace{\ln 1}_{0})\right) = \boxed{4-2\ln 3}$$

OR

$$A=\int_0^4\frac{1}{1+\sqrt{x}}\,dx$$

Let $\underline{u=1+\sqrt{x}}$ so $x=(u-1)^2$

$dx=2(u-1)\,du$

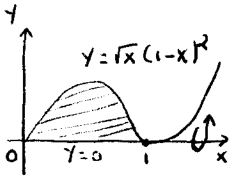
if $x=0$, $u=1$

$x=4$, $u=3$

$$=\int_1^3\frac{1}{u}\cdot 2(u-1)\,du = 2\int_1^3\frac{u-1}{u}\,du = 2\int_1^3\left(1-\frac{1}{u}\right)du = 2\left[u-\ln u\right]_1^3$$

$$= 2\left((3-\ln 3)-(1-\underbrace{\ln 1}_{0})\right) = 2(2-\ln 3) = \boxed{4-2\ln 3}$$

6.1 – 60



$\sqrt{x}(1-x)^2 = 0 \qquad \underline{x=0} \text{ or } \underline{x=1}$

$V = \int_0^1 \pi\left(\sqrt{x}(1-x)^2\right)^2 dx = \pi\int_0^1 x(1-x)^4\,dx$ LET $u = 1-x$, $x = 1-u$, $dx = -du$ IF $x=0,\ u=1$; $x=1,\ u=0$

$= \pi\int_1^0 (1-u)u^4(-1)\,du = \pi\int_0^1 (u^4 - u^5)\,du$

$= \pi\left[\frac{u^5}{5} - \frac{u^6}{6}\right]_0^1 = \pi\left(\left(\frac{1}{5}-\frac{1}{6}\right) - 0\right) = \boxed{\frac{\pi}{30}}$

CH. 6 RE – 22 $\int_2^3 x^2\sqrt{x-2}\,dx$ LET $u = \sqrt{x-2}$, so $x = u^2+2$, $dx = 2u\,du$ IF $x=2,\ u=0$; $x=3,\ u=1$

$= \int_0^1 (u^2+2)^2\cdot u\cdot 2u\,du = 2\int_0^1 u^2(u^4+4u^2+4)\,du = 2\int_0^1 (u^6+4u^4+4u^2)\,du$

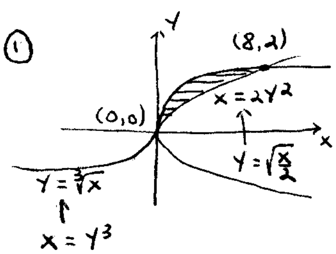
$= 2\left[\frac{u^7}{7} + \frac{4}{5}u^5 + \frac{4}{3}u^3\right]_0^1 = 2\left(\left(\frac{1}{7}+\frac{4}{5}+\frac{4}{3}\right)-0\right) = 2\left(\frac{15+84+140}{105}\right) = \boxed{\frac{478}{105}}$

OR $\int_2^3 x^2\sqrt{x-2}\,dx$ LET $u = x-2$, so $x = u+2$, $dx = du$ IF $x=2,\ u=0$; $x=3,\ u=1$

$= \int_0^1 (u+2)^2\sqrt{u}\,du = \int_0^1 (u^2+4u+4)u^{1/2}\,du = \int_0^1 (u^{5/2}+4u^{3/2}+4u^{1/2})\,du$

$= \left[\frac{2}{7}u^{7/2} + \frac{8}{5}u^{5/2} + \frac{8}{3}u^{3/2}\right]_0^1 = \left(\frac{2}{7}+\frac{8}{5}+\frac{8}{3}\right) - 0 = \boxed{\frac{478}{105}}$

WHITE SHEET – 1



$x = y^3,\ x = 2y^2 \qquad y^3 = 2y^2 \qquad y^3 - 2y^2 = 0 \qquad y^2(y-2) = 0 \qquad \underline{y=0},\ \underline{y=2}$

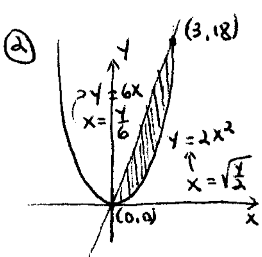
a) $\boxed{A = \int_0^8 \left(\sqrt[3]{x} - \sqrt{\frac{x}{2}}\right)dx}$

b) $\boxed{A = \int_0^2 (2y^2 - y^3)\,dy}$

c) $\boxed{V = \int_0^8 \pi\left((\sqrt[3]{x})^2 - \left(\sqrt{\frac{x}{2}}\right)^2\right)dx}$ (AROUND X-AXIS)

d) $\boxed{V = \int_0^2 \pi\left((2y^2)^2 - (y^3)^2\right)dy}$ (AROUND Y-AXIS)

2



$6x = 2x^2 \qquad 0 = 2x^2 - 6x \qquad 2x(x-3) = 0 \qquad \underline{x=0},\ \underline{x=3}$

a) $\boxed{A = \int_0^3 (6x - 2x^2)\,dx}$

b) $\boxed{A = \int_0^{18} \left(\sqrt{\frac{y}{2}} - \frac{y}{6}\right)dy}$

c) $\boxed{V = \int_0^3 \pi\left((6x)^2 - (2x^2)^2\right)dx}$ (AROUND X-AXIS)

d) $\boxed{V = \int_0^{18} \pi\left(\left(\sqrt{\frac{y}{2}}\right)^2 - \left(\frac{y}{6}\right)^2\right)dy}$ (AROUND Y-AXIS)

Sec. 6.1 – Solutions

(29) $\displaystyle\int \frac{x}{(3x-1)^2}\,dx$ Let $u = 3x-1$, so $x = \frac{1}{3}(u+1)$ and $dx = \frac{1}{3}\,du$

$$= \int \frac{\frac{1}{3}(u+1)}{u^2}\cdot\frac{1}{3}\,du = \frac{1}{9}\int \frac{u+1}{u^2}\,du = \frac{1}{9}\int\left(\frac{1}{u}+\frac{1}{u^2}\right)du = \frac{1}{9}\left[\ln|u| - \frac{1}{u}\right] + C = \boxed{\frac{1}{9}\left[\ln|3x-1| - \frac{1}{3x-1}\right] + C}$$

(31) $\displaystyle\int \frac{1}{\sqrt{t}-1}\,dt$ Let $u = \sqrt{t}$, so $t = u^2$ and $dt = 2u\,du$

$$= \int \frac{1}{u-1}\cdot 2u\,du = 2\int \frac{u}{u-1}\,du = 2\int\left(1 + \frac{1}{u-1}\right)du$$

$$\begin{array}{r} 1 \\ u-1\,\overline{)\,u} \\ \underline{u-1} \\ 1 \end{array}$$

$$= 2\left[u + \ln|u-1|\right] + C = \boxed{2\left[\sqrt{t} + \ln|\sqrt{t}-1|\right] + C}$$

(OR Let $u = \sqrt{t}-1$, $t = (u+1)^2$, $dt = 2(u+1)\,du$ To get $\displaystyle\int \frac{1}{u}\cdot 2(u+1)\,du = 2\int \frac{u+1}{u}\,du$)

(35) $\displaystyle\int \frac{x}{\sqrt{2x+1}}\,dx$ Let $u = \sqrt{2x+1}$, so $x = \frac{1}{2}(u^2-1)$, $dx = \frac{1}{2}(2u)\,du = u\,du$

$$= \int \frac{\frac{1}{2}(u^2-1)}{\cancel{u}}\cdot\cancel{u}\,du = \frac{1}{2}\int (u^2-1)\,du = \frac{1}{2}\left[\frac{u^3}{3} - u\right] + C = \boxed{\frac{1}{6}(2x+1)^{3/2} - \frac{1}{2}(2x+1)^{1/2} + C}$$

OR Let $u = 2x+1$, $x = \frac{1}{2}(u-1)$, $dx = \frac{1}{2}\,du$ To get

$$\int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\cdot\frac{1}{2}\,du = \frac{1}{4}\int \frac{u-1}{\sqrt{u}}\,du = \frac{1}{4}\int (u-1)\,u^{-1/2}\,du = \frac{1}{4}\int \left(u^{1/2} - u^{-1/2}\right)du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right] + C = \boxed{\frac{1}{6}(2x+1)^{3/2} - \frac{1}{2}(2x+1)^{1/2} + C}$$

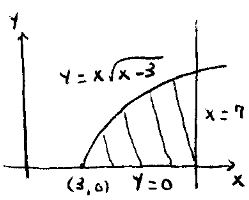
---

(43) $\displaystyle\int_0^4 \frac{x}{(x+4)^2}\,dx$ Let $u = x+4$, $x = u-4$, $dx = du$ If $x=0$, $u=4$; $x=4$, $u=8$

$$= \int_4^8 \frac{u-4}{u^2}\,du = \int_4^8\left(\frac{u}{u^2} - \frac{4}{u^2}\right)du = \int_4^8\left(\frac{1}{u} - 4u^{-2}\right)du = \left[\ln u + 4u^{-1}\right]_4^8 = \left(\ln 8 + \frac{4}{8}\right) - \left(\ln 4 + \frac{4}{4}\right)$$

$$= \ln 8 - \ln 4 + \frac{1}{2} - 1 = \ln\frac{8}{4} - \frac{1}{2} = \boxed{\ln 2 - \frac{1}{2}}$$

(47)



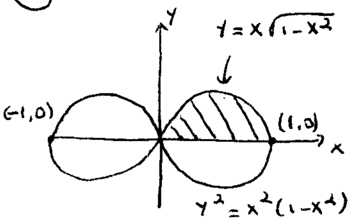
$$A = \int_3^7 x\sqrt{x-3}\,dx$$ Let $u = \sqrt{x-3}$, $x = u^2+3$, $dx = 2u\,du$ If $x=3$, $u=0$; $x=7$, $u=2$

$$= \int_0^2 (u^2+3)\,u\,(2u)\,du = 2\int_0^2 (u^4+3u^2)\,du = 2\left[\frac{u^5}{5} + u^3\right]_0^2$$

$$= 2\left(\frac{32}{5} + 8\right) = 2\left(\frac{72}{5}\right) = \boxed{\frac{144}{5}} = \boxed{28.8}$$

(OR Let $u = x-3$, $x = u+3$, $dx = du$)

(57)



$$A = 4\int_0^1 x\sqrt{1-x^2}\,dx$$ Let $u = 1-x^2$, $du = -2x\,dx$ If $x=0$, $u=1$; $x=1$, $u=0$

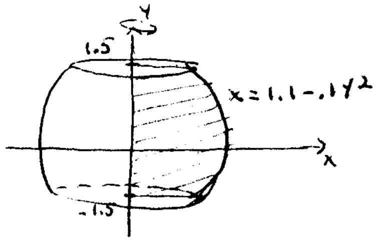
$$= \frac{4}{-2}\int_0^1 \sqrt{1-x^2}\,(-2)\,x\,dx = -2\int_1^0 \sqrt{u}\,du = 2\int_0^1 u^{1/2}\,du$$

$$= 2\left[\frac{2}{3}u^{3/2}\right]_0^1 = \frac{4}{3}(1-0) = \boxed{\frac{4}{3}}$$

MATH 16B

SOLUTIONS - WHITE SHEET (SEC. 5.5 AND 5.7)

(5)



$$V = \int_{-1.5}^{1.5} \pi(1.1 - .1y^2)^2\,dy = 2\int_0^{1.5} \pi(1.1 - .1y^2)^2\,dy$$

$\uparrow$ EVEN FUNCTION

$$= 2\pi \int_0^{1.5} (1.21 - .22y^2 + .01y^4)\,dy$$

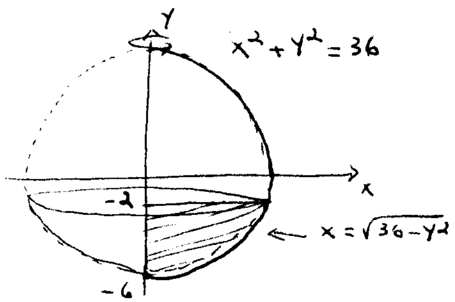
$$= 2\pi\left[1.21y - \frac{.22}{3}y^3 + \frac{.01}{5}y^5\right]_0^{1.5}$$

$$= 2\pi\left(1.21(1.5) - \frac{.22}{3}(1.5)^3 + \frac{.01}{5}(1.5)^5 - 0\right)$$

$$= \pi\left(1.21(3) - \frac{.22}{\cancel{3}}(\cancel{3})(1.5)^2 + (.01)\left(\frac{1.5}{5}\right)(3)(1.5)^3\right)$$

$$= \pi(3.63 - .495 + .030375) = \boxed{(3.165375)\pi \text{ FT}^3} \approx \boxed{9.9 \text{ FT}^3}$$

(6)



$$V = \int_{-6}^{-2} \pi(f(y))^2\,dy = \int_{-6}^{-2} \pi\left(\sqrt{36 - y^2}\right)^2 dy$$

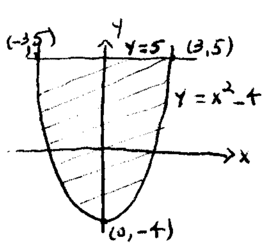
$$= \pi\int_{-6}^{-2} (36 - y^2)\,dy = \pi\int_2^6 (36 - y^2)\,dy$$

$\uparrow$ EVEN FUNCTION

(CAN LET $u = -y$, $du = -dy$)

$$= \pi\left[36y - \frac{y^3}{3}\right]_2^6 = \pi\left(\left(36\cdot 6 - \frac{6\cdot 36}{3}\right) - \left(72 - \frac{8}{3}\right)\right) = \pi\left(144 - 72 + \frac{8}{3}\right) = \boxed{\frac{224\pi}{3} \text{ IN}^3}$$

(7)



$y = x^2 - 4$
$x^2 = y + 4$
$x = \pm\sqrt{y + 4}$

$x^2 - 4 = 5$, $x^2 = 9$, $x = \pm 3$

(EVEN FUNCTION)

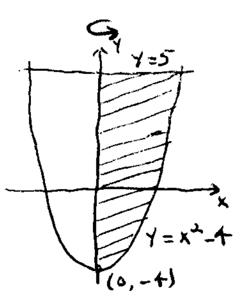
(a) $$A = \int_{-3}^{3} (5 - (x^2 - 4))\,dx = \int_{-3}^{3} (9 - x^2)\,dx = 2\int_0^3 (9 - x^2)\,dx = 2\left[9x - \frac{x^3}{3}\right]_0^3$$

$$= 2(27 - 9) = 2(18) = \boxed{36}$$

OR $$A = \int_{-4}^{5} \left(\sqrt{y+4} - (-\sqrt{y+4})\right)dy = 2\int_{-4}^{5} \sqrt{y+4}\,dy$$

LET $u = y + 4$, $du = dy$. IF $y = -4$, $u = 0$; $y = 5$, $u = 9$

$$= 2\int_0^9 \sqrt{u}\,du = 2\int_0^9 u^{1/2}\,du = 2\left[\frac{2}{3}u^{3/2}\right]_0^9 = \frac{4}{3}(9^{3/2}) = \frac{4}{3}(3^3) = 4\cdot 9 = \boxed{36}$$



(b) $$V = \int_{-4}^{5} \pi(f(y))^2\,dy = \int_{-4}^{5} \pi\left(\sqrt{y+4}\right)^2 dy = \pi\int_{-4}^{5} (y+4)\,dy$$

LET $u = y + 4$, $du = dy$. IF $y = -4$, $u = 0$; $y = 5$, $u = 9$

$$= \pi\int_0^9 u\,du = \pi\left[\frac{u^2}{2}\right]_0^9 = \boxed{\frac{81\pi}{2}}$$

(OR use $\pi\int_{-4}^{5} (y+4)\,dy = \pi\left[\frac{y^2}{2} + 4y\right]_{-4}^{5} = \pi\left(\frac{25}{2} + 20 - (8 - 16)\right)$

$= \pi\left(\frac{25}{2} + 28\right) = \boxed{\frac{81\pi}{2}}$)